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# Nonlinear Aircraft Ground Dynamics

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The dynamics of aircraft manoeuvring on the ground is nonlinear due to nonlinearities in tyres, in other landing gear components, and potentially in the aerodynamics of the airframe. Existing linear approaches are effective in respect of local behaviour around specific operating conditions, but in order to evolve a global understanding of the dynamics of an aircraft on the ground other techniques are needed.

We demonstrate here that bifurcation analysis and continuation methods can be used for the purpose of building up a global stability diagram in a computationally efficient manner. Specifically, we perform a continuation study of a nonlinear tricycle model implemented in the SimMechanics environment and then coupled to the continuation software AUTO. Our study yields unexpected phenomena, namely a complicated structure of branches with hysteresis loops and instabilities leading to oscillations and even chaotic dynamics.

## I. Introduction

Landing gears need to satisfy many conflicting requirements. For example, weight and pavement loading need to be minimised, and shock absorption maximised. Landing gears contain highly nonlinear components such as tyres, brakes, and oleos (shock absorbers), so that the overall system is inherently nonlinear. Figure 1 shows the main contributors to nonlinearity. The (lateral) stability of the aircraft on the ground is determined chiefly by the position of the gears, along with the tyre and oleo characteristics. Experience has shown that the use of different tyres can mean the difference between a stable and an unstable aircraft. Due to their complexity, the analysis of landing gears is usually done at some very specific design conditions, without characterising the behaviour of the system over a wide variety of parameters.

The Federal Aviation Regulations (FARs) contain minimal detail regarding the characteristics that are needed for adequate ground handling of an aircraft, only stipulating that no exceptional skill should be needed by the pilot to steer the aircraft. There is very little literature regarding the analysis of aircraft on the ground, although it is known that simulators are used to gain an understanding of how the aircraft will behave in this operating region. A recent study on a navy jet trainer showed how simple models could be used to identify dominant ground handling characteristics.<sup>1</sup> A bicycle model was used to identify the dominant dynamics, and then more detailed nonlinear models were used in areas of interest. The form of the bicycle model was similar to that used in the automotive industry, and differed only in the regions of the parameter space that were occupied, as well as the degree to which aerodynamics played a part. As far as we know, no attempt has been made to move to a three-dimensional environment.

On the other hand, the need for more detailed studies of nonlinear aircraft dynamics on the ground becomes more and more pressing. For example, over the last decade it has become apparent that some required lateral load maxima during ground manoeuvres are rather conservative, which leads to the addition of weight to the landing gear structure. Part 25.495 of the airworthiness regulations, stipulates that a *g-level* of  $0.5g$  should be applied at the CG position for ground load calculations. However, experience from taxi tests has shown that a lateral *g-level* of even  $0.35g$  is hard to achieve on large aircraft. Hence a large discrepancy exists between what airlines experience during operations and the requirement stated in the regulations. Above a critical value the tyre loses traction and the aircraft skids out of control. It is therefore known that a critical *g-level* exists beyond which traction is lost. Interestingly, this critical value is, as yet, undetermined.

