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Area Spectral Efficiency of a Channel Adaptive Cellular Mobile Radio System in a Correlated Shadowed Environment

M F Tariq and A Nix

Centre for Communication Research
Merchant Venturers Building, Woodland Road
University of Bristol, BS8 1UB, UK
E-mail: M.F.Tariq@bristol.ac.uk , Andy.Nix@bristol.ac.uk

Abstract — In this paper, the average Area Spectral Efficiency (ASE) in bits/Sec/Hz/Km^2 of a variable transmission rate, high capacity cellular communication system is studied in a correlated shadowed environment. The effect on average ASE of the normalised reuse distance, shadowing variance, correlation between radio signals, coverage radius and cell sectorization is investigated by computer simulations. It is shown mathematically that the average ASE of an interference limited mobile radio channel increases with the increased shadowing variance. Downlink simulation results indicate that, the increase in shadowing correlation between the desired and the sum of the undesired signals reduces the average ASE whereas mutual correlation between individual interfering signals increases the average ASE. The variation of cell size on average ASE is also investigated. The results suggest small cell sizes in heavily shadowed areas are well suited for high-speed variable data rate transmissions.

I. Introduction

The introduction of high-speed wireless data services require a highly spectrum efficient communication system. Cell splitting and reusing the same frequencies at distant cells increases the spectrum efficiency to a large extent but is not enough to meet ever-increasing traffic demand. Spectral efficiency can be increased further by taking into account the wireless channel variability. Instead of deploying a fixed rate modem, a variable rate channel adaptive modem has been proposed in recent years [1]. The enhancement in spectrum efficiency is realised from the flexibility of adapting the transmission rate according to several fading and interference conditions encountered in the wireless channel.

The conventional definition to measure the Area Spectral Efficiency (ASE) of a fixed data rate cellular system is Channels/MHz/Km^2 [2]. If this definition is used to calculate the ASE of a variable data rate communication system, it does not take into account the time variant multiple transmission rates of the channel. The average ASE of a channel adaptive cellular system is calculated by Alouni & Goldsmith [3] in bits/sec/Hz/Km^2 for a TDMA/FDMA mobile radio channel by using the well-known Shannon capacity theorem. In their analysis, uplink channel was considered and the mobile radio channel was characterised by the propagation path loss only.

In densely populated areas, high obstacles shadow the MS antenna and cause variations in the mean received signal power. The purpose of this paper is to develop a computer simulation model for calculating the average ASE of a channel adaptive cellular system in a correlated shadowed environment. Results are presented for the downlink only because of the excessive demand to broadcast high speed data in most of the emerging communication services.

The rest of the paper is organised as follows. Section II describes the propagation and system modelling. In Section III a mathematical formula is derived for the average ASE and a computer simulation model is stated. Section IV discusses the computer simulation results followed by the conclusions in Section V.

II. Propagation and System Models

A. Cellular layout

The geographical area is described by regular hexagons. For mathematical convenience hexagons are approximated by circles of radius R. The minimum distance between cells using a common set of frequencies is D. Therefore, the total area served by each set of frequencies is given by $\pi (D/2)^2$. It is assumed that the system is interference limited and six interfering BS surround an MS. The BS and MS antennas are assumed to be omni directional with equal gains, heights and transmit power in each cell respectively. With 120° sectoring, the maximum number of interfering BS reduces to 2 [2].

B. Traffic distribution

It is assumed that all the MS are independent and uniformly distributed in a cell. The probability density function of an MS location relative to a BS in cylindrical co-ordinates is

$$f_{r,\theta}(r,\theta) = \frac{r}{\pi R^2} \qquad \text{where } 0 \leq \theta \leq 2\pi, 0 \leq r \leq R \qquad (1)$$

C. Channel loading

A cellular TDMA/FDMA communication system has been assumed in the analysis with a fixed number of downlink channels of equal bandwidth available in each cell. Fixed channel allocation is considered with fully loaded cells.

