



Patcharamaneepakorn, P., Doufexi, A., & Armour, S. M. D. (2013). Weighted sum capacity maximization using a modified leakage-based transmit filter design. *IEEE Transactions on Vehicular Technology*, 62(3), 1177-1188. 10.1109/TVT.2012.2230202

Link to published version (if available):  
[10.1109/TVT.2012.2230202](https://doi.org/10.1109/TVT.2012.2230202)

[Link to publication record in Explore Bristol Research](#)  
PDF-document

## University of Bristol - Explore Bristol Research

### General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:  
<http://www.bristol.ac.uk/pure/about/ebr-terms.html>

### Take down policy

Explore Bristol Research is a digital archive and the intention is that deposited content should not be removed. However, if you believe that this version of the work breaches copyright law please contact [open-access@bristol.ac.uk](mailto:open-access@bristol.ac.uk) and include the following information in your message:

- Your contact details
- Bibliographic details for the item, including a URL
- An outline of the nature of the complaint

On receipt of your message the Open Access Team will immediately investigate your claim, make an initial judgement of the validity of the claim and, where appropriate, withdraw the item in question from public view.

# Weighted Sum Capacity Maximisation using a Modified Leakage-based Transmit Filter Design

Piya Patcharamaneepakorn, Angela Doufexi, *Member, IEEE*, and Simon Armour

**Abstract**—This paper proposes a linear transmit filter design to maximise Weighted Sum Capacity (WSC) in multiuser Multiple-Input Multiple-Output (MU-MIMO) systems. The proposed scheme is based on a modified signal-to-leakage-plus-noise ratio (SLNR) criterion, which integrates receiver structures and power allocations into the precoder design and can efficiently exploit unused receiver subspaces. Based on the proposed transmitter design with receive matched filters, the WSC maximisation problem can be simplified to power allocation and data stream selection problems. A power allocation algorithm for finding a local optimal solution is also proposed and is shown to be obtained by the iteration of closed-form water-filling solutions. Furthermore, a low-complexity user and substream selection is proposed as an alternative solution to maximise WSC. Simulation results show that the proposed algorithms outperform the conventional scheme and achieve comparable performance to a joint transceiver design, despite requiring simpler receiver structures.

**Index Terms**—Multiuser MIMO, linear precoding, signal-to-leakage-plus-noise ratio (SLNR), iterative water-filling, user selection.

## I. INTRODUCTION

MU-MIMO schemes have attracted considerable interest in recent years due to the capability of multiplexing multiple users' data streams into the same frequency and time resources and offering high system throughput. The theoretical sum capacity of MU-MIMO is known to be achieved by Dirty Paper Coding (DPC) [1]–[3]. In practice, however, the implementations of DPC are generally difficult as they involve high complexity in nonlinear coding/decoding of users' data streams. The search for practical transceiver designs to achieve the capacity limit is, therefore, still ongoing. To this end, linear precoding techniques are often of interest due to their simplicity. In addition, the weighted sum capacity (WSC) is normally adopted as an optimisation criterion since it incorporates users' priority and fairness into consideration [4].

The problem of finding linear transceiver designs to maximise the WSC is known to be non-convex and is generally difficult to solve. Existing works thus aim to obtain a local optimum point. These works may be classified into two major frameworks. The first framework involves the joint design of transmit and receive filters using iterative algorithms.

The principle is to iterate the computation of transmit filters, power allocations and receive filters by assuming that the other components are fixed during each update. In [5], based on the uplink-downlink duality theory, the design of downlink transmit filters is assisted by using a virtual uplink system, in addition to solving associated power allocations by a Geometric Programming (GP) in each iteration. The concept of duality is also exploited in [6] in a multi-carrier context, where the power allocation problem is shown to be a signomial programming (SP) problem. Another approach is proposed in [7] based on the relationship between WSC and weighted minimum mean square error (WMMSE) problems. In this case, transmit filters with power allocation can be obtained via an MSE weights update, whereby GP problems can be avoided. Nevertheless, in the joint transceiver design approach, the computation of receive filters generally requires the knowledge of other users' transmit filters. Although this can be done by an estimation at the receivers or directly feeding forward the decoding matrices to the receivers, it may eventually require additional processing at receivers and/or large control overhead. It should also be noted that user and data substream selections are generally embedded into the joint transceiver design process. Specifically, only users and data substreams with nonzero allocated power can be scheduled for data transmission.

The second framework focuses on low-complexity transmit filter designs by introducing specific criteria on the design algorithms. Zero-Forcing (ZF) [8] and Block-Diagonalization (BD) [9], for instance, impose zero multi-user interference (MUI) constraints when users are equipped with single and multiple antennas, respectively. Despite zero MUI, the stringent requirement of ZF and BD leads to a transmit power boost issue [8], causing poor performance in the low signal-to-noise ratio (SNR) regime. Minimising mean square error (MMSE) [8], [10] is another well-known criterion to improve the shortcomings of ZF by incorporating noise power into the transmit filter design. This idea has been generalised to multi-antenna cases in Regularised Block Diagonalization (RBD) [11] and Generalised MMSE Channel Inversion (GMI) [12] schemes. Maximising the signal-to-leakage-plus-noise ratio (SLNR) [13] is also an attractive criterion, providing an alternative approach to the signal-to-interference-plus-noise ratio (SINR) maximisation problem. Due to the imposed design criterion, the transmitter and receiver structures are generally simplified and are usually obtained by closed-form solutions at the expense of suboptimal performance. Consequently, the above precoding schemes are generally not optimal with respect to the WSC maximisation problems.

This paper aims to apply a low-complexity transmit filter design approach to the WSC maximisation problems. One

Copyright (c) 2012 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending a request to pubs-permissions@ieee.org.

Manuscript received July 13, 2012; revised October 12, 2012; accepted November 16, 2012. The review of this paper was coordinated by Prof. W. Hamouada.

The authors are with the Centre for Communications Research, University of Bristol, Bristol, BS8 1UB, UK (e-mail: {eezpp, A.Doufexi, Simon.Armour}@bristol.ac.uk).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 00.0000/TVT.0000.000000

major issue inhibiting this approach occurs when users have multiple antennas and the number of transmission layers is not fully utilised, i.e. when the number of scheduled data streams is less than the number of receive antennas. In this case, there exists unused receiver subspaces which are not exploited by the transmitter design. The system performance can thus be improved by an efficient use of these receiver subspaces. One simple approach is to utilise only a subset of receive antennas [14], [15] which, however, may lead to an ineffective exploitation of the available receive diversity. Another approach involves an estimation of effective receiver subspaces and uses them to refine the transmit beamforming vectors which are later used to update the effective receiver subspaces in an iterative manner. Examples of this include the Coordinated Tx-Rx BD [9], Iterative nullspace-directed SVD (iNuSVD) [16], and Iterative RBD (IRBD) [11]. These techniques deploy the left singular vectors of the equivalent channel, obtained by the product of the transmit filter and the channel matrix, as an estimation of the receiver filter. Another concern regarding WSC maximisation is how to choose a proper precoding design criterion in order to meet the desired objective.

This paper adopts the SLNR criterion as it exhibits a potential alternative to the SINR maximisation. A modified definition of SLNR is proposed in order to incorporate effective receiver subspaces into the precoding design criteria. The proposed scheme inherits the simple transceiver structures from the conventional SLNR scheme. Consequently, the problem of maximising WSC can be simplified to the power allocation and substream selection problems, which are separately treated in this paper. On one hand, the problem of finding a local optimal power allocation is formulated as a nonlinear optimisation problem and is solved using the Lagrange Multiplier Method [17]. Similar to the joint transceiver approach, the solution of power allocation leads to an implicit user and substream selection as users with zero power are prohibited from data transmission. On the other hand, a suboptimal algorithm for substream selection is proposed based on the assumption of equal power per substream (EPS). It is shown that the proposed algorithms outperform the receive antenna selection scheme and the performance is comparable to the joint transceiver approach, despite requiring simpler receiver structures. In addition, the proposed suboptimal substream selection can serve as a highly reliable initial condition for the proposed power allocation scheme, which can bring further improvement to the system performance.

The paper is organised as follows. Section II describes the general model of MU-MIMO systems. In Section III, the conventional SLNR scheme is briefly reviewed, and the modified definition of SLNR and the proposed iterative precoding scheme are presented. The problem of finding a local optimal power allocation and its convergence properties as well as the proposed suboptimal substream selection are elaborated in Section IV. Section V studies the complexity of the proposed algorithms and their comparisons with existing algorithms. Simulation results are presented in Section VI and, finally, a concise summary is given in Section VII.

*Notation:*  $\text{Tr}(\cdot)$ ,  $(\cdot)^T$ ,  $(\cdot)^H$  denote the trace, transpose and Hermitian operations, respectively.  $\|\cdot\|_F$  represents the Frobe-

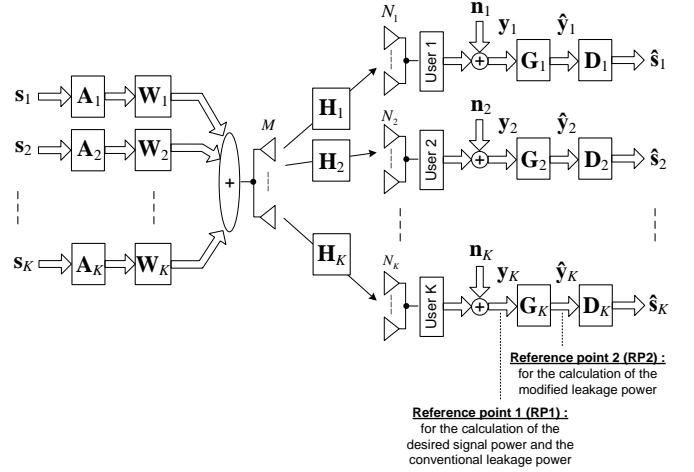


Fig. 1. Block diagram of a downlink MU-MIMO system.

nius norm.  $(\cdot)^+$  denotes the maximum value between the input argument and zero.  $\text{diag}(\cdot)$ ,  $\text{blkdiag}\{\cdot\}$  represent a diagonal matrix and a block diagonal matrix, respectively.  $\mathbf{I}_M$ ,  $\mathbf{0}_{M \times N}$  denote an identity matrix of size  $M \times M$  and a zero matrix of size  $M \times N$ , respectively.