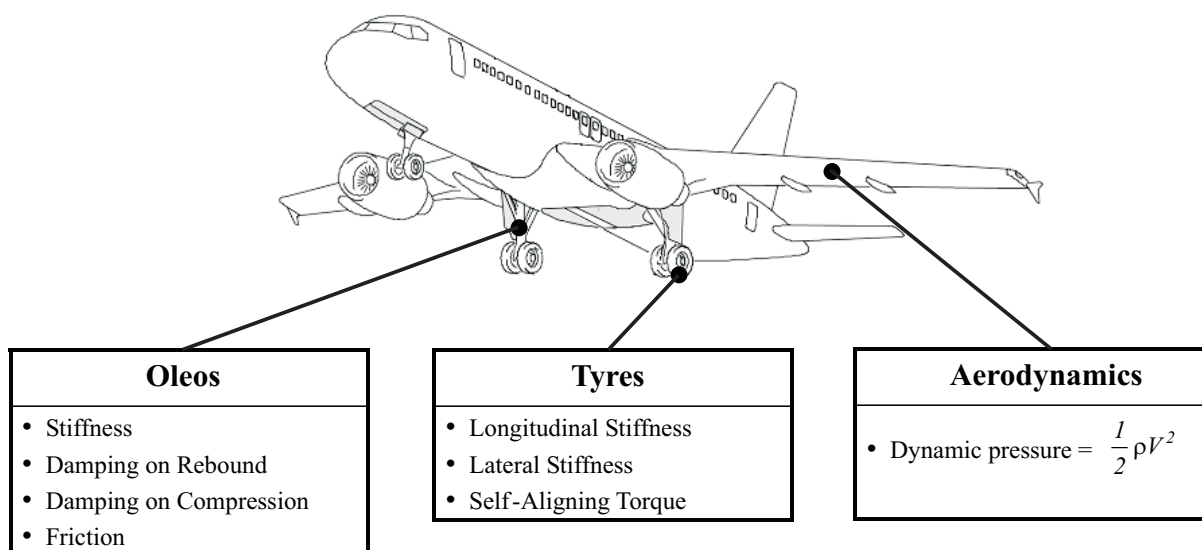


Figure 1: Nonlinear components affecting aircraft ground dynamics.

In this paper we present results of a study of the turning characteristics as determined from a (simplified) three-dimensional and nonlinear SimMechanics<sup>2</sup> model of the Airbus A320. We use the mathematical technique of bifurcation analysis to follow specific solutions in parameters. This is done by coupling the SimMechanics model to the continuation software AUTO.<sup>3</sup> In this way we can reveal the nonlinear character of the system, which expresses itself in the presence of multiple branches of stable turning solutions. Specifically, we find different types of stability boundaries, hysteresis loops and even chaotic aircraft motion.

The SimMechanics model considered here is based on a well-established ADAMS model that has been developed and used within the Landing Gear Group of Airbus UK. The design tool ADAMS<sup>4</sup> has been widely used in the automotive industry over the last decade. It is also being used for ground manoeuvrability studies, mechanism design and failure analysis within the Landing Gear Group of Airbus UK, who have responsibility within Airbus to verify that the landing gear systems will provide the correct performance to manoeuvre around specific airports.

Parameters, such as the tyre characteristics, track width, CG position, velocity, and steering angle, all influence the obtainable (lateral) loading conditions that the aircraft can experience. It would be of great benefit if the influence of these parameters can be studied during the preliminary design phases of a project, where some analysis is indeed already done. Due to the cost of detailed nonlinear simulations, at present only localised areas of the design space can be explored. A rapid method of analysis that could give a global view of the stability would be of great benefit. Continuation methods as presented here have the potential to provide this capability.

## II. Background on the model

The starting point was an ADAMS model that was developed at the Landing Gear Group of Airbus for ground handling studies. The main program of ADAMS can analyse kinematic, quasi-static and dynamic mechanical systems. The first step of the modelling is to describe the rigid parts and the joints connecting the parts,<sup>4</sup> where a part is described by its mass, inertia and orientation. Specifically, in the tricycle model considered here the nose gear is constrained by a cylindrical joint, which is driven by an angular motion, and the main gears are constrained by translational joints. The next step is the addition of internal force elements, known as line of sight forces, to represent the shock absorbers and tyre forces. The tyres are modelled with impact functions that switch on as soon as the distance between the wheel centre and the tyre becomes less than the wheel radius. External forces such as thrust and aerodynamic forces are then added, and are known as action-only forces. All geometric aspects were parameterised, from the axle widths, wheel dimensions, gear positions, to the rake angles on the gears. This means that all joint definitions and forces