D. Propagation path loss

Attenuation of radio signals in a typical mobile environment consists of propagation path loss, shadowing variations and fast fading. Propagation path loss is the attenuation in the mean received power (ξ) of a signal as the separation between the mobile station and the base station increases. It is also known as area mean power and is estimated in this paper by a simple two-ray path loss model for both desired and undesired signals. It is stated in [3] as

$$\xi = \frac{A}{r^a(1 + r/g)^b} \qquad (2)$$

where A is a constant, r is the separation between the MS and the BS, and g is the break point on the loss curve. g is further defined as $4h_b h_m / \lambda_c$, where h_b, h_m and λ_c are the BS antenna height, MS antenna height and wave length of the carrier frequency, respectively.

E. Shadowing

When a mobile station is in the vicinity of high obstacles, the mean received power starts to fluctuate on top of the area mean power ξ. This phenomenon is also known as slow fading. The variation in mean received power follows a lognormal distribution $f_P(P)$ with mean ξ and variance σ^2.

In some propagation environments, the same obstacles may cast overlapping shadows to signals transmitted from various base stations (BS). This results in a shadowing correlation ρ_{du} between the desired and undesired signals at the mobile receiver [4]. As there are six significant interferers affecting an MS, the sum of the six lognormal variables can be represented by a single lognormal variable with a new mean and variance [5]. Therefore the joint probability of the local mean powers of the desired and the sum of the undesired signals can be written as

$$f_{P_{od},P_{ou}}(P_{od},P_{ou}) = \frac{1}{2\pi\sigma_d\sigma_u P_{od} P_{ou}\sqrt{1-\rho_{du}^2}} \cdot \\ \exp\left(-\frac{\tau_d^2 + \tau_u^2 - 2\rho_{du}\tau_d\tau_u}{2(1-\rho_{du}^2)}\right) \qquad (3)$$

where $\tau_d = \dfrac{1}{\sigma_d}\ln\left(\dfrac{P_{od}}{\xi_d}\right)$ and $\tau_u = \dfrac{1}{\sigma_u}\ln\left(\dfrac{P_{ou}}{\xi_u}\right)$

where ξ_d, P_{od}, σ_d and ξ_u, P_{ou}, σ_u are the area mean power, local mean power and standard deviation of the desired and the sum of the undesired signals respectively.

Besides the shadowing correlation, the angular separation of the interfering signals at the MS antenna generates a mutual correlation between them. It is characterised by another correlation factor ρ_{yy}. The realistic values of such a correlation lie between 0.4 and 0.6 [4].

F. Fast fading

The movement of the MS in the multipath environment causes the local mean power to fade by 30 dB or more over distances as close as $\lambda/2$ m. This phenomenon is sometimes described as Rayleigh, or short term, fading.

In this paper our adaptation parent process does not take into account the effects of fast fading. The MS only adapts to the local mean power of the received signal to change its transmission rate.

G. Carrier to Interference ratio (C/I)

ASE depends upon the cochannel interference and noise present at the mobile receiver. This analysis is aimed at high capacity cellular communication, therefore the effect of thermal noise can be neglected [2]. The local mean CIR (v) of the desired and undesired signals can be written as

$$v = \frac{P_{od}}{P_{ou}} = \frac{P_{od}}{\sum_{n=1}^{N} I_n} \qquad (4)$$

where N is the total number of interfering BS and I_n represents the local mean power of an individual interferer.

III. Area Spectral Efficiency

The ASE of a channel adaptive cellular system is defined as bits/Sec/Hz/Km^2. For a TDMA/FDMA communication system having equal bandwidth channels, the average ASE is given by [3] as

$$\langle A_e \rangle = \frac{4\langle C \rangle}{\pi D^2} = \frac{4\langle C \rangle}{\pi R^2 R_u^2} \qquad (5)$$

where R_u is defined as the normalised reuse distance and is given by D/R. $\langle C \rangle$ is the average bit rate of a mobile radio channel with variable transmission rate. It is derived from the well-known Shannon capacity theorem with the assumption that the transmission rate is continuously adapted to the local mean CIR in such a manner that the bit error rate (BER) goes to zero asymptotically [3]. It can be defined as

$$\langle C \rangle = \int_0^R \int_0^{2\pi} \langle C(r,\theta) \rangle f_{r,\theta}(r,\theta) \, dr \, d\theta \qquad (6)$$

where $\langle C(r,\theta) \rangle$ is the site specific average bit rate of a mobile user normalised by the channel band width at a distance r and angle θ from the central BS. It is defined as

$$\langle C(r,\theta) \rangle = \int_0^\infty \log_2(1+v) f_V(v) \, dv \qquad (7)$$