## II. SYSTEM MODEL

Consider a single-cell single-carrier downlink MU-MIMO system with  $M$  transmit antennas at the base station (BS) and  $K$  users, each with  $N_k$  receive antennas as depicted in Fig. 1. The user  $k$ 's channel matrix is denoted as  $\mathbf{H}_k \in \mathbb{C}^{N_k \times M}$ . The transmitted signal at the BS can be given by  $\mathbf{x} = \mathbf{W}\mathbf{A}\mathbf{s} = \sum_k \mathbf{W}_k \mathbf{A}_k \mathbf{s}_k$ . The vector  $\mathbf{s} = [\mathbf{s}_1^T, \mathbf{s}_2^T, \dots, \mathbf{s}_K^T]^T$  denotes the overall data vector, where  $\mathbf{s}_k \in \mathbb{C}^{N_k}$  and  $E\{\mathbf{s}_k \mathbf{s}_k^H\} = \mathbf{I}_{N_k}$ .  $\mathbf{W} = [\mathbf{W}_1, \mathbf{W}_2, \dots, \mathbf{W}_K]$  is the transmit precoding matrix, where  $\mathbf{W}_k \in \mathbb{C}^{M \times N_k}$  and each column is normalised to unit norm.  $\mathbf{A}$  is the power loading matrix defined by  $\mathbf{A} = \text{blkdiag}\{\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_K\}$  with  $\mathbf{A}_k = \text{diag}(\mathbf{a}_k)$  and  $\mathbf{a}_k = (a_{k1}, a_{k2}, \dots, a_{kN_k})^T \in \mathbb{R}^{N_k}$ , such that the total transmission power  $P = \sum P_k = \sum \text{Tr}(\mathbf{A}_k \mathbf{A}_k^H)$ . The additive Gaussian noise vector for each user  $k$ , denoted as  $\mathbf{n}_k$ , has zero mean and covariance matrix  $E\{\mathbf{n}_k \mathbf{n}_k^H\} = \sigma_k^2 \mathbf{I}_{N_k}$ , and is assumed to be statistically independent to data and noise from the other users. The received signal at user  $k$  can be given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{W}_k \mathbf{A}_k \mathbf{s}_k + \mathbf{H}_k \sum_{j \neq k} \mathbf{W}_j \mathbf{A}_j \mathbf{s}_j + \mathbf{n}_k. \quad (1)$$

At user  $k$ , the receive processing can be decomposed as  $\mathbf{R}_k = \mathbf{D}_k \mathbf{G}_k$ , where  $\mathbf{G}_k \in \mathbb{C}^{N_k \times N_k}$  is the receive filter normalised such that each row has unity norm and  $\mathbf{D}_k \in \mathbb{R}^{N_k \times N_k}$  is a diagonal matrix, wherein the diagonal entries represent the norms of the associated rows in  $\mathbf{G}_k$ . Assuming  $\mathbf{G}_k$  and  $\mathbf{D}_k$  are known by the receiver, the received signal at the output of the receive filter,  $\hat{\mathbf{y}}_k$ , and the estimated data sequence,  $\hat{\mathbf{s}}_k$ , can be written as

$$\hat{\mathbf{y}}_k = \mathbf{G}_k \mathbf{y}_k, \quad (2)$$

$$\hat{\mathbf{s}}_k = \mathbf{R}_k \mathbf{y}_k = \mathbf{D}_k \hat{\mathbf{y}}_k. \quad (3)$$

### III. ITERATIVE SLNR PRECODING SCHEME

#### A. The conventional definition of SLNR

In conventional SLNR precoding scheme [13], the SLNR is defined as

$$\text{SLNR}_k = \frac{E\|\mathbf{H}_k \mathbf{W}_k \mathbf{A}_k \mathbf{s}_k\|_F^2}{\sum_{j \neq k} E\|\mathbf{H}_j \mathbf{W}_k \mathbf{A}_k \mathbf{s}_k\|_F^2 + E\|\mathbf{n}_k\|_F^2} \quad (4)$$

$$= \frac{\text{Tr}[\mathbf{H}_k \mathbf{W}_k \mathbf{A}_k \mathbf{A}_k^H \mathbf{W}_k^H \mathbf{H}_k^H]}{\sum_{j \neq k} \text{Tr}[\mathbf{H}_j \mathbf{W}_k \mathbf{A}_k \mathbf{A}_k^H \mathbf{W}_k^H \mathbf{H}_j^H] + \text{Tr}(\sigma_k^2 \mathbf{I}_{N_k})}. \quad (5)$$

Notice that the leakage power in (4)-(5) is calculated at the receive antenna output before the receive filter (RP1 in Fig. 1). By assuming equal power for each data substream (denoted as  $P_s$ ), i.e.  $\mathbf{A}_k = \sqrt{P_s} \mathbf{I}_{N_k}$ , the SLNR in (5) can be rewritten as

$$\text{SLNR}_k = \frac{\text{Tr}[\mathbf{W}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{W}_k]}{\text{Tr}\left[\mathbf{W}_k^H \left(\sum_{j \neq k} \mathbf{H}_j^H \mathbf{H}_j + \frac{\sigma_k^2}{P_s} \mathbf{I}_M\right) \mathbf{W}_k\right]}. \quad (6)$$

As proposed in [13], the objective of precoding design is to maximise the above SLNR metric. This leads to the following optimisation problem:

$$\begin{aligned} \mathbf{W}_k^{opt} &= \arg \max_{\mathbf{W}_k \in \mathbb{C}^{M \times N_k}} \text{SLNR}_k \\ \text{s.t. } &\text{Tr}(\mathbf{W}_k^H \mathbf{W}_k) = N_k \end{aligned} \quad (7)$$

where  $\text{SLNR}_k$  is defined as in (6).

#### B. A Modified Definition of SLNR

As noted earlier, the conventional definition of SLNR considers the leakage power at the receive antenna output (RP1 in Fig. 1). In this case, the precoding algorithm tries to maximise the received signal power of the intended user, while keeping the sum of leakage power received at the receive antennas of the other users as low as possible. However, it is seen that the leakage signals are later steered by the receive beamforming vectors. As a result, parts of the leakage signals may be nulled out by the receive filters. This motivates a new approach to consider the leakage power at the receive filters' outputs (RP2 in Fig. 1), which potentially offers a better approximation of leakage powers. Thus, a modified definition of SLNR can be given by

$$\text{SLNR}_k = \frac{E\|\mathbf{H}_k \mathbf{W}_k \mathbf{A}_k \mathbf{s}_k\|_F^2}{\sum_{j \neq k} E\|\mathbf{G}_j \mathbf{H}_j \mathbf{W}_k \mathbf{A}_k \mathbf{s}_k\|_F^2 + E\|\mathbf{n}_k\|_F^2}. \quad (8)$$

Notice that the above definition only alters the reference point for the calculation of leakage powers (the denominator), while the desired signal power (the nominator) is considered at the original reference point (RP1). By doing so, the precoding and receive matrices can be obtained in the same way as the conventional SLNR scheme. This avoids a precision loss in the computation of the precoding matrix  $\mathbf{W}_k$  as a result of using an estimation of the receive filter  $\mathbf{G}_k$ . Similar to the

conventional scheme, assuming EPS, the SLNR in (8) can be rewritten as

$$\begin{aligned} \text{SLNR}_k &= \frac{\text{Tr}[\mathbf{W}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{W}_k]}{\text{Tr}\left[\mathbf{W}_k^H \left(\sum_{j \neq k} \mathbf{H}_j^H \mathbf{G}_j^H \mathbf{G}_j \mathbf{H}_j + \frac{\sigma_k^2}{P_s} \mathbf{I}_M\right) \mathbf{W}_k\right]} \quad (9) \end{aligned}$$

$$= \frac{P_s \text{Tr}[\mathbf{W}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{W}_k]}{\text{Tr}\left[\mathbf{W}_k^H \left(\sum_{j \neq k} \mathbf{H}_j^H \mathbf{G}_j^H (P_s \mathbf{I}_{N_j}) \mathbf{G}_j \mathbf{H}_j + \sigma_k^2 \mathbf{I}_M\right) \mathbf{W}_k\right]} \quad (10)$$

$$= \frac{P_s \text{Tr}[\mathbf{W}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{W}_k]}{\text{Tr}\left[\mathbf{W}_k^H \left(\sum_{j \neq k} \mathbf{H}_j^H \mathbf{G}_j^H \mathbf{A}_j^H \mathbf{A}_j \mathbf{G}_j \mathbf{H}_j + \sigma_k^2 \mathbf{I}_M\right) \mathbf{W}_k\right]}. \quad (11)$$

Note that (9) resembles the conventional definition of SLNR (6), with a modification of the equivalent leakage channel to user  $j$  defined by  $\mathbf{G}_j \mathbf{H}_j$ . It is also noticed that although the definition in (9) considers the receive beamforming vectors in the computation of leakage power, each leakage stream contributes with equal significance regardless of its power allocation. This has been made explicit in (10) and (11), where  $\mathbf{A}_j = \sqrt{P_s} \mathbf{I}_{N_j}$  according to the assumption of equal power. In general cases, to take into account different priorities of each leakage streams in the precoding design, a modified definition of SLNR is proposed by the inspection of (11) as given by

$$\text{mSLNR}_k = \frac{\text{Tr}[\mathbf{W}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{W}_k]}{\text{Tr}\left[\mathbf{W}_k^H \left(\sum_{j \neq k} \bar{\mathbf{H}}_j^H \bar{\mathbf{H}}_j + \sigma_k^2 \mathbf{I}_M\right) \mathbf{W}_k\right]}. \quad (12)$$

In this case, the equivalent leakage channel (ELC) to a user  $j$  is defined as

$$\bar{\mathbf{H}}_j = \mathbf{\Omega}_j \mathbf{G}_j \mathbf{H}_j \quad (13)$$

where  $\mathbf{\Omega}_j$  is a diagonal matrix, in which each diagonal entry indicates a weighting factor (priority) of each leakage stream associated to the user  $j$ .

It can be seen that the weight matrix  $\mathbf{\Omega}_j$  controls the amount of leakage power to each substream of user  $j$ . The precoding algorithm pays little attention to a substream with a small weighting factor, allowing other substreams to gain benefits by adjusting their beamforming vectors although causing high interference to this substream, while it gives high priority to a substream with a high weight, e.g. other substreams may be sacrificed to guarantee low interference to this substream. Notice that this effect conforms to the water-filling (WF) power allocation strategy for sum-capacity maximisation. Thus, it is seen that, by setting weights equal to allocated powers i.e.  $\mathbf{\Omega}_j = \mathbf{A}_j$ , the proposed scheme can facilitate WF strategies. Hence,  $\mathbf{\Omega}_j = \mathbf{A}_j$  is assumed throughout this paper.

Notice that the modified definition (12) also supports user substream selection, i.e. when some data substreams are allocated zero power (some diagonal entries of  $\mathbf{A}_j$  become zero). In this case, the equivalent leakage channel (13) contains zero row vectors, resulting in zero leakage power in the computation of modified SLNR (12). In other words, unused receiver subspaces due to unallocated data substreams do not contribute to any signal leakage. They can, therefore, be exploited by the precoding algorithm to improve its transmit beamforming vectors.

