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# Current practice in the modelling of Age, Period and Cohort effects with panel data: a commentary on Tawfik et al (2012), Clarke et al (2009), and McCulloch (2012)

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## Abstract

This comment assesses how age, period and cohort (APC) effects are modelled with panel data in the social sciences. It considers variations on a 2-level multilevel model which has been used to show apparent evidence for simultaneous APC effects. We show that such an interpretation is often misleading, and that the formulation and interpretation of these models requires a better understanding of age, period and cohort effects and the exact collinearity present between them. This interpretation must draw on theory to justify the claims that are made. By comparing two papers which over-interpret such a model, and another that in our view interprets it appropriately, we outline best practice for researchers aiming to use panel datasets to find APC effects, with an understanding that it is impossible for any statistical model to find and separate all three effects.

# 1 Introduction

This comment assesses the possibility of measuring age, period, and cohort (APC) effects using panel data. In particular, we consider the use of a multilevel (or random effects) model which uses additive linear effect for, and interactions between, any two of the elements of APC. Whilst we do not object to the use of the model in all situations, a number of recent articles have used such a model that is poorly formulated when compared with the theory that it is supposedly based on; these models are then over-interpreted, or interpreted it in such a way that their conclusions are confused and often unjustified. The model is certainly not able to identify and disentangle APC effects without making the assumption that at least one of the three has no effect, because of the APC 'identification problem' which is often misunderstood and underemphasised, despite being relatively well known. The papers which are considered here are from different disciplines in the social sciences (electoral studies, epidemiology and sociology), and we hope that the methodological issues that are discussed can feed into a variety of research areas where panel data is used and where there is a particular interest in both trends over time *and* trends over the life course of individuals.

The article proceeds as follows. First, we summarize what APC effects mean conceptually and the identification problem inherent to them. Second we outline the specification and characteristics of the statistical models which are the subject of this comment. The main body of the paper then critically assesses three papers that have used such models: two papers which we argue have mis- or over-interpreted their results, and one which has in our view used their model well to find interesting conclusions whilst neither claiming nor attempting to model age, period and cohort together simultaneously. We hope that the paper will act as a warning to anyone hoping to conduct research into APC effects: that a statistical model should reflect the theory that it is based on and the research questions it is supposed to be answering, and that the assumptions of the model must be carefully understood and explicated in light of the APC identification problem.

## 2 Age, Period and Cohort: concepts and identification

Age, period and cohort are conceptually distinct ways in which things can change over time (Suzuki 2012). Age is usually measured as time since birth, period as time since a given calendar date, and cohort as when a given individual was born, again in calendar time. The conceptual difference between the three effects is clear in this fictional dialogue (in Suzuki 2012 p1):

> A: I can't seem to shake off this tired feeling. Guess I'm just getting old. [Age effect]
>
> B: Do you think it's stress? Business is down this year, and you've let your fatigue build up. [Period effect]
>
> A: Maybe. What about you?
>
> B: Actually, I'm exhausted too! My body feels really heavy.
>
> A: You're kidding. You're still young. I could work all day long when I was your age.
>
> B: Oh, really?
>
> A: Yeah, young people these days are quick to whine. We were not like that. [Cohort effect]

The problem is that this clear conceptual distinction is muddied by the statistical dependency of the three effects on each other, which can be expressed simply as

$$Age = Period - Cohort \tag{1}$$

This identification problem has been known for many years (Mason et al. 1973). Including all three terms in a statistical model will lead to exact multicollinearity, making such a model unidentifiable. It is also often impossible to tell what assumptions are unreasonable based on statistical measures, such as model fit. Glenn (1989 p755) makes this point clearly:

> When the effects are linear, several different combinations of the different kinds of effects can produce the same pattern of variation in the data. In other words, different

combinations of age, period, and cohort influences can have identical empirical consequences.

Whilst techniques have been suggested to get around this problem, all rely on certain assumptions which may or may not be tenable and if unreasonable can lead to incorrect interpretations (Osmond and Gardner 1989; Glenn 2005). Any APC model which claims to show all three effects in one model must make those assumptions explicit and justify them with theory. A model that is able to show APC effects accurately, without assumptions and without theoretical forethought, is a logical and mathematical impossibility (Glenn 1976; 2005 p6; Goldstein 1979; Bell and Jones 2013).

It is common, however, to see models in which one of the three APC terms is left out of the model and it is these models that are the subject of this paper. We argue that terms should be omitted on the basis of theory and not just statistical necessity, and interpretation must be conducted with care. If one of age, period or cohort is omitted erroneously (and one must be omitted for the model to be identified), it is likely that the other two terms will absorb that effect in line with equation (1), showing apparent effects that are highly misleading. An awareness of these pitfalls is essential in any analysis that deals with age, period and/or cohort effects and it is when researchers attempt to apply models mechanically and without critical forethought that problems arise.