$f_V(v)$ is the probability density function (pdf) of the local mean CIR (v). It can be defined as [4]

$$f_V(v) = \int_0^\infty f_{P_{od},P_{ou}}(P_{od},P_{ou}) \left| \frac{\partial(P_{od},P_{ou})}{\partial(v,w)} \right| dw \qquad (8)$$

where $w = P_{ou}$ and $\left| \dfrac{\partial(P_{od},P_{ou})}{\partial(v,w)} \right|$ is evaluated as P_{ou} [4]. $f_V(v)$ can be derived from Eq. (8) as

$$f_V(v) = \frac{1}{\sqrt{2\pi}\sigma_e v} \exp\left(-\frac{\ln^2\left(\frac{v\,\xi_u}{\xi_d}\right)}{2\sigma_e^2}\right) \qquad (9)$$

where σ_e^2 is called the effective variance and is defined as

$$\sigma_e^2 = \sigma_d^2 + \sigma_u^2 - 2\rho_{du}\sigma_d\sigma_u \qquad (10)$$

The site-specific average capacity of a channel adaptive communication system with power sums of multiple correlated lognormal signals can be written as

$$\langle C(r,\theta)\rangle = \frac{1}{\sqrt{2\pi}\sigma_e \ln(2)} \int_0^\infty \frac{\ln(1+v)}{v} \exp\left(-\frac{\ln^2\left(\frac{v\xi_u}{\xi_d}\right)}{2\sigma_e^2}\right) dv \quad (11)$$

The analytical solution of the above equation is not available. However a close approximation is possible if we assume that $\ln(1+v) \approx \ln(v)$ for a sufficiently large v given that $v \geq z$, where z is an arbitrary value. In this case the probability density function $f_V(v)$ has to be modified as [6]

$$f_V(v \mid v \geq z) = \frac{f_V(v)}{\int_z^\infty f_V(v)dv} \quad (12)$$

$\langle C(r,\theta)\rangle$ can then be simplified to

$$\langle C(r,\theta)\rangle' \approx \frac{1}{\ln(2)}\left(\ln\left(\frac{\xi_d}{\xi_u}\right) + \sqrt{\frac{2}{\pi}}\frac{\sigma_e}{e^{s^2} erfc(s)}\right) \quad (13)$$

where $s = \ln\left(z\xi_u / \xi_d\right)/\sqrt{2}\sigma_e$ and $erfc(.)$ is the complimentary error function.

The average channel capacity $\langle C_e(r,\theta)\rangle'$ in bits/sec/Hz is plotted in Fig. (1). It is an increasing function of effective shadowing variance of the desired and the sum of undesired signals. The results are somewhat optimistic but it should be noted that Eq. (13) gives the theoretical Shannon upper bound on the channel capacity of the variable rate communication system in correlated shadowed environment. In practical implementation, a discrete set of thresholds and channel noise would be the prime factors to limit the high rate transmissions.
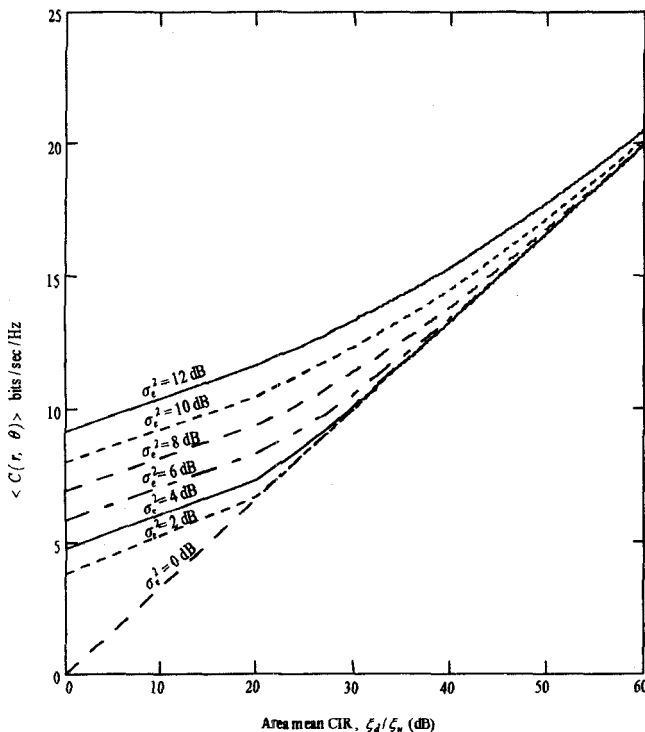