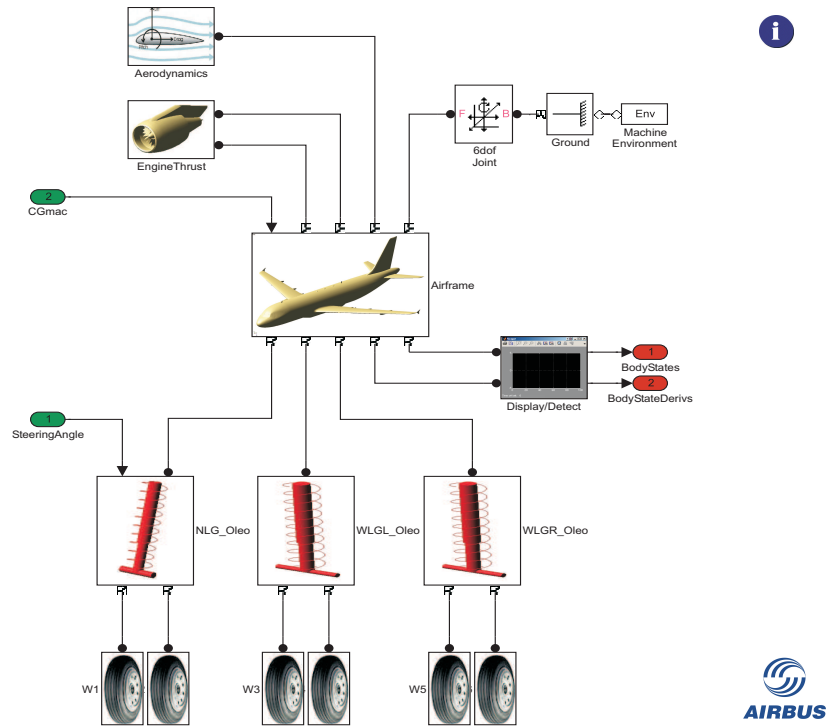


Figure 2: SimMechanics representation of A320.

are automatically updated when the design variables are changed.

Unfortunately, an ADAMS model cannot be coupled to our main bifurcation analysis tool, the continuation software AUTO. However, it has been demonstrated that Matlab models can be linked to AUTO.<sup>5</sup> Therefore, we decided to use SimMechanics, which is a recent Simulink blockset of Matlab. SimMechanics uses spatial operator algebra to solve the equations of motion and is ideal for real-time applications, due to the efficiency of the algorithm.<sup>6</sup> The method of building the SimMechanics model is easier than representing the same mechanical system in Simulink. A similar approach to the ADAMS modelling is followed when building a SimMechanics model, where parts, joints and forces need to be defined. Additional work was needed in the development of the tyre model, along with the definitions of the coordinate systems. While there are important differences between the two modelling packages, the development of the SimMechanics model followed that of the ADAMS model. It has the same characteristics and the two models have been compared extensively to ensure consistency. Figure 2 shows the top-level depiction of the SimMechanics model.

The first step in linking the SimMechanics model to AUTO is to simulate the model until the states are in equilibrium. The body states and steering angle are then written into an initialisation file that is used as starting data for the continuation analysis. Continuation parameters are set based on the number of states that are being changed. The SimMechanics/Simulink model is compiled during the initialisation phase for further use during function calls, after which continuation with AUTO commences. An axis transformation from the body reference frame to the world reference frame is implemented, since Simulink expects the states in the world reference frame. The states are then set and the derivatives are obtained. To be able to run several cases automatically, script files were written that adjust the input files depending on the type of continuation analysis, as well as the initial data from the equilibrium runs. A report containing the continuation results, along with tables of the maximum  $g$ -levels, is then generated.

## A. Modelling hierarchy

In order to identify the causes of different behaviours of an aircraft on the ground, we actually developed a hierarchy of models. The key idea is to capture the relevant dynamics in as simple a model as possible. In this way, more extensive parameter studies become feasible. The specific aim of the analysis is to see what

effect the tyres, oleos and aerodynamics have on the dynamic behaviour of a 2D and 3D aircraft model. These components can be ‘activated’ by setting specific model parameters.