### C. Iterative Algorithms and Choices of Receive Filter

The precoding designs for the modified SLNR criterion can be obtained by replacing the objective function in (7) with the modified definition (12). Similar to the conventional scheme, the optimal precoding matrix can be given by [13]:

$$\mathbf{W}_k^{opt} = \mathbf{T}_k [\Phi_k; \mathbf{0}_{(M-N_k) \times N_k}] \quad (14)$$

where the columns of  $\mathbf{T}_k \in \mathbb{C}^{M \times M}$  defines the generalised eigenspace of the pair  $\left\{ \mathbf{H}_k^H \mathbf{H}_k, \left( \sum_{j \neq k} \bar{\mathbf{H}}_j^H \bar{\mathbf{H}}_j + \sigma_k^2 \mathbf{I}_M \right) \right\}$  and  $\Phi_k \in \mathbb{R}^{N_k \times N_k}$  is a diagonal matrix, the diagonal entries are nonzero and are chosen to satisfy the power constraint  $\text{Tr}(\mathbf{W}_k^H \mathbf{W}_k) = N_k$  and each column is normalised to unity norm.

Notice that the computation of the precoding matrix  $\mathbf{W}_k$  depends on the equivalent leakage channel  $\bar{\mathbf{H}}_j$  of the other users. This requires *a priori* knowledge of the receive filters, which would technically be known after the precoding matrix is obtained. To this end, iterative algorithms are normally used to cope with this situation as also seen in [9], [11], [16]. Thus, an *iterative SLNR (iSLNR)* scheme based on the modified SLNR definition is proposed as summarised in Algorithm 1. Notice that the definition in (13) is valid for any types of receive filter. In this paper, for simplicity, matched filters (MF) are assumed as in the conventional scheme. Hence, the receive filter for any user  $j$  can be given by

$$\mathbf{G}_j = \Psi_j \mathbf{W}_j^H \mathbf{H}_j^H \quad (15)$$

where  $\Psi_j \in \mathbb{R}^{N_j \times N_j}$  is a diagonal matrix, each diagonal entry is chosen so that each row is normalised to unity norm. Note that a study of the iSLNR scheme with other types of receive filters will be pursued in another work.

---

#### Algorithm 1 Iterative SLNR Precoding Scheme (iSLNR)

---

- 1: **Initialise:** Define a user ordering  $\mathcal{U}$  (e.g. ascending order) and the number of iteration ( $n_{iter}$ ). Set the power loading vectors  $\{\mathbf{a}_k\}$  and initialise ELC, e.g.  $\{\bar{\mathbf{H}}_k\} \leftarrow \{\mathbf{H}_k\}$ .
  - 2: **procedure** iSLNR( $\{\bar{\mathbf{H}}_k\}, \{\mathbf{a}_k\}, \mathcal{U}, n_{iter}$ )
  - 3:   **for**  $i \leftarrow 1, n_{iter}$  **do**
  - 4:     **for**  $j \leftarrow 1, K$  **do**
  - 5:        $k \leftarrow \mathcal{U}(j)$                             $\triangleright$  Get user indices
  - 6:       compute  $\mathbf{W}_k$  using (14)
  - 7:       update  $\bar{\mathbf{H}}_k$  using (13) and (15)
  - 8:     **end for**
  - 9:   **end for**
  - 10: **end procedure**
- 

### D. iSLNR with imperfect channel information

In previous subsections, it is assumed that the full channel state information (CSI) is available at the BS. In practice, however, the CSI is obtained either by reverse channel estimation (e.g. using uplink-downlink reciprocity) in time-division duplexing (TDD) or by quantised feedback in frequency-division duplexing (FDD) systems. This leads to channel estimation errors causing the degradation of the system performance.

Similar to the conventional SLNR scheme [13], the proposed iSLNR scheme can be modified to take into account the channel estimation errors in the presence of imperfect channel information. In this case, the channel matrix of each user  $k$  can be modelled as [18], [19]

$$\mathbf{H}_k = \mathbf{H}'_k + \mathbf{E}_k \quad (16)$$

where  $\mathbf{H}_k$ ,  $\mathbf{H}'_k$ , and  $\mathbf{E}_k$  represent the actual channel matrix, the estimated channel matrix and the estimation error matrix, respectively. Each elements of  $\mathbf{E}_k$  are assumed to be i.i.d. zero-mean complex Gaussian variables with variance  $\sigma_e^2$  and are spatially uncorrelated. In addition, it is assumed that  $\mathbf{H}'_k$  and  $\mathbf{E}_k$  are independent and are uncorrelated to the data and noise vectors. Assuming  $\sigma_e^2$  is known to the BS, the modified SLNR definition (12) can be re-evaluated as

$$\begin{aligned} \text{mSLNR}_k &= \frac{E \left\{ \text{Tr} \left[ \mathbf{W}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{W}_k \right] / \mathbf{H}'_k \right\}}{E \left\{ \text{Tr} \left[ \mathbf{W}_k^H \left( \sum_{j \neq k} \bar{\mathbf{H}}_j^H \bar{\mathbf{H}}_j + \sigma_k^2 \mathbf{I}_M \right) \mathbf{W}_k \right] / \{\mathbf{H}'_j\} \right\}} \quad (17) \end{aligned}$$

where the expectation is conditional on the estimated channel matrices of the user  $k$ ,  $\mathbf{H}'_k$ , and of the other users,  $\{\mathbf{H}'_j\}$ . Assuming that  $\Omega_j$  and  $\mathbf{G}_j$  are known and constant during the evaluation of the modified SLNR values, it can be shown in Appendix A that (17) can be rewritten as

$$\begin{aligned} \text{mSLNR}_k &= \frac{\text{Tr} \left[ \mathbf{W}_k^H \left( \mathbf{H}'_k \mathbf{H}'_k + N_k \sigma_e^2 \mathbf{I}_M \right) \mathbf{W}_k \right]}{\text{Tr} \left[ \mathbf{W}_k^H \left( \sum_{j \neq k} \bar{\mathbf{H}}_j^H \bar{\mathbf{H}}_j + \left( \sum_{j \neq k} \theta_j \sigma_e^2 + \sigma_k^2 \right) \mathbf{I}_M \right) \mathbf{W}_k \right]} \quad (18) \end{aligned}$$

with  $\bar{\mathbf{H}}_j = \Omega_j \mathbf{G}_j \mathbf{H}'_j$  and  $\theta_j = \text{Tr} \left[ \mathbf{G}_j^H \Omega_j^H \Omega_j \mathbf{G}_j \right]$ . It follows that the proposed algorithms in previous sections remain applicable with the objective function in (7) being replaced with (18).

## IV. WEIGHTED SUM CAPACITY MAXIMISATION USING THE iSLNR PRECODING SCHEME

In this section, the proposed iSLNR scheme is applied to WSC maximisation problems, assuming perfect channel estimation.

### A. Power Allocation to Maximise WSC

Given the set of precoding matrices  $\{\mathbf{W}_k\}$  and let  $p_{kb} = a_{kb}^2$ , where  $k = 1, \dots, K$  and  $b = 1, \dots, N_k$ , denote the allocated power to the  $b^{\text{th}}$  stream of the  $k^{\text{th}}$  user, the WSC maximisation problems can be formulated as:

$$\begin{aligned} \arg \max_{\{p_{kb}\}, \forall k, \forall b} & C_{ws}(\{p_{kb}\}) \quad (19) \\ \text{subject to} & \sum_k \sum_b p_{kb} \leq P \\ \text{and} & p_{kb} \geq 0 \end{aligned}$$

where the weighted sum capacity associated to the (real and positive) users' weights  $\alpha_k$  can be written as

$$C_{ws} = \sum_{k=1}^K \alpha_k \log_2 \det(\mathbf{I}_{N_k} + \mathbf{H}_k \mathbf{W}_k \mathbf{P}_k \mathbf{W}_k^H \mathbf{H}_k^H \mathbf{Z}_k^{-1}) \quad (20)$$

with  $\mathbf{Z}_k = \sigma_k^2 \mathbf{I}_{N_k} + \mathbf{H}_k \left( \sum_{j \neq k} \mathbf{W}_j \mathbf{P}_j \mathbf{W}_j^H \right) \mathbf{H}_k^H$  and  $\mathbf{P}_k = \mathbf{A}_k \mathbf{A}_k^H = \text{diag}(a_{k1}^2, \dots, a_{kN_k}^2)$ .

Using the Lagrange Multiplier method [17], it is shown in Appendix B that a local optimal solution of (19) can be obtained as

$$p_{kb} = \frac{\alpha_k}{\nu - \lambda_{kb} + t_{kb}} - \frac{1}{\gamma_{kb}} \quad (21)$$

where

$$\begin{aligned} \gamma_{kb} &= \mathbf{w}_{kb}^H \mathbf{H}_k^H \left( \sigma_k^2 \mathbf{I}_{N_k} + \mathbf{H}_k \tilde{\mathbf{W}}_{kb} \tilde{\mathbf{P}}_{kb} \tilde{\mathbf{W}}_{kb}^H \mathbf{H}_k^H \right)^{-1} \\ &\quad \times \mathbf{H}_k \mathbf{w}_{kb}, \quad (22) \\ t_{kb} &= \sum_{j \neq k} \alpha_j \mathbf{w}_{kb}^H \mathbf{H}_j^H \left( \sigma_j^2 \mathbf{I}_{N_j} + \mathbf{H}_j \mathbf{W}_j \mathbf{P}_j \mathbf{W}_j^H \mathbf{H}_j^H \right)^{-1} \\ &\quad \times \left( \mathbf{H}_j \mathbf{W}_j \mathbf{P}_j \mathbf{W}_j^H \mathbf{H}_j^H \right) \\ &\quad \times \left( \sigma_j^2 \mathbf{I}_{N_j} + \mathbf{H}_j \tilde{\mathbf{W}}_j \tilde{\mathbf{P}}_j \tilde{\mathbf{W}}_j^H \mathbf{H}_j^H \right)^{-1} \mathbf{H}_j \mathbf{w}_{kb} \quad (23) \end{aligned}$$

with the *Lagrange multipliers*  $\nu \geq 0$  and  $\lambda_{kb} \geq 0$ ;  $\forall k, \forall b$  satisfying the complementary slackness conditions:  $\nu (\sum_k \sum_b p_{kb} - P) = 0$  and  $\lambda_{kb} p_{kb} = 0$ , respectively.  $\tilde{\mathbf{W}}_{kb}$  and  $\tilde{\mathbf{W}}_j$  denote submatrices of  $\mathbf{W}$  obtained by removing columns and/or rows associated to the  $b^{\text{th}}$  substream of user  $k$  and all substreams of user  $j$  from  $\mathbf{W}$ , respectively.  $\tilde{\mathbf{P}}_{kb}$  and  $\tilde{\mathbf{P}}_j$  are also defined in a similar way and  $\mathbf{P} = \text{blkdiag}\{\mathbf{P}_1, \dots, \mathbf{P}_K\}$ . Notice that (22) and (23) are quadratic forms associated to a positive definite matrix and a positive semi-definite matrix, respectively, thus  $\gamma_{kb} > 0$  and  $t_{kb} \geq 0$ .