## 3 The model

As previously stated, the models used by the papers that are the basis of this discussion leave out one of the three APC effects: In addition to the two included additive linear effects, they incorporate into the model an interaction effect between the two terms. Tawfik et al. (2012) leave out age, and so include period, cohort and an interaction between period and cohort. Clarke et al. (2009) leave out cohort, including age, period, and their interaction. Finally McCulloch (2012) leaves out period, and so include in their models age, cohort and an interaction between the two.

All three papers use a multilevel model, with occasions at level 1 nested within individuals at level 2. Each allows for random slopes, allowing each individual to have a different trajectory of age (in the case of McCulloch), or period (in the case of Tawfik et al), or both age and period (in the case of Clarke et al).

Two of the three examples have a binary dependent variable, and therefore use a logit transformation in their model. However for the sake of clarity, below we outline the model specification for a model with a continuous dependent variable $y_{ij}$ and Normally distributed residuals. In this example, period and cohort are included in the model and age is excluded (as in Tawfik et al. (2012)).

Micro model:

$$y_{ij} = \beta_{0j} + \beta_{1j}Period_{ij} + e_{ij}$$

Macro Models:

$$\beta_{0j} = \beta_0 + \beta_2 Cohort_j + u_{0j}$$

$$\beta_{1j} = \beta_1 + \beta_3 Cohort_j + u_{1j}$$

Giving a combined model:

$$y_{ij} = \beta_0 + \beta_1 Period_{ij} + \beta_2 Cohort_j + \beta_3 Period_{ij} * Cohort_j + (u_{0j} + u_{1j}Year_{ij} + e_{ij})$$

(2)

With distributional assumptions:

$$\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} \sim N(0, \begin{bmatrix} \sigma_{u0}^2 & \\ \sigma_{u01} & \sigma_{u1}^2 \end{bmatrix})$$

$$e_{ij} \sim N(0, \sigma_e^2)$$

(3)

Here, $y_{ij}$ is the dependent variable, varying by individual j and occasion i. $\beta_0$ is the intercept of the model, and $\beta_1$, $\beta_2$, and $\beta_3$ are the coefficients associated with period, cohort and the interaction respectively. At the individual level (level 2), there are individual random effects ($u_{0j}$, with a variance of $\sigma^2_{u0}$), as well as slope residuals $u_{1j}$, which have a variance of $\sigma^2_{u1}$ and covary with $u_{0j}$ according to the parameter $\sigma_{u01}$. The occasion level (level 1) residuals $e_{ij}$ are assumed to vary normally with a variance of $\sigma^2_e$.

We argue that the use of such a model is fine, so long as (1) the omitted APC term is left out of the model because of reliable theory rather than just statistical convenience, and (2) the model is interpreted only in terms of the variables present in the model. The following examples show the risks of over-interpretation where either of these two conditions are not satisfied.

# 4 Trends in voter turnout: Tawfik et al 2012

Tawfik et al look at trends in voter turnout, using a large dataset of over 800,000 observations of 16,000 citizens, voting (or not) in 58 ballots over 12 years from 1996 to 2007. Their model is similar to that in equation (2); they do not include an age variable in their model, instead including a period term, a cohort term, and an interaction between the two. The ease with which APC effects can be confused is increased by the terminology that the authors use. They describe APC effects as different types of 'age' effects, whilst the variable used to measure the cohort effect is called 'age' (because it measures age at the time of the first measurement). This is an unfortunate start for a model that doesn't include an age effect (as defined earlier) at all.

The authors state in their abstract that they "find suggestive evidence for three age effects: period, aging, and cohort." This is an over-interpretation of a model which is not specified to be able to reach such conclusions. The period effect found is based on the significance of the coefficient associated with period (equivalent to $\beta_1$ in equation 2). However, this could easily be the result of an age effect, because age is not controlled for in the model and is obviously highly collinear with the period variable as individuals age as time (periods) passes. Moreover, their evidence for an age effect is based on the significance of the period-cohort interaction effect (equivalent to $\beta_3$ above) – despite age not being present they argue that the interaction shows differential aging effects for individuals in different cohorts. Whilst this might be the case, and they do make a theoretical case for both effects, they suggest no reason why they choose to interpret the additive period coefficient as a period effect and the interactive period coefficient as a (differential) aging effect (rather than a period effect as the model formulation would imply). Further, the period interaction effect cannot simply be translated into an age effect – it must be decomposed into an age effect *and a cohort effect* according to the dependency in equation 1. As such, if it is assumed that there is this differential age effect on the basis of the interaction in the model, then it must also be assumed that the cohort effect has been underestimated. Their use of a cohort variable, and interactions between that variable and other covariates, to indicate a cohort effect may be justified, but only based on a theoretical understanding of the social processes involved. Mathematically there is no reason why the cohort effect that they find could not have been partly or wholly the result of a combination of age and period effects.