We used a bottom-up approach where the complexity of the model is increased by adding additional complexity. Specifically, we consider four separate models:

1. ***Bicycle model - tyres only*** A parameterised non-linear bicycle ADAMS model was built as a reference model and then constructed in SimMechanics using the equations from automotive examples,<sup>7</sup> and the geometric data from the ADAMS model. This model, which includes the nonlinearity of the tyres, is already more complex than the linearised navy jet trainer model developed by Klyde *et al.*<sup>1,8–11</sup>
2. ***Tricycle model - tyres only*** This SimMechanics model has six degrees of freedom and includes the nonlinearity of the tyres. It can be compared directly with the bicycle model.
3. ***Tricycle model with tyres and oleos*** The stiffness and damping curves for the oleos (shock absorbers) were approximated by smooth functions for the used double-stage oleo. Each oleo adds one degree of freedom to the system, increasing the degrees of freedom from six to nine, and the states from 12 to 18.
4. ***Tricycle model with tyres, oleos and aerodynamics*** Aerodynamics was incorporated based on the data that was provided by the GARTEUR group.<sup>12</sup>

### III. Nonlinear stability analysis and continuation methods

Dynamical systems theory provides a methodology for studying systems of nonlinear ordinary differential equations (ODEs). A key method is that of bifurcation analysis, where one identifies different ways in which the dynamics of the system can change. In combination with the numerical technique of continuation, one can perform a nonlinear stability analysis by following solutions and detecting their stability changes (bifurcations). The bifurcations can then be followed in more parameters to identify regions in parameter space that correspond to different behaviour of the system. See, for example, the textbooks<sup>13,14</sup> as entry points to the literature.

To summarize some basic ideas consider an ODE model of the form

$$\dot{u} = f(u, \lambda) \quad (1)$$

where  $u$  is an  $n$ -dimensional state vector,  $\lambda$  a (multidimensional) control parameter, and  $f$  a sufficiently smooth (typically nonlinear) function. In terms of standard equations of motion for an aircraft on the ground, the state vector  $u$  contains the aircraft translational and rotational states, along with the translational states of the oleos. The control parameter  $\lambda$  consists of the steering angle, thrust, the position of the CG, and possibly other relevant parameters. Equilibrium solutions of (1), also known as trim conditions, satisfy

$$f(u_0, \lambda) = 0. \quad (2)$$

The implicit function (2) defines a solution locus of equilibria, which is a one-dimensional solution curve when a single parameter, such as the steering angle, is varied. The stability of the equilibria can be determined from the  $(n \times n)$  Jacobian matrix  $Df$  of partial derivatives of the function  $f$  with respect to the state  $u$ .

Continuation software, such as the package AUTO<sup>3</sup> used here, is able to follow curves of equilibria while monitoring their stability. Changes of stability, that is, bifurcations are automatically detected and can then be followed in additional parameters. In this paper we find saddle-node (fold) and Hopf bifurcations (onset of oscillations).<sup>13,14</sup> Similarly, periodic solutions can be followed and their stability changes detected. The continuation of suitable solution curves allows one to build up a comprehensive picture of the overall dynamics in a systematic way.

Bifurcation analysis is now a standard and powerful tool that is being used extensively in application areas as diverse as fluid dynamics, laser physics and mathematical biology, to name but a few. It is now also proving its use in engineering applications, notably in control engineering and in hybrid testing. In terms of aerospace applications, bifurcation analysis has been used successfully in flight mechanics.<sup>15</sup> To our knowledge there have been as yet no bifurcation studies of the behaviour of aircraft on the ground.

Here we apply bifurcation analysis to the study of the turning dynamics of the tricycle model from Section IIIA. Our approach is similar to that taken in a recent study of handling characteristics of an automotive model,<sup>16</sup> but the aircraft model is more nonlinear, chiefly due to the tyre characteristics.