Figure 1: *Average Capacity of a mobile radio channel versus area mean CIR for different values of effective variance (z =10)*

The site specific average ASE at a distance r and angle θ from the central BS is given by

$$\langle A_e(r,\theta)\rangle \approx \frac{4}{\ln(2)\pi R^2 R_u^2}\left(\ln\left(\frac{\xi_d}{\xi_u}\right) + \sqrt{\frac{2}{\pi}}\frac{\sigma_e}{e^{s^2} erfc(s)}\right) \quad (14)$$

It can be easily recognised that the first term on the right hand side of the Eq. (14) represents the average ASE without shadowing and the second term is the addition due to correlated shadowing.

The final equation can be written as

$$\langle A_e\rangle \approx \frac{8}{\ln(2)\pi R^4 R_u^2}\int_0^R\left(\ln\left(\frac{\xi_d}{\xi_u}\right) + \sqrt{\frac{2}{\pi}}\frac{\sigma_e}{e^{s^2} erfc(s)}\right)r\,dr \quad (15)$$

Computer Simulation: It is clear from the Eq. (15) that the average ASE mainly depends on the area mean CIR (ξ_d/ξ_u) and effective variance (σ_e). The area mean CIR is a function of random locations of the MS, which makes the ASE mathematically intractable to solve. We have done a computer simulation in which the MS is randomly located following a uniform distribution as described in the section II. The area mean powers ξ_d, ξ_u and variances σ_d, σ_u are used to calculate the average ASE of a cellular system by forming an empirical average as

$$\langle A_e\rangle \approx \lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^L \langle A_e(r_l,\theta_l)\rangle \quad (16)$$

Evaluation of ξ_u and σ_u : In the previous sections we have used ξ_u and σ_u as the area mean power and standard deviation of the sum of interfering signals respectively, each having a lognormal distribution. In this paper, a method proposed by [5] is used to calculate the mean and variance of the power sums of N interfering lognormal signals each having area mean power ξ_y and shadowing standard deviation σ_y.

IV. Simulation Results

The results produced by computer simulations are plotted in Fig. 2-6. They are discussed as below.

Fig 2 shows the variation in average ASE in a shadowed environment versus normalised reuse distance for different values of shadowing variances $(\sigma_d \& \sigma_y)$. It can be seen that average ASE increases with the increase in shadowing variances of both desired and the sum of undesired signals. This is also evident from Eq. (15), which states the increase in average ASE due to the increase in effective variance. Fig. 2 also shows that the difference in average ASE becomes less significant with larger values of normalised reuse distance. The results confirm the fact that highly spectral efficient communication systems require small reuse distances.

The plots of Fig. 3 show the average ASE as a function of normalised reuse ratio for different values of correlation values between the desired and the sum of the undesired signals. It can be noted that the average ASE decreases with an increase in shadowing correlation. This change is less significant at high reuse distances. At a normalised reuse distance of 2, a

correlation between desired and the undesired signals can make a difference as large as 5 bits/sec/Hz/Km^2.

Fig. 4 shows the variation in the average ASE versus the normalised reuse distance for different correlation values between the individual interfering signals. It is evident from Fig. 4 that the average ASE slightly increases with increasing correlation between individual interferers. This result verifies that the increased correlation between many lognormal signals decreases the average mean power of their sum if they are correlated.

In Fig. 5, the effect of cell size on average ASE is shown for different values of shadowing standard deviations. Results indicate that the increase in average ASE is significant for small cell sizes as compared to big larger cells.

Finally Fig. 6 compares the average ASE versus normalised reuse distance for different values of shadowing standard deviation for two cases; a typical cellular system with 6 interferers, and a 120° sectored cell with two interferers only. Fig. 6 shows that the average ASE increases with decreasing number of interferers. This increase is more significant for smaller shadowing standard deviations and reuse distances.