From (21), it can be seen that an optimal power allocation of a data stream  $p_{kb}$  depends on power allocations of the others through the terms  $\gamma_{kb}$  and  $t_{kb}$ . Thus, solving (21)-(23) in general is a complicated task. However, similar techniques to those in [20] can be applied by iteratively solving the above conditions. In this case,  $\gamma_{kb}$  and  $t_{kb}$  are assumed to be fixed in each iteration and (21) can be recognised as a modified water-filling problem. By solving the complementary slackness conditions, the solution to (21) can be written as

$$p_{kb} = \left( \frac{\alpha_k}{\nu + t_{kb}} - \frac{1}{\gamma_{kb}} \right)^+ \quad (24)$$

where the value of  $\nu$  is determined from the power constraint:

$$\sum_{k=1}^K \sum_{b=1}^{N_k} \left( \frac{\alpha_k}{\nu + t_{kb}} - \frac{1}{\gamma_{kb}} \right)^+ = P. \quad (25)$$

As  $\alpha_k$ ,  $\gamma_{kb}$  and  $t_{kb}$  are nonnegative, (25) is a monotonic function of  $\nu$  and can be efficiently solved by a one-dimensional search (e.g. bisection). Note that when no positive real number satisfies (25), indicating that  $\sum_k \sum_b p_{kb} \leq P$ ,  $\nu$  becomes zero in conformity with the complementary slackness. Once the allocated power for all users' data streams

are obtained, the values of  $\gamma_{kb}$  and  $t_{kb}$  can be updated. This procedure can be re-iterated until predefined convergence criteria are satisfied. Notice that the above iterative procedure can be considered as a *generalisation of the modified iterative water-filling (GIWF)* in [20] from the case of single-antenna receivers to multi-antenna receivers.

It is worth noting that (21)-(23) can be computed for any set of precoding matrices  $\{\mathbf{W}_j\}$  with multiple data streams regardless of precoding design criteria. In the case of BD, for instance, it can be shown that  $t_{kb}$  becomes zero as  $\mathbf{H}_j \mathbf{w}_{kb} = 0$  due to the zero-forcing constraint.  $\gamma_{kb}$  can also be simplified to  $\|\mathbf{H}_k \mathbf{w}_{kb}\|^2 / \sigma_k^2$ , which is independent to power allocation of the other streams. Then, the above algorithm reduces to the conventional water-filling [9], [21] as a result.

This paper focuses on the power allocation strategy for the modified SLNR precoding (12). Notice that this scheme requires the knowledge of power allocation (due to the assumption of  $\mathbf{\Omega}_j = \mathbf{A}_j$ ) in the calculation of the precoding matrix (14). Thus, the precoding design and power allocation can be integrated into the same process as proposed in Algorithm 2. Since the WSC maximisation problem is non-convex, the performance of the algorithm largely depends on the initial condition. This paper assumes EPS as the initial power allocation, which performs generally well as can be seen in Section VI-C. In addition, a better initial condition can be obtained by a user and substream selection algorithm, potentially with low complexity implementation. This enables a systematic approach to finding a reliable initial condition, which is one of the main advantages compared to a joint transceiver design scheme.

---

#### Algorithm 2 iSLNR with GIWF (version 1)

---

- 1: **Initialise:** Assume EPS and compute  $\mathbf{W}_k^{(0)}$ ,  $\forall k$  based on the conventional SLNR scheme.  
 $i \leftarrow 0$
  - 2: **repeat**
  - 3:      $i \leftarrow i + 1$
  - 4:      $\forall k, \forall b$  : compute  $p_{kb}^{(i)}$  using (24)
  - 5:      $\forall k$  : update  $\tilde{\mathbf{H}}_k^{(i)}$  using (13) and (15)
  - 6:      $\forall k$  : update  $\mathbf{W}_k^{(i)}$  using (14)
  - 7: **until**
  - 8:     (1) Max. iterations exceed **OR**
  - 9:     (2)  $|C_{ws}^{(i)} - C_{ws}^{(i-1)}| \leq \epsilon_1$
- 

#### B. Discussion on Convergence Property of iSLNR GIWF

As also mentioned in [20], the convergence property of iterative water-filling is rather hard to establish with full generality. However, simulation results suggest that Algorithm 2 usually converges when  $\sum N_k \leq M$ . For  $\sum N_k > M$ , oscillation between different power allocation states has been observed occasionally. Although a mathematical proof is not given in this paper, it can be shown by simulation (Section VI-C) that the oscillation issue can be solved by introducing an update step size (confidence weight)  $\eta$ , where  $0 < \eta < 1$ , to slow down the update of power allocation as given in Algorithm 3 (line 8). A normal update (line 6) may be performed in the first  $T$  iterations before the weighting takes

effect, to avoid the slow convergence. Notice that Algorithm 3 reduces to Algorithm 2 when  $\eta$  is set to 1. In addition, an extra exit condition may be added to detect the oscillation. In this case, the value of the objective function swings around a certain average value. By comparing the average WSC of the current iteration, i.e.  $C_{av}^{(i)} = \frac{1}{W} \sum_{n=0}^{W-1} C_{ws}^{(i-n)}$  ( $W$  denotes an Averaging Window Size), with the previous iteration, the algorithm can be terminated if a sufficiently small change is observed. The condition  $C_{ws}^{(i)} \geq C_{ws}^{(i-1)}$  is also checked to ensure that the algorithm exits when it swings to the better solution.

---

**Algorithm 3** iSLNR with GIWF (version 2)

---

```

1: Initialise: Assume EPS and compute  $\mathbf{W}_k^{(0)}$ ,  $\forall k$  based on
   the conventional SLNR scheme.
    $i \leftarrow 0$ 
2: repeat
3:    $i \leftarrow i + 1$ 
4:    $\forall k, \forall b$ : compute  $p_{kb}$  using (24)
5:   if  $i \leq T$  then ▷ e.g.  $T = 5$ 
6:      $\forall k, \forall b$ : update  $p_{kb}^{(i)} \leftarrow p_{kb}$ 
7:   else
8:      $\forall k, \forall b$ : update  $p_{kb}^{(i)} \leftarrow \eta \cdot p_{kb} + (1 - \eta) \cdot p_{kb}^{(i-1)}$ 
9:   end if
10:   $\forall k$ : update  $\bar{\mathbf{H}}_k^{(i)}$  using (13) and (15)
11:   $\forall k$ : update  $\mathbf{W}_k^{(i)}$  using (14)
12: until
13:  (1) Max. iterations exceed OR
14:  (2)  $|C_{ws}^{(i)} - C_{ws}^{(i-1)}| \leq \epsilon_1$  OR
15:  (3)  $|C_{av}^{(i)} - C_{av}^{(i-1)}| \leq \epsilon_2$  and  $C_{ws}^{(i)} \geq C_{ws}^{(i-1)}$ 

```

---

### C. Suboptimal Transmission Rank Selection Algorithm

This subsection provides an alternative method for solving the WSC problem. Unlike the power allocation strategy where the user and substream selection is obtained implicitly, an explicit data substream selection is proposed under EPS assumption. The algorithm is based on a sequential search algorithm, similar to [22], [15]. The pseudo code of the proposed algorithm is summarised in Algorithm 4. The principle is to choose substreams in order, starting from a substream with the maximum weighted capacity. Then, a substream providing the best WSC with the previously selected ones is added until no further improvement is attained. The outer *for* loop involves the successive addition of substreams, while the inner *for* loop is concerned with the search for the best candidate substream. Line 7 in the inner loop represents the iSLNR precoding design under the EPS assumption, as described in Section III-C. Notice that the equivalent leakage channels are updated at the end of each outer loop, providing a good initial condition for subsequent inner loops. The sufficient number for inner-loop iterations  $n_{iter}$  tends to be reduced as a result. In addition, unlike [22], [15], the performance of the proposed algorithm depends on the user ordering, especially when  $n_{iter}$  is small. It is also proposed to update the precoding matrices in reverse selection order, which appears to further lower the required number of inner-loop iterations as observed from experimental simulations.

Among substreams for a particular user, the dominant eigenmode is known to have highest effective channel gain [23]. To reduce complexity of the algorithm, only the largest eigenmode from each user may be included in the initial candidate list ( $\mathcal{D}$ ). Only when the dominant eigenmode of a user is selected can the next strongest substream of this user participate in the candidate selection. This reduces the number of candidate search to the order of  $K$ , compared to the order of  $N = \sum_k N_k$  for the algorithm in [15], as discussed in Section V. Furthermore, the proposed algorithm can be viewed as a *Transmission Rank Selection (TRS)*, whereby the output of the algorithm indicates the number of eigenmodes (in descending order) selected for each user.

---

**Algorithm 4** Transmission Rank Selection (TRS)

---

```

1: Initialise:
    $\mathcal{S} \leftarrow \emptyset$ ,  $C_{max} \leftarrow 0$  ▷ Initialise the selection set
    $\mathcal{D} \leftarrow$  the set of indices associated to dominant eigenmodes
    $B \leftarrow \min(M, \sum N_k)$  ▷ Number of Spatial Layers
    $\{\bar{\mathbf{H}}_k\} \leftarrow \{\mathbf{H}_k\}$  ▷ Initialise ELC
2: for  $i \leftarrow 1, B$  do
3:   for all  $d \in \mathcal{D}$  do
4:      $\mathcal{U} \leftarrow \text{Reverse}([\mathcal{S}, d])$  ▷ Sort users in reverse order
5:     set  $\{\mathbf{a}_k\}$  ▷ Assuming EPS for active substreams
6:      $\{\bar{\mathbf{H}}_k^{(d)}\} \leftarrow \{\bar{\mathbf{H}}_k\}$ 
7:      $[\{\mathbf{W}_k^{(d)}\}, \{\bar{\mathbf{H}}_k^{(d)}\}]$  ▷ See Algorithm 1
        $\leftarrow \text{iSLNR}(\{\bar{\mathbf{H}}_k^{(d)}\}, \{\mathbf{a}_k\}, \mathcal{U}, n_{iter})$ 
8:     compute  $C_{ws}^{(d)}$ 
9:   end for
10:   $\bar{d} \leftarrow \arg \max C_{ws}^{(d)}$  ▷ Choose the best substream
11:  if  $C_{ws}^{(\bar{d})} > C_{max}$  then
12:     $C_{max} \leftarrow C_{ws}^{(\bar{d})}$ 
13:     $\{\mathbf{W}_k\} \leftarrow \{\mathbf{W}_k^{(\bar{d})}\}$ ,  $\{\bar{\mathbf{H}}_k\} \leftarrow \{\bar{\mathbf{H}}_k^{(\bar{d})}\}$ 
14:     $\mathcal{S} \leftarrow [\mathcal{S}, \bar{d}]$ 
15:     $\mathcal{D} \leftarrow$  replacing  $\bar{d}$  with the subsequent eigenmode
16:  else
17:    break
18:  end if
19: end for

```

---

## V. COMPLEXITY ANALYSIS

In this section, the computational complexity of the proposed algorithms is approximated in terms of the number of floating point operations (flops) [24]. It is, in general, rather tedious and complicated to calculate the exact number of operations for various algorithms. Hence, for comparison purposes and simplicity, the complexity is estimated for the case of real matrices. Although this may not lead the exact computational complexity, it suffices to illustrate the degree of complexity of each algorithm. In this case, the complexity of typical matrix operations can be assumed as follows [24], [25]:

- Multiplication of an  $m \times n$  matrix and an  $n \times p$  matrix:  $2mnp$ .
- Inversion of an  $m \times m$  matrix:  $m^3$ .