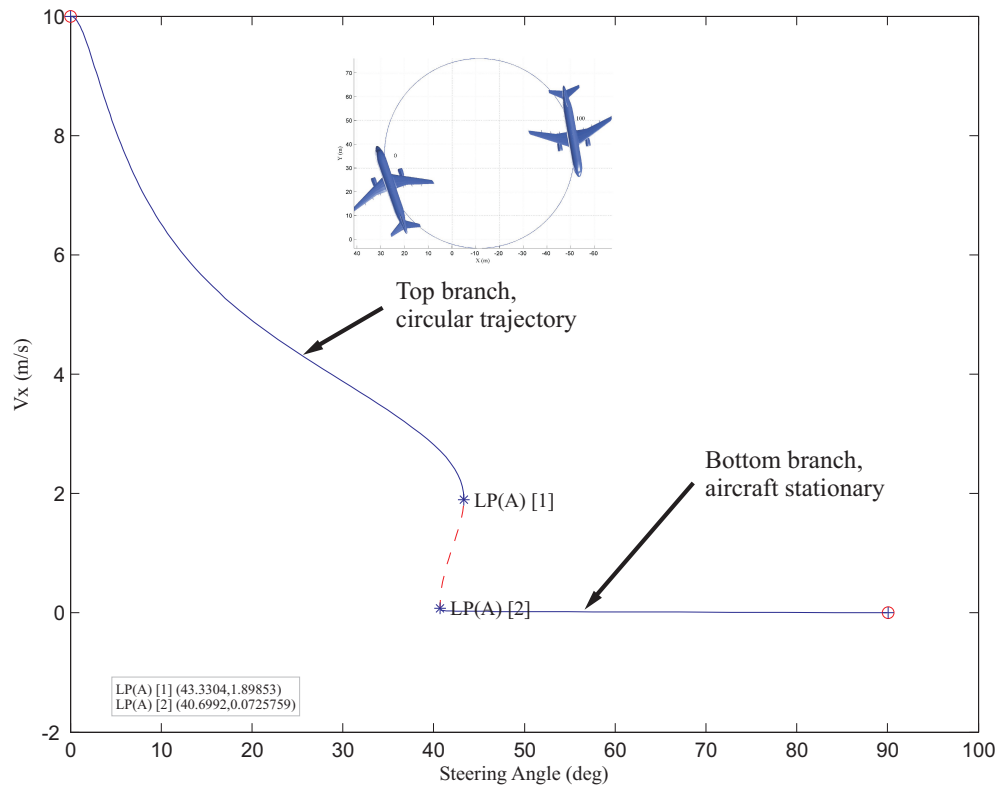


Figure 3: Bifurcation diagram for aircraft at 5% of max thrust; plotted is the equilibrium forward velocity change as the steering angle is varied.

#### IV. Bifurcation analysis of aircraft turning

Ground operations tend to occur with constant thrust settings, where the thrust is occasionally adjusted by the pilot, with the aim of altering the velocity. Experience has shown that the aircraft velocity will decrease during a turn, due to a higher projected side force from the tyre towards the rear of the aircraft. If the steering angle is changed at a slow rate, it can be assumed that the aircraft is subjected to quasi-steady loads, where the accelerations can be ignored. This is equivalent to a continuation analysis that provides a set of curves representing either an equilibrium position, or an indication of where oscillatory or even chaotic behaviour is present.

The first step is to find an equilibrium condition for the aircraft. A velocity controller can be used to accelerate the aircraft to a specific velocity, where an equilibrium state should be reached after a few seconds. The thrust is then held constant at this equilibrium value, while the aircraft states at this condition are used as the starting conditions for the continuation analysis. The continuation analysis will then follow the equilibrium positions while the steering angle is varied. Specific tests are conducted to detect limit points, other branches, and Hopf bifurcations.

Figure 3 shows the result of a continuation. The horizontal branch for zero velocity corresponds to the aircraft being stationary. The other solid branch corresponds to turning at the given velocity and at an associated specific radius. An example of the resulting circular trajectory of the aircraft is shown in the inset. At a steering angle of about 43 degrees the turning stability is lost in a saddle-node bifurcation (LP label of AUTO). The dashed unstable branch connects to the branch of stationary solutions. Notice the bistability between stable turning and a stopped aircraft, which results in a hysteresis loop with respect to ramping up and down the steering angle.