As such, the researcher's claim that the model suggests the presence of all three of age, period and cohort, is rather misleading. It is based on a model that only measures two effects (period and cohort), and as such they effectively double-count these effects, finding three effects when there is only enough information to discern two. And the decision about which two effects drive the processes at hand can only be made on the basis of theory; it cannot be made on the basis of a statistical model.

## 5 Trends in levels of obesity: Clarke et al 2009

While Tawfik et al do not even acknowledge the existence of the identification problem in attempting to find all three APC effect, Clarke et al show some awareness to the problem and make the clear statement that they “do not focus on cohort effects in this paper” (501). Unfortunately, this clarity is swiftly blurred.

Clarke et al look at trajectories in body mass index (BMI) score, a measure of obesity, on about 2400 individuals measured at multiple occasions between 1986 and 2004. Their model includes an age and a period term (and an interaction between them), and excludes any cohort effect. They find positive coefficients associated with age and period, whilst the interaction term is positive but small. As it is, the model suggests the following: (1) that individuals are generally more overweight in 2004 than in 1986, (2) individuals get more overweight as they age, and (3) that age effect appears to be getting slightly bigger over time.

The problem here is two-fold. First, if there is in fact no period effect, and rather a cohort effect, then the cohort effect will manifest itself as an age and a period effect in line with the dependency expressed in equation 1. So, not only will there appear to be a period effect when there is no such thing, the age effect will also be underestimated to a degree similar to the size of the underlying cohort effect. Second, it is not clear from the paper whether the authors conceive of periods or cohorts causing the significant period effect that they find. Having stated that they are not focusing on cohort effects, figure 2 in the paper (p. 504) appears to do exactly that. It shows the differences in the aging effect of three cohort groups, with the earlier cohorts having a shallower increase in BMI than later cohorts. The authors also argue that “successive cohorts become overweight at increasingly earlier [sic] points in the life course” (p. 499).

To summarise, Clarke et al argue in places that a period effect is behind the change over time, and at other times they claim the change is the result of a cohort effect. This is not a problem necessarily with the model but with the lack of theory behind both the model formulation and model interpretation, and confusion between APC effects that leaves important questions unanswered in the readers’ minds. If the authors have theory that cohorts are the driving force of change over time, then they should include cohorts, rather than periods, in their model. If they believe that the effect is driven by period effects, then the model that they use model is appropriate, but then cohort effects must be assumed to be non-existent. It is not possible to tease out three separate APC effects from a model that only deals with two of them, or indeed any model because of the dependency in equation 1. A choice needs to be made (to exclude one of APC) on the basis of theory, and that choice should be reflected in the terms included in the model and made clear in the conclusions that are drawn from it.

# 6 Trends in levels of volunteering: McCulloch forthcoming

A rather more suitable use of these models is exemplified in McCulloch's forthcoming paper which looks at trends in volunteering between 1991 and 2007. He only considers cohort and age effects (and their interaction) and leaves out period effects from the model. He finds that (1) later cohorts are less likely to be members of voluntary organisations, and (2) that whilst the earliest cohort has no age effect for men (and a small positive age effect for women), later cohorts have an increasingly negative age effect, where individuals volunteer less as they get older.

It is mathematically possible that these apparent cohort and age effects are confounded with a period effect. However, the decision to assume that there is no period effect, and to argue that societal change is being driven by cohort change, is backed up by theory in the paper. He points to past research that claims that union membership has declined with cohort succession, and argues that this is linked to membership to voluntary organisations for various reasons. Moreover, because the model is structured to reflect that theory, the terms in the model can be interpreted as the effects themselves (an age coefficient represents an age effect), and not as confounded combinations of other effects, as they are in the other two papers. Because period effects are able to be dismissed on the basis of theory, there is no attempt to model all of age, period and cohort, and as such the model can come to interesting and robust conclusions.

# 7 Conclusions

This comment argues that researchers modelling panel data must continuously be aware of the mathematical dependency of APC effects. It is impossible to tease apart these effects without assumptions, and models that claim to do so need to make those assumptions explicit. The papers that have been considered here do not use models that attempt to model all three of the APC effects. Yet by over-interpreting these models, two of the papers see age, period and cohort effects that are not necessarily present. Their models are not structured on the basis of theory and as such the interpretations of the models are uncertain and confused. However, McCulloch's paper shows that when the model is properly specified, and the assumptions made are explicitly stated and based on strong theory, models such as these are able to provide a useful interpretation of social processes over time.

Finally, it is worth reiterating that models which attempt to find all three APC effects are subject to similar risks of confounded effects, despite claims to the contrary in the literature. Recent work (Bell and Jones 2013) on one such model (Yang and Land 2006) has shown that the ability of such a model to converge and fit does not mean that the results that are found by it are correct. Different combinations of APC effects can produce identical datasets, and no statistical trick will be able to tell two identical datasets apart. For such models to be mathematically identified, certain assumptions must be made, and the theory that makes those assumptions justifiable is rarely present. We hope that the message is clear: researchers hoping to model age, period and/or cohort effects should proceed with caution.

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