TABLE I  
AN APPROXIMATION OF COMPUTATIONAL COMPLEXITY OF THE PROPOSED AND EXISTING ALGORITHMS.

Algorithm	iSLNR	iSLNR GIWF	iSLNR TRS
Complexity	$\bar{K}n_{iter}(2nM^2 + 2BM^2 + 14M^3)$	$K^2(4nM^2) + (L-1)Kn(2BM^2 + 2nM^2 + 4n^2M + n^3)$	$BKn_{iter}(\frac{2}{3}B^2M^2 + nBM^2 + 7BM^3)$
Algorithm	Conventional SLNR [13]	WSRBF-WMMSE [7]	URAS1 [15]
Complexity	$\bar{K}(2n\bar{K}M^2 + 14M^3)$	$LK^2(2n^2M + 2n^3)$	$BKn(\frac{2}{3}B^2M^2 + nBM^2 + 7BM^3)$

- Generalised eigenvalue decomposition (GED) of a pair of an  $m \times m$  symmetric matrix and an  $m \times m$  symmetric positive definite matrix:  $14m^3$ .

Consider the case of equal number of receive antennas per user, denoted as  $n$ , with a high number of users, i.e. the total number of receive antennas  $N = \sum_k N_k = nK > M$ . Let  $\bar{K} \leq K$  denote the number of users selected for data transmission, which is assumed to be in the order of the number of available spatial layers,  $B = \min(M, N)$ . Based on these assumptions, the computational complexity of the proposed algorithms can be estimated in the following subsections.

#### A. Estimated complexity of iSLNR (Algorithm 1)

For the conventional SLNR scheme, the computation of each user's precoding matrix involves matrix multiplications (for  $\mathbf{H}_k^H \mathbf{H}_k$  and  $\sum_{j \neq k} \mathbf{H}_j^H \mathbf{H}_j$ ; the matrix addition with  $\sigma_k^2 \mathbf{I}_M$  is omitted as low complexity order) and one GED, which requires the complexity of  $\mathcal{O}(2n\bar{K}M^2)$  and  $\mathcal{O}(14M^3)$ , respectively. For iSLNR, on one hand, the complexity is slightly increased in the computation of equivalent leakage channel (ELC) (13), e.g. in the order of  $\mathcal{O}(2nM^2) < \mathcal{O}(2n\bar{K}M^2)$ . On the other hand, ELC may contain several zero row vectors, reducing the complexity of the matrix multiplications to  $\mathcal{O}(2nM^2 + 2BM^2)$ . Thus, it may be concluded that each iteration of iSLNR requires approximately a comparable complexity order to that of the conventional SLNR scheme. This number, however, increases with the number of iteration of iSLNR,  $n_{iter}$ , as given in Table I.

#### B. Estimated complexity of iSLNR GIWF (Algorithm 2, 3)

Compared to Algorithm 1, Algorithm 2 involves additional tasks in the computation of  $\gamma_{kb}$  and  $t_{kb}$ , and a bisection for water-filling. It can be seen that the complexity of the former dominates the latter as it involves several matrix multiplications and inversions. Thus, the additional complexity order can be estimated from the complexity of the calculation of  $\gamma_{kb}$  and  $t_{kb}$ . At the initial stage, all substreams are assigned non-zero powers, resulting in the estimated overall complexity of  $\mathcal{O}(K^2(4nM^2))$ . In subsequent iterations, however, the complexity reduces to approximately  $\mathcal{O}(Kn(2BM^2 + 2nM^2 + 4n^2M + n^3))$  as several substreams are given zero powers. Thus, denoting  $L$  as the number loops required for the algorithm to converge, the complexity of Algorithm 2 can be estimated as given in Table I. Note that Algorithm 3 differs from Algorithm 2 only in the power-updating step, thus it has approximately the same complexity order as Algorithm 2.

#### C. Estimated complexity of iSLNR TRS (Algorithm 4)

In Algorithm 4, the outer loop involves a successive increment of substreams, which requires at most  $B$  iterations. The inner loop involves the precoding designs and the searches for the best eigenmode from the candidate list of size  $K$ , therefore requiring approximately  $Kn_{iter}$  iterations (including  $n_{iter}$  iterations for iSLNR precoding loops). Thus, the overall complexity is proportional to  $BKn_{iter}$  as estimated in Table I.

#### D. Comparison with existing algorithms

The estimated complexity of an antenna selection method (URAS1) [15] and a joint transceiver design based on WMMSE (WSRBF-WMMSE) [7] are also given in Table I for comparison purposes. URAS1 requires to search over the entire unselected antennas during the inner loop, resulting in the overall complexity proportional to  $BN = BKn$ , compared to  $BKn_{iter}$  for the case of iSLNR TRS. Therefore, the advantage of iSLNR TRS over URAS1 depends on the ratio  $n/n_{iter}$ . Thus, assuming a fixed  $n_{iter}$ , the complexity of iSLNR TRS can be significantly reduced compared to URAS1 if users are equipped with a large number of receive antennas,  $n$ , as a result of the reduced candidate search from the entire antenna list in URAS1 to the dominant eigenmode list in iSLNR TRS.

The complexity of iSLNR GIWF in the first iteration (the term  $K^2(4nM^2)$ ) increases quadratically with the number of user  $K$ , similar to the case of WSRBF-WMMSE (i.e.  $K^2(2n^2M + 2n^3)$ ). However, due to zero allocated-power in most substreams, the complexity of iSLNR GIWF greatly reduces to a linear growth, i.e.  $Kn(2BM^2 + 2nM^2 + 4n^2M + n^3)$ , in subsequent loops. Furthermore, it is seen that the quadratic complexity order of iSLNR GIWF can be improved if a good initial power allocation is given, e.g. obtained by iSLNR TRS, for which the complexity linearly increases with  $K$ . In this case, the overall complexity of iSLNR TRS+GIWF can be approximated as a linear function of  $K$ .

## VI. SIMULATION RESULTS

The performance of the proposed algorithms is evaluated in this section. For all simulations, spatially uncorrelated MIMO channels generated as i.i.d. Gaussian random variables  $\mathcal{CN}(0, 1)$  and equal noise variance for all receivers ( $\sigma_k^2 = \sigma^2, \forall k$ ) are assumed. The SNR is defined as  $P/\sigma^2$ . The thresholds  $\epsilon_1, \epsilon_2$  and the maximum number of iterations are set to 0.001, 0.0001 and 100, respectively, in all simulations.



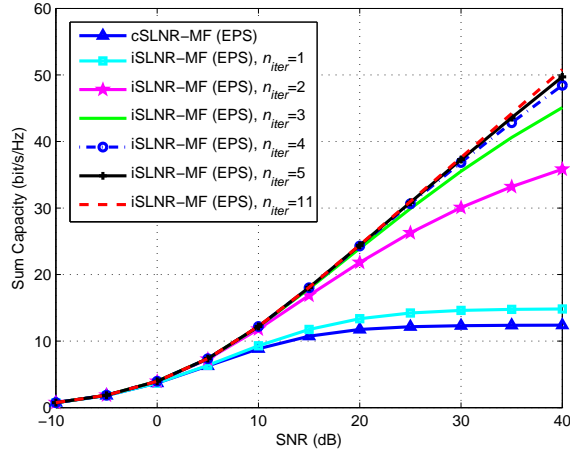


Fig. 2. Ergodic sum capacity for Dominant Eigenmode Transmission, with  $M = 4$ ;  $N_k = 2$ ,  $\forall k$ ;  $K = 4$ .

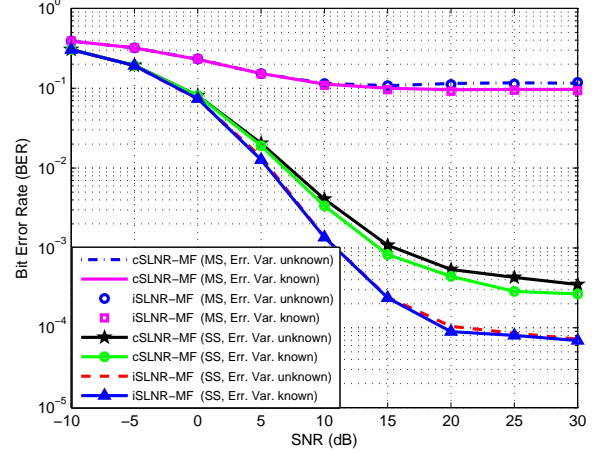


Fig. 4. BER performance of iSLNR-MF ( $n_{iter} = 5$ ) compared to cSLNR-MF with imperfect channel estimation,  $\sigma_e^2 = 0.1$ , and  $M = 4$ ;  $N_k = 2$ ,  $\forall k$ ;  $K = 2$ .

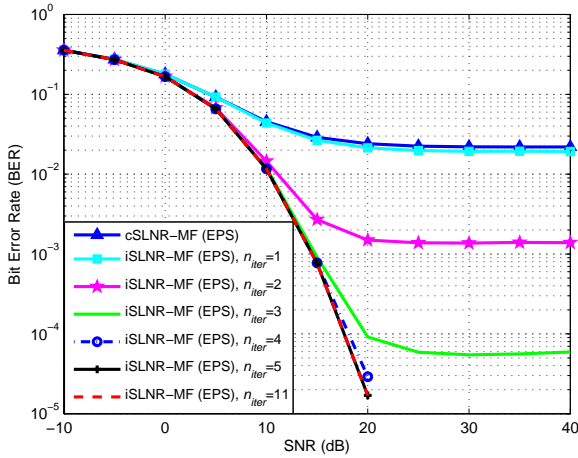


Fig. 3. Average uncoded BER (QPSK modulation) for Dominant Eigenmode Transmission, with  $M = 4$ ;  $N_k = 2$ ,  $\forall k$ ;  $K = 4$ .