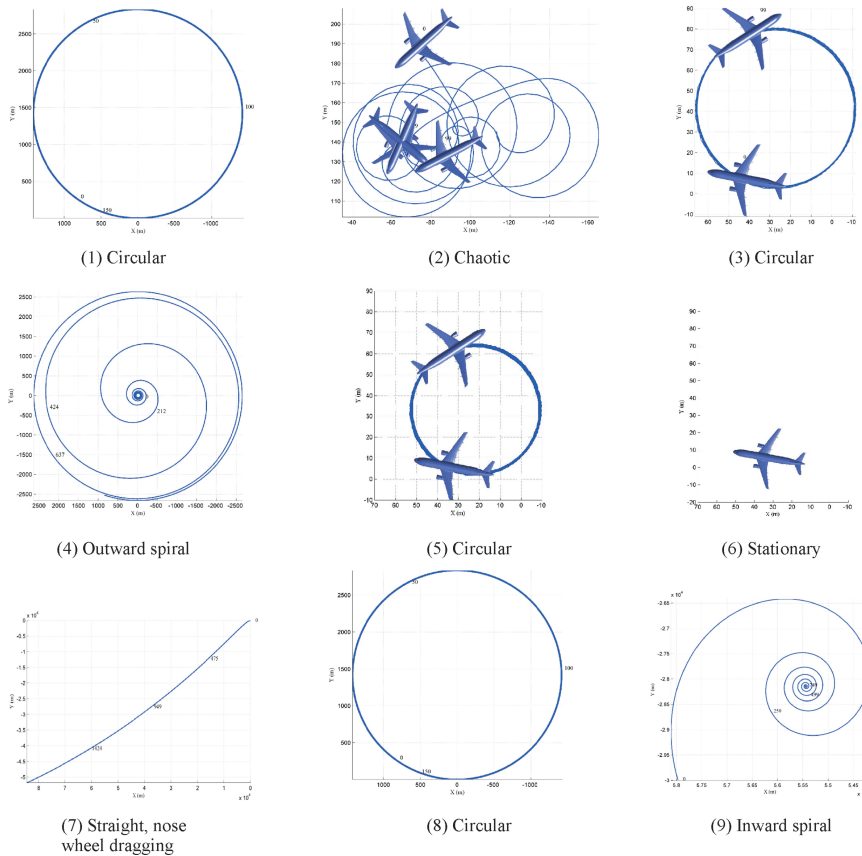
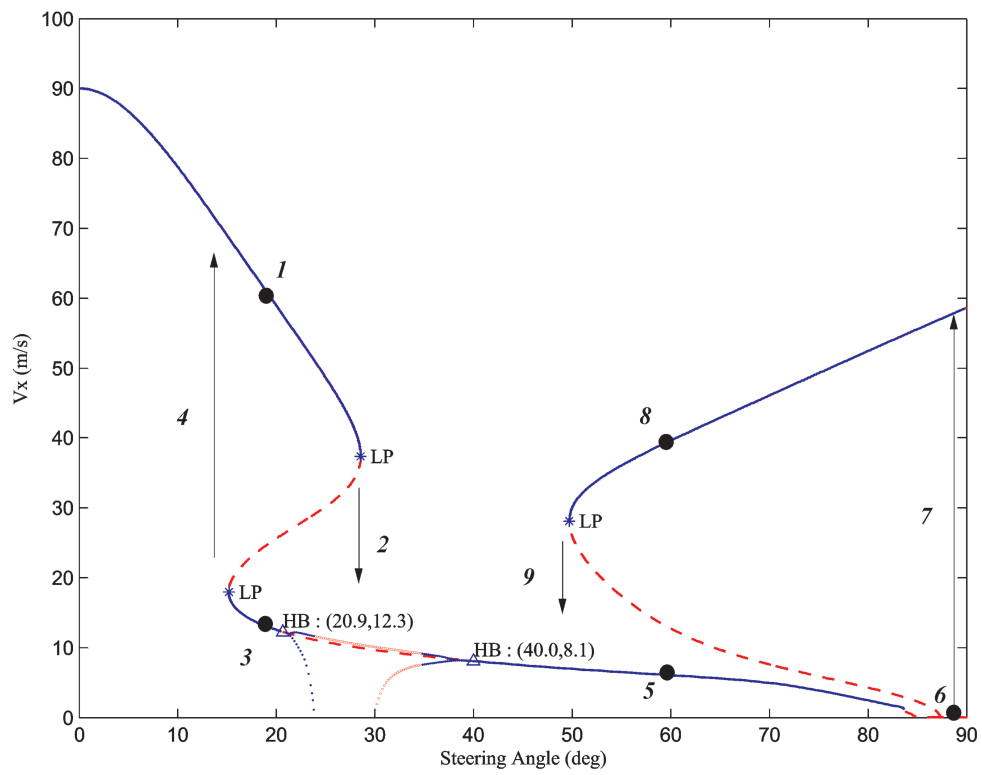


Figure 4: Bifurcation diagram for aircraft at 20% of max thrust; plotted is the equilibrium forward velocity change as the steering angle is varied.