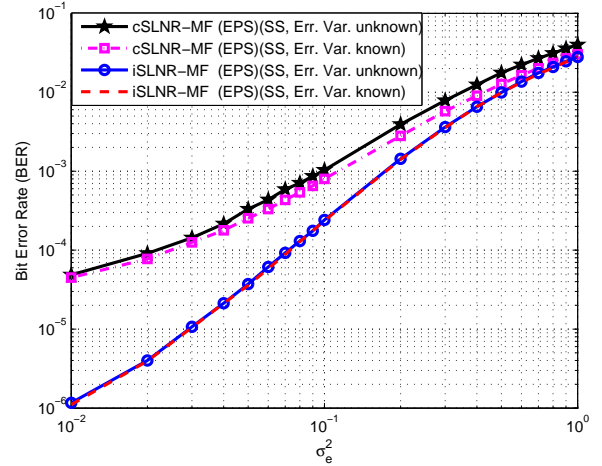


Fig. 5. BER performance of iSLNR-MF vs. estimation error variance at SNR = 15 dB, with  $M = 4$ ;  $N_k = 2$ ,  $\forall k$ ;  $K = 2$ .

### A. iSLNR with EPS

This subsection examines the proposed iSLNR precoding scheme with matched filters (iSLNR-MF), assuming the Dominant Eigenmode Transmission (only the strongest eigenmode for each user is transmitted) and equal power allocation for each active substream. Compared to the conventional SLNR scheme (cSLNR-MF), the iSLNR-MF can efficiently exploit the receiver subspaces by taking into account the receiver structures in the transmit filter design. This leads to the improvement in the ergodic sum capacity and the average uncoded BER as depicted in Fig. 2 and Fig. 3, respectively. Note that the performance of iSLNR-MF improves as the number of iterations ( $n_{iter}$ ) increases due to the higher accuracy of the receiver subspaces estimation. Nevertheless, no significant gain can be obtained after a sufficient high number of iterations. It is also observed that an optimal number of iterations seems to increase with the operating SNR. For practical purposes, 3-5 iterations are recommended as suggested by the simulation results.

### B. iSLNR with EPS and imperfect channel estimation

The performance of iSLNR-MF EPS with imperfect channel estimation is given in Fig. 4. Compared to the case of perfect CSI, error floors can be observed in the BER performance at high SNR due to CSI estimation errors. It can also be seen that iSLNR-MF leads to identical performance as cSLNR-MF in the multi-stream (MS) case (no unused receiver subspaces as a result of full-eigenmode transmission). In contrast, the advantage of iSLNR-MF over cSLNR-MF can again be observed in the single-stream (SS) case. In addition, it is noticed that the improvement of iSLNR-MF is mainly attributed to the exploitation of unused receiver subspaces; the knowledge of estimation error variance marginally contributes to the overall improvement. Compared to cSLNR-MF, the statistical knowledge of estimation errors (i.e. error variance) at the transmitter has less influence to the performance of iSLNR-MF. Therefore, iSLNR-MF tends to be more robust to the channel estimation errors than the cSLNR-MF scheme.

The performance with various estimation errors is also plotted in Fig. 5. The performance gap between iSLNR-MF

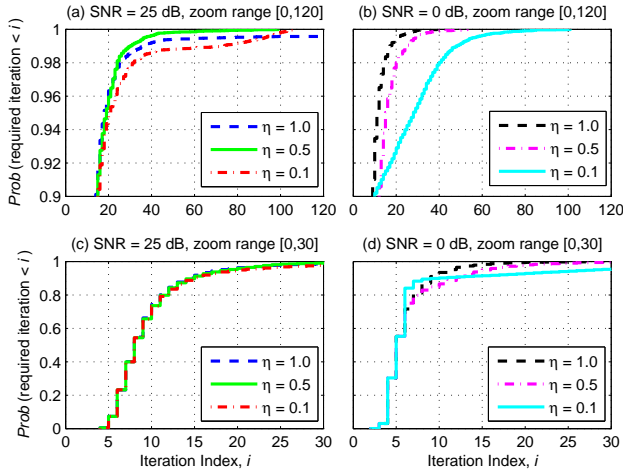


Fig. 6. Cumulative distribution function of iSLNR-MF GIWF for 10,000 channel realisations with  $M = 4$ ,  $N_k = 2\forall k$ ,  $K = 5$  and (a) SNR = 25 dB, zoom range [0,120] (b) SNR = 0 dB, zoom range [0,120] (c) SNR = 25 dB, zoom range [0,30] (d) SNR = 0 dB, zoom range [0,30].

and cSLNR-MF can be clearly seen for low error variance, while it becomes smaller as error variance increases. For very high error variance (e.g. about the same order as channel gain), no significant gain of iSLNR-MF can be observed as no information about unused receiver subspaces can be reliably extracted from the available channel knowledge.

### C. iSLNR with GIWF

This subsection evaluates the performance of iSLNR with the proposed power allocation algorithm. Firstly, the convergence property for the case of  $\sum N_k > M$  is presented in Fig. 6(a)-(b). At high SNR, it is seen that iSLNR-MF GIWF1 ( $\eta = 1$ ) usually converges in most cases (more than 99.5%), while iSLNR-MF GIWF2 ( $\eta = 0.5, 0.1$ ) converges in all cases. In addition, both algorithms always converge at low SNR, with slow convergence for  $\eta = 0.1$  due to small updating steps. This suggests that choosing  $0.5 \leq \eta \leq 0.9$  tends to be a reasonable setting. It is also noticed in Fig. 6(c)-(d) that the proposed algorithms converge within 10 iterations in most cases, with a slightly faster rate at low SNR than that at high SNR. This demonstrates a good convergence property of the proposed algorithms. For high SNR, a typical converged case can be presented by *Channel Realisation 1* in Fig. 7, whereby the algorithms converge after a few iterations. In this case, iSLNR-MF GIWF1 and iSLNR-MF GIWF2 have almost identical performance. In contrast, an oscillation may occasionally occur for iSLNR-MF GIWF1 as can be represented by *Channel Realisation 2*. Clearly, iSLNR-MF GIWF2 can avoid the oscillation issue by incorporating the update step size  $\eta$ , which is set to 0.5 in this simulation. Consequently, iSLNR-MF GIWF2 is assumed in subsequent simulations.

Secondly, the sum capacity is evaluated for the case of  $\sum N_k \leq M$  in Fig. 8. In this case, iSLNR-MF GIWF serves as a typical power allocation strategy as no user selection is required. An improvement over cSLNR-MF can clearly be seen at low SNR, where the algorithm tends to allocate fewer substreams than the available spatial layers as also shown

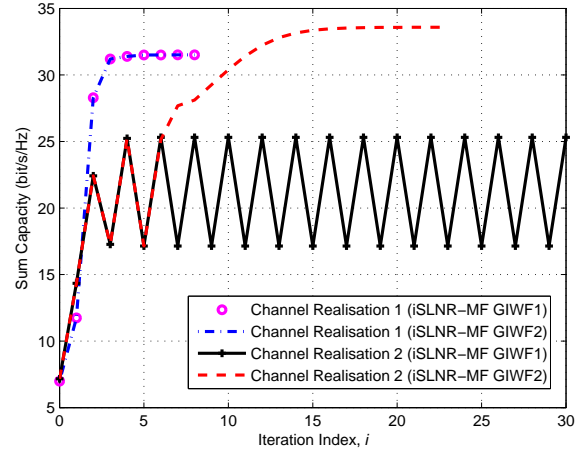


Fig. 7. Examples of convergence properties of iSLNR-MF GIWF (two cases of channel realisations) at SNR = 25 dB, with  $M = 4$ ;  $N_k = 2, \forall k$ ;  $K = 5$ .

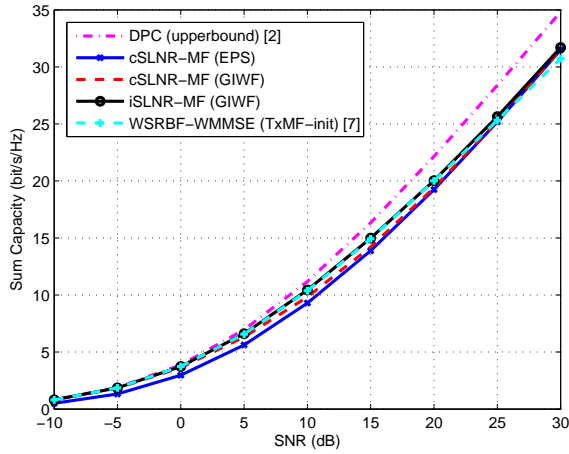
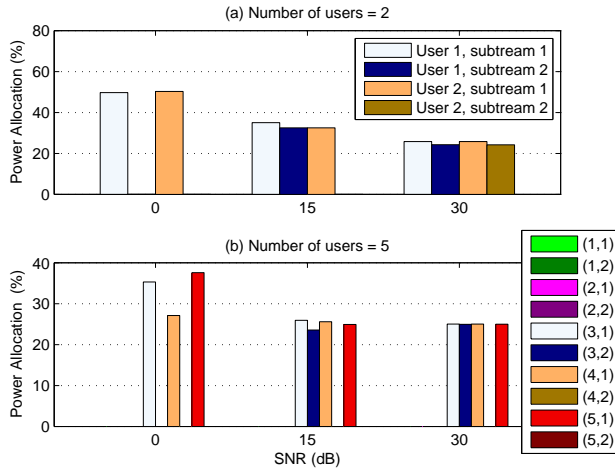
in Fig. 9(a). This allows more power for each substream to overcome noise. Notice that the selected substreams are often the dominant eigenmodes as high channel gain can be expected, and the power is almost equally distributed among the selected substreams.

For  $\sum N_k > M$ , iSLNR-MF GIWF exhibits both user and substream selection as depicted in Fig. 9(b). The pair  $(k, b)$  denotes the  $b^{\text{th}}$  substream of the  $k^{\text{th}}$  user and substreams are sorted from the largest to the smallest eigenvalues. Notice that the number of selected data streams respects the number of available spatial layers (i.e. four, in this simulation) and it tends to be fewer at low SNR as discussed earlier. The ergodic sum capacity is also given in Fig. 10. It is seen that iSLNR-MF GIWF attains significant gain compared with the conventional scheme (cSLNR-MF GIWF) as the modified scheme efficiently considers the receiver subspaces and power allocations in the precoding design. In addition, cSLNR-MF GIWF seems to suffer from high fluctuations during iteration process, causing poor selection outcomes especially at high SNR.

### D. iSLNR-MF with TRS

As noted in Section IV-C, the performance of iSLNR-MF with TRS depends on the number of inner-loop iterations  $n_{iter}$ , as can be seen from Fig. 10. However, a sufficient number of iterations can be fewer than that of required by iSLNR-MF EPS due to the update of equivalent leakage channels at the end of each outer loop. In Fig. 10, the majority of potential capacity gain can be achieved with only 2 iterations, compared to approximately 4 iterations required by iSLNR-MF EPS. In addition, the algorithm only needs to search over at most  $B$  (the number of available spatial layers) outer loops. This strongly suggests that iSLNR-MF TRS yields an efficient low complexity algorithm. Furthermore, iSLNR-MF TRS can be complemented by GIWF (power allocation) which can bring further improvement, although only slight gain can be observed as almost equally distributed power can be expected at a local optimal point.

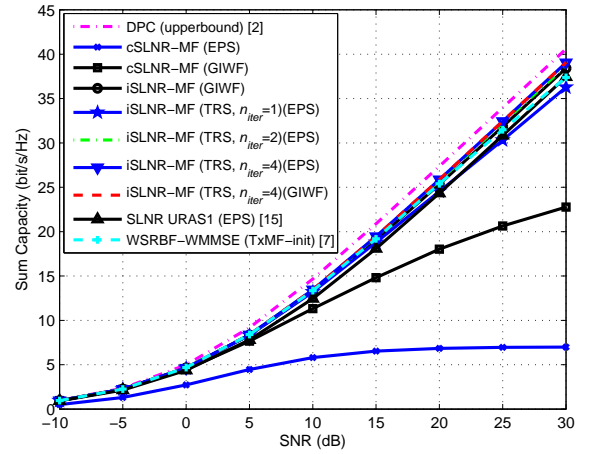
The proposed methods achieve a significant gain compared to an antenna selection scheme (URAS1 EPS) in [15] and

Fig. 8. Ergodic sum capacity with  $M = 4$ ;  $N_k = 2$ ,  $\forall k$ ;  $K = 2$ .Fig. 9. Power allocation of iSLNR-MF GIWF for a randomly selected channel realisation at SNR = 0, 15 and 30 dB, with  $M = 4$ ;  $N_k = 2$ ,  $\forall k$ ; and (a)  $K = 2$ , (b)  $K = 5$ .

attain comparable performance to a joint transceiver design (WSRBF-WMMSE with transmit matched filter [TxMF] initialisation) in [7], albeit with a simpler receiver structure. Note that the initialisation of the transmit filters remains an issue for WSRBF-WMMSE [7], while iSLNR-MF GIWF seems to work reasonably well with EPS initialisation and can also be accompanied by iSLNR-MF TRS as a highly reliable initial condition.

## VII. CONCLUSIONS

This paper proposed an iterative transmit filter design based on a modified definition of SLNR. In contrast to the conventional scheme, the proposed method considers the receiver filters and power allocations into the transmit filter design in order to efficiently use and estimate the receiver subspaces at user terminals. The resulting transceiver structures remain simple and inherit the closed-form solutions from the conventional scheme. As a result, the WSC maximisation problems can be simplified to power allocation and user/substream selection problems. A power allocation algorithm was proposed in this paper, which implicitly integrates the user and substream selection functionalities into its algorithm. An explicit user and substream selection was also proposed as an alternative method

Fig. 10. Ergodic sum capacity with  $M = 4$ ;  $N_k = 2$ ,  $\forall k$ ;  $K = 5$ .

to maximise WSC. Both proposed methods can potentially be combined, which could bring further improvement, subjected to the complexity trade-off. Numerical results show that the proposed algorithms outperform the conventional scheme and the antenna selection approach. They also achieve a comparable performance to a joint transceiver design approach, despite using simple receive matched filters. The proposed schemes, therefore, provide potential alternatives for practical implementations.

## APPENDIX A

### DERIVATION OF THE MODIFIED SLNR WITH IMPERFECT CSI

By substituting (16) into (17), it can be shown that

$$\begin{aligned} \text{mSLNR}_k &= \frac{\text{Tr} \left[ \mathbf{W}_k^H (\mathbf{H}_k^H \mathbf{H}_k' + E \{ \mathbf{E}_k^H \mathbf{E}_k \}) \mathbf{W}_k \right]}{\text{Tr} \left[ \mathbf{W}_k^H \left( \sum_{j \neq k} (\hat{\mathbf{H}}_j^H \hat{\mathbf{H}}_j + E \{ \mathbf{E}_j^H \mathbf{B}_j \mathbf{E}_j \}) + \sigma_k^2 \mathbf{I}_M \right) \mathbf{W}_k \right]} \quad (26) \end{aligned}$$

with  $\mathbf{B}_j = \mathbf{G}_j^H \Omega_j^H \Omega_j \mathbf{G}_j$ . Let  $b_{rc}$  be the  $(r, c)^{th}$  element of  $\mathbf{B}_j$  and  $\mathbf{e}_c$  denote the  $c^{th}$  column vector of  $\mathbf{E}_j$ , it follows that the  $(m, n)^{th}$  element of  $E \{ \mathbf{E}_j^H \mathbf{B}_j \mathbf{E}_j \}$  can be expressed as

$$[E \{ \mathbf{E}_j^H \mathbf{B}_j \mathbf{E}_j \}]_{mn} = E \{ \mathbf{e}_m^H \mathbf{B}_j \mathbf{e}_n \} \quad (27)$$

$$= \sum_{r=1}^{N_j} \sum_{c=1}^{N_j} b_{rc} E \{ e_{rm}^* e_{cn} \} \quad (28)$$

$$= \begin{cases} \text{Tr}(\mathbf{B}_j) \sigma_e^2 & \text{if } m = n, \\ 0 & \text{otherwise} \end{cases} \quad (29)$$

where (29) follows from the uncorrelated property of  $\mathbf{E}_j$ , i.e.  $E \{ e_{rm}^* e_{cn} \}$  equal to  $\sigma_e^2$  if  $r = c$  and  $m = n$ , and equal to 0 otherwise. Thus,  $E \{ \mathbf{E}_j^H \mathbf{B}_j \mathbf{E}_j \}$  can be written as  $\theta_j \sigma_e^2 \mathbf{I}_M$ , with  $\theta_j = \text{Tr}(\mathbf{B}_j) = \text{Tr}[\mathbf{G}_j^H \Omega_j^H \Omega_j \mathbf{G}_j]$ . Similarly,  $E \{ \mathbf{E}_k^H \mathbf{E}_k \} = N_k \sigma_e^2 \mathbf{I}_M$ . It can be easily seen that (18) is obtained from (26) accordingly.

APPENDIX B  
 DERIVATIVE OF THE LAGRANGIAN

The Lagrangian of the problem (19) can be written as (in this case, the base-2 logarithmic function in (19) can be replaced by the natural logarithm without loss of generality, due to the monotonic property of logarithmic functions):

$$\mathcal{L} = \sum_{j=1}^K \alpha_j \ln \det(\mathbf{C}_j) - \nu \left( \sum_j \sum_s p_{js} - P \right) + \sum_j \sum_s \lambda_{js} p_{js} \quad (30)$$

with  $\mathbf{C}_j = \mathbf{I}_{N_j} + \mathbf{H}_j \mathbf{W}_j \mathbf{P}_j \mathbf{W}_j^H \mathbf{H}_j^H \mathbf{Z}_j^{-1}$ . Recall the following relationships for Matrix derivatives with respect to a scalar  $t$  [26]:

$$\frac{d}{dt} \ln \det \mathbf{A} = \text{Tr} \left[ \mathbf{A}^{-1} \left( \frac{d}{dt} \mathbf{A} \right) \right], \quad (31)$$

$$\frac{d}{dt} \mathbf{A}^{-1} = -\mathbf{A}^{-1} \left( \frac{d}{dt} \mathbf{A} \right) \mathbf{A}^{-1}. \quad (32)$$

Taking the derivative of the Lagrangian (30) with respect to the allocated power  $p_{kb}$  (for the  $b^{\text{th}}$  stream of the  $k^{\text{th}}$  user) and equating it to zero result in

$$\sum_{j=1}^K \alpha_j \text{Tr} \left( \mathbf{C}_j^{-1} \frac{\partial \mathbf{C}_j}{\partial p_{kb}} \right) - \nu + \lambda_{kb} = 0. \quad (33)$$

For user  $j = k$ ,

$$\begin{aligned} & \text{Tr} \left( \mathbf{C}_k^{-1} \frac{\partial \mathbf{C}_k}{\partial p_{kb}} \right) \\ &= \text{Tr} \left( \mathbf{C}_k^{-1} \mathbf{H}_k \frac{\partial}{\partial p_{kb}} \left( \sum \mathbf{w}_{kb} p_{kb} \mathbf{w}_{kb}^H \right) \mathbf{H}_k^H \mathbf{Z}_k^{-1} \right) \\ &= \text{Tr} \left( \mathbf{C}_k^{-1} \mathbf{H}_k \mathbf{w}_{kb} \mathbf{w}_{kb}^H \mathbf{H}_k^H \mathbf{Z}_k^{-1} \right) \\ &= \text{Tr} \left( \mathbf{H}_k \mathbf{w}_{kb} \mathbf{w}_{kb}^H \mathbf{H}_k^H (\mathbf{C}_k \mathbf{Z}_k)^{-1} \right) \\ &= \mathbf{w}_{kb}^H \mathbf{H}_k^H (\mathbf{C}_k \mathbf{Z}_k)^{-1} \mathbf{H}_k \mathbf{w}_{kb}. \end{aligned} \quad (34)$$

Define

$$\begin{aligned} \mathbf{T}_k &= \mathbf{C}_k \mathbf{Z}_k \\ &= (\mathbf{I}_{N_k} + \mathbf{H}_k \mathbf{W}_k \mathbf{P}_k \mathbf{W}_k^H \mathbf{H}_k^H \mathbf{Z}_k^{-1}) \mathbf{Z}_k \\ &= \mathbf{Z}_k + \mathbf{H}_k \mathbf{W}_k \mathbf{P}_k \mathbf{W}_k^H \mathbf{H}_k^H \\ &= \sigma_k^2 \mathbf{I}_{N_k} + \mathbf{H}_k \left( \sum_{j=1}^K \mathbf{W}_j \mathbf{P}_j \mathbf{W}_j^H \right) \mathbf{H}_k^H \\ &= \sigma_k^2 \mathbf{I}_{N_k} + \mathbf{H}_k \tilde{\mathbf{W}}_{kb} \tilde{\mathbf{P}}_{kb} \tilde{\mathbf{W}}_{kb}^H \mathbf{H}_k^H + \mathbf{H}_k \mathbf{w}_{kb} p_{kb} \mathbf{w}_{kb}^H \mathbf{H}_k^H. \end{aligned} \quad (35)$$