Figure 4 shows that one may encounter more complicated situations (depending on the thrust level). There are four separate stable parts of branches and Hopf bifurcation points HB that give rise to stable oscillations. We now discuss the possible aircraft behaviour as represented by the numbering on the bifurcation diagram.

The stable section **1** of the branch represents circular trajectories of the aircraft, where the radius of the circle decreases as the steering angle is increased. The limit point LP (a saddle-node bifurcation) marks the loss of stability of the turn. In fact, as is indicated by **2**, past the limit point the aircraft loses control without any warning. Further analysis of the state variables shows that the inner tyre saturates and the load is shifted to the outer gear. The outer tyre then saturates with an accompanying load redistribution to the nose and inner gears. The trajectory of the steering angle versus the forward velocity, past the limit point, also determines the final behaviour. For large enough forward velocity the aircraft may settle into a forward equilibrium (after a momentarily loss of control), but otherwise it may actually end up in chaotic motion.

The stable branch **3** also represents a circular trajectories of the aircraft but, compared to solutions on branch **1**, the radius is much smaller. In other words, for the same steering angle and thrust settings there are two different simultaneously stable turns. When the steering angle is decreased past the limit point the aircraft performs an outward spiral, as is shown by **4**, until an equilibrium at a larger circular trajectory is found. In fact, the inner or the nose gear tyres saturate, and then the particular tyres stay close to saturation, without any significant load shift to any of the other gears.

The stable branch **5** represents a circular trajectories of the aircraft with very small radius. The aircraft actually becomes stationary at very high steering angles, and **6** is an example. In this stopped position a force impulse (e.g., a thrust impulse or wind gust) causes the nose gear to start dragging across the ground. Hence, the aircraft accelerates in a straight line, as is shown by **7**, until a velocity on the top branch **8** is reached. This stable branch **8** is somewhat counterintuitive in the sense that the radius of the circular trajectory decreases as the steering angle is decreased. When the stability is lost by decreasing the steering angle to below the limit point LP the aircraft performs an inward spiral, as is shown by **9**, until an equilibrium at a smaller circular trajectory is found. Here the inner gear tyres saturate, which is accompanied with a pitch down motion. Then the nose gear tyres saturate, with a load shift to the outer gear.

In the regions between the two Hopf points (HB) limit cycle and chaotic motion can be found. When starting from points **3** or **5** decreasing or increasing the steering angle, respectively, will result in the onset of (initially small) stable oscillations. The aircraft will start shaking, giving the pilot a warning that the steering angle or thrust needs to be altered to move out of this region. If the steering angle is brought into the region around 30 degrees, then even chaotic aircraft motion is possible. As is indicated with the example in panel **2**, this manifests itself in a complicated trajectory on the ground.

## V. Conclusions

We demonstrated that bifurcation analysis and continuation methods can be used to study the turning characteristics of aircraft. Specifically we developed from an existing ADAMS model, which has been validated against test results, a hierarchy of (equivalent) models in the SimMechanics environment. This allowed us to link the model to the continuation software AUTO, which was then used to follow branches of solutions. Already in this initial study for constant thrust and varying steering angle we found a complicated structure of branches, including hysteresis loops and instabilities leading to oscillations and even chaotic dynamics. This highlights the nonlinear nature of the problem, as well as the capability of bifurcation analysis for determining the dynamics of aircraft on the ground.

Future work will involve a more extensive investigation of turning dynamics and, in particular, the influence of the thrust level. Of special interest will also be a more detailed analysis of how stability boundaries depend on other system parameters. In the longer term we plan to address questions of practical importance for aircraft handling and landing gear design, including the value of the maximum lateral g-level that can occur and safe margins for tyre characteristics.



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