Let  $\mathbf{A} = \sigma_k^2 \mathbf{I}_{N_k} + \mathbf{H}_k \tilde{\mathbf{W}}_{kb} \tilde{\mathbf{P}}_{kb} \tilde{\mathbf{W}}_{kb}^H \mathbf{H}_k^H$  (invertible),  $\mathbf{u} = \mathbf{H}_k \mathbf{w}_{kb}$  and  $\mathbf{v}^H = p_{kb} \mathbf{w}_{kb}^H \mathbf{H}_k^H$ . Applying the matrix inversion lemma:  $(\mathbf{A} + \mathbf{u} \mathbf{v}^H)^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1} \mathbf{u} \mathbf{v}^H \mathbf{A}^{-1}}{1 + \mathbf{v}^H \mathbf{A}^{-1} \mathbf{u}}$  to (35) and substituting into (34) results in

$$\text{Tr} \left( \mathbf{C}_k^{-1} \frac{\partial \mathbf{C}_k}{\partial p_{kb}} \right) = \gamma_{kb} - \frac{p_{kb} \gamma_{kb}^2}{1 + p_{kb} \gamma_{kb}} \quad (36)$$

where  $\gamma_{kb}$  is defined as in (22). Similarly, for users  $j \neq k$ ,

$$\begin{aligned} & \text{Tr} \left( \mathbf{C}_j^{-1} \frac{\partial \mathbf{C}_j}{\partial p_{kb}} \right) \\ &= \text{Tr} \left( \mathbf{C}_j^{-1} \mathbf{H}_j \mathbf{W}_j \mathbf{P}_j \mathbf{W}_j^H \mathbf{H}_j^H \mathbf{Z}_j^{-1} \left( -\frac{\partial \mathbf{Z}_j}{\partial p_{kb}} \right) \mathbf{Z}_j^{-1} \right) \\ &= \text{Tr} \left( \mathbf{H}_j \mathbf{W}_j \mathbf{P}_j \mathbf{W}_j^H \mathbf{H}_j^H \mathbf{Z}_j^{-1} (-\mathbf{H}_j \mathbf{w}_{kb} \mathbf{w}_{kb}^H \mathbf{H}_j^H) (\mathbf{C}_j \mathbf{Z}_j)^{-1} \right) \\ &= \text{Tr} \left( \mathbf{H}_j \mathbf{W}_j \mathbf{P}_j \mathbf{W}_j^H \mathbf{H}_j^H \mathbf{Z}_j^{-1} (-\mathbf{H}_j \mathbf{w}_{kb} \mathbf{w}_{kb}^H \mathbf{H}_j^H) \mathbf{T}_j^{-1} \right) \\ &= -\mathbf{w}_{kb}^H \mathbf{H}_j^H \mathbf{T}_j^{-1} \mathbf{H}_j \mathbf{W}_j \mathbf{P}_j \mathbf{W}_j^H \mathbf{H}_j^H \mathbf{Z}_j^{-1} \mathbf{H}_j \mathbf{w}_{kb}. \end{aligned} \quad (37)$$

Substituting (36)-(37) into (33) leads to

$$\frac{\alpha_k}{p_{kb} + \frac{1}{\gamma_{kb}}} = \nu - \lambda_{kb} + t_{kb} \quad (38)$$

where  $t_{kb}$  is defined as in (23). It follows that (21) is obtained from (38).

## ACKNOWLEDGMENT

The authors would like to thank the editor and the anonymous reviewers for their thorough comments that greatly improved this paper.

## REFERENCES

- [1] G. Caire and S. Shamai, "On the Achievable Throughput of a Multi-antenna Gaussian Broadcast Channel," *IEEE Trans. Inf. Theory*, vol. 49, no. 7, pp. 1691–1706, Jul. 2003.
- [2] N. Jindal, W. Rhee, S. Vishwanath, S. A. Jafar, and A. Goldsmith, "Sum Power Iterative Water-Filling for Multi-Antenna Gaussian Broadcast Channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 4, pp. 1570–1580, Apr. 2005.
- [3] H. Weingarten, Y. Steinberg, and S. S. (Shitz), "The Capacity Region of the Gaussian MIMO Broadcast Channel," in *Proc. IEEE International Symp. Inform. Theory (ISIT 2004)*, Jun. 2004.
- [4] M. Kobayashi and G. Caire, "An Iterative Water-Filling Algorithm for Maximum Weighted Sum-Rate of Gaussian MIMO-BC," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1640–1646, Aug. 2006.
- [5] S. Shi, M. Schubert, and H. Boche, "Rate Optimization for Multiuser MIMO Systems With Linear Processing," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 4020–4030, Aug. 2008.
- [6] G. Zheng, K. Wong, and T. Ng, "Throughput Maximization in Linear Multiuser MIMO-OFDM Downlink Systems," *IEEE Trans. Veh. Technol.*, vol. 57, no. 3, pp. 1993–1998, May 2008.
- [7] S. Christensen, R. Agarwal, E. Carvalho, and J. Cioffi, "Weighted Sum-Rate Maximization using Weighted MMSE for MIMO-BC Beamforming Design," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 4792–4799, Aug. 2008.
- [8] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, "A Vector-Perturbation Technique for Near-Capacity Multiantenna Multiuser Communication - Part I: Channel Inversion and Regularization," *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 195–202, Jan. 2005.
- [9] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-Forcing methods for Downlink Spatial Multiplexing in Multiuser MIMO channels," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 461–471, Feb. 2004.
- [10] M. Joham, W. Utschick, and J. A. Nossek, "Linear transmit processing in MIMO communications systems," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 2700–2712, Aug. 2005.
- [11] V. Stankovic and M. Haardt, "Generalized Design of Multi-User MIMO Precoding Matrices," *IEEE Trans. Wireless Commun.*, vol. 7, no. 3, pp. 953–961, Mar. 2008.
- [12] H. Sung, S. Lee, and I. Lee, "Generalized Channel Inversion Methods for Multiuser MIMO Systems," *IEEE Trans. Commun.*, vol. 57, no. 11, pp. 3489–3499, Nov. 2009.

- [13] M. Sadek, A. Tarighat, and A. Sayed, "A Leakage-Based Precoding Scheme for Downlink Multi-User MIMO Channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1711–1721, May 2007.
- [14] M. Sadek, A. Tarighat, and A. H. Sayed, "Active Antenna Selection in Multiuser MIMO Communications," *IEEE Trans. Signal Process.*, vol. 55, no. 4, pp. 1498 – 1510, Apr. 2007.
- [15] P. Patcharamaneepakorn, A. Doufexi, and S. Armour, "Reduced Complexity Joint User and Receive Antenna Selection Algorithms for SLNR-based Precoding in MU-MIMO systems," in *IEEE 75th Vehicular Technology Conference (VTC2012-Spring)*, Yokohama, Japan, May 2012.
- [16] Z. Pan, K. Wong, and T. Ng, "Generalized Multiuser Orthogonal Space-Division Multiplexing," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1969–1973, Nov. 2004.
- [17] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.
- [18] T. Yoo and A. J. Smith, "Capacity and power allocation for fading MIMO channels with channel estimation error," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 2203–2214, May 2006.
- [19] A. D. Dabbagh and D. J. Love, "Multiple Antenna MMSE Based Downlink Precoding with Quantized Feedback or Channel Mismatch," *IEEE Trans. Commun.*, vol. 56, no. 11, pp. 1859–1868, Nov. 2008.
- [20] W. Yu, "Multiuser Water-filling in the Presence of Crosstalk," *Information Theory and Applications Workshop, 2007*, pp. 414–420, Jan. 2007.
- [21] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, June 27, 2005.
- [22] Z. Shen, R. Chen, J. Andrews, R. Heath, and B. Evans, "Low Complexity User Selection Algorithms for Multiuser MIMO Systems With Block Diagonalization," *IEEE Trans. Signal Process.*, vol. 54, no. 9, pp. 3658–3663, Sep. 2006.
- [23] P. Cheng, M. Tao, and W. Zhang, "A New SLNR-Based Linear Precoding for Downlink Multi-User Multi-Stream MIMO Systems," *IEEE Commun. Lett.*, vol. 14, no. 11, pp. 1008–1010, Nov. 2010.
- [24] G. H. Golub and C. F. V. Loan, *Matrix Computations*, 3rd ed. The Johns Hopkins University Press, 1989.
- [25] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes: The Art of Scientific Computing*, 3rd ed. Cambridge University Press, 2007.
- [26] R. Blum, "MIMO Capacity With Interference," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 5, pp. 793–801, Jun. 2003.



**Simon M. D. Armour** received the B.Eng. degree from the University of Bath, Bath, U.K., in 1996 and the Ph.D. degree from the University of Bristol, Bristol, U.K., in 2001.

He has been an academic at the University of Bristol since 2001 and has held the post of Senior Lecturer since 2007. He has published over 100 papers in the field of baseband PHY layer and MAC layer techniques for wireless communications, with particular focus on orthogonal frequency-division multiplexing, coding, multiple-input-multiple-output, and cross layer multiuser radio resource management strategies. He has investigated such techniques in general terms as well as specific applications to 802.11, 802.16, and Third-Generation Partnership Project Long-Term Evolution.



**Piya Patcharamaneepakorn** received the B.Eng. degree in electrical engineering from Chulalongkorn University, Bangkok, Thailand, in 1999, and the M.Eng. degree in telecommunications from the Asian Institute of Technology, Pathumthani, Thailand, in 2001. He is currently pursuing the Ph.D. degree with the Department of Electrical and Electronics Engineering, University of Bristol, Bristol, U.K.

His research interests include Long-Term Evolution (LTE/LTE-advanced), enhanced multiple-input multiple-output (MIMO) techniques, multi-user MIMO (MU-MIMO) precoding schemes, and multi-cell MIMO cooperation techniques.



**Angela Doufexi** received the B.Sc. degree in physics from the University of Athens, Athens, Greece, in 1996, the M.Sc. degree in electronic engineering from Cardiff University, Cardiff, U.K., in 1998, and the Ph.D. degree from the the Centre for Communications Research, University of Bristol, Bristol, U.K., in 2002.

She is currently Senior Lecturer in Wireless Networks with the University of Bristol. Her research interests include orthogonal frequency-division multiplexing and orthogonal frequency-division multiple access systems, multiuser diversity and resource allocation, wireless LANs, WPANs, and WiMax (PHY and MAC), space time coding and multiple-input-multiple-output, Long-Term Evolution and Fourth-Generation communications systems, vehicular ad hoc networks and vehicular communications, cognitive radio, and multimedia transmission. She has published over 100 journal and conference papers in these areas.