V. Conclusion

In this paper, the average ASE of a variable data rate cellular communication system is evaluated in a correlated shadowed environment. The average ASE is analysed as a function of normalised reuse distance, shadowing standard deviation, correlation between desired and the sum of undesired signals, cell radius and cell sectorization.

The presence of shadowing increases the average ASE of a variable rate communication system. Densely shadowed areas with higher shadowing variance are capable of receiving higher data rates as compared to areas with less shadowing variance. The increase in average ASE is more significant at low values of normalised reuse distance than at higher ones.

It is noted that the average ASE decreases with increased correlation between desired and undesired signals. This decrease is less at larger values of reuse distance. In contrast, mutual correlation between the individual signals increases the average ASE. This increase is lower when the normalised reuse distance is increased.

When cell sizes are varied with different shadowing variances, it is noted that the increase in spectral efficiency is more pronounced with small rather than large cells.

The comparison between a 120° sectored cell and an omni directional case shows an increased average ASE for the sectored case. This increase becomes small when the shadowing variance and the normalised reuse distance is increased.

It can thus be concluded that a channel adaptive mobile cellular system is more spectrally efficient in areas which are densely populated, have higher shadowing variances, have small cell sizes and use small reuse distances.

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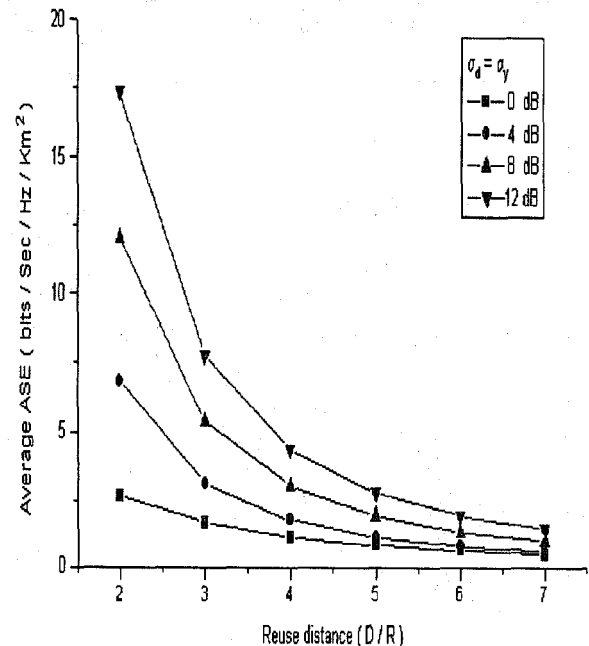
Figure 2: *Average ASE versus normalised reuse distance for different values of shadowing standard deviations. (R = 800 m; h_b = 10m; h_m = 2 m; f_c = 900 MHz; ρ_{du} = ρ_{uu} = 0.0; N = 6; a = b=2;z = 10)*
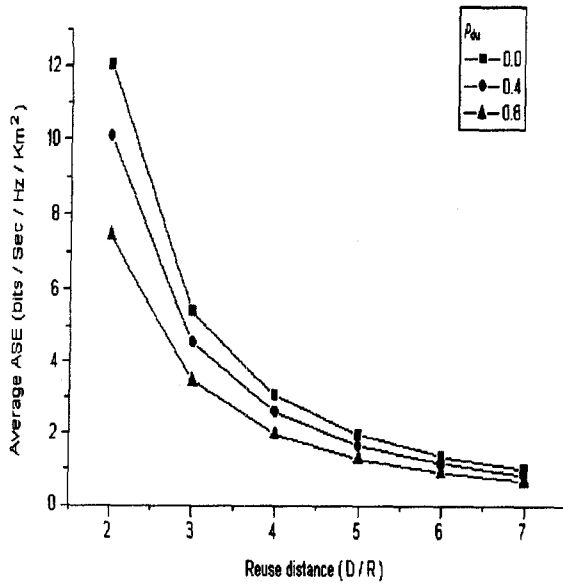
Figure 3: *Average ASE versus normalised reuse distance for different values of correlation coefficients between the desired and the sum of undesired signals. (R = 800 m; h_b = 10m; h_m = 2 m; f_c = 900 MHz; $\sigma_d = \sigma_y$ = 8 dB; ρ_{uu} = 0.0; N = 6; a = b =2; z = 10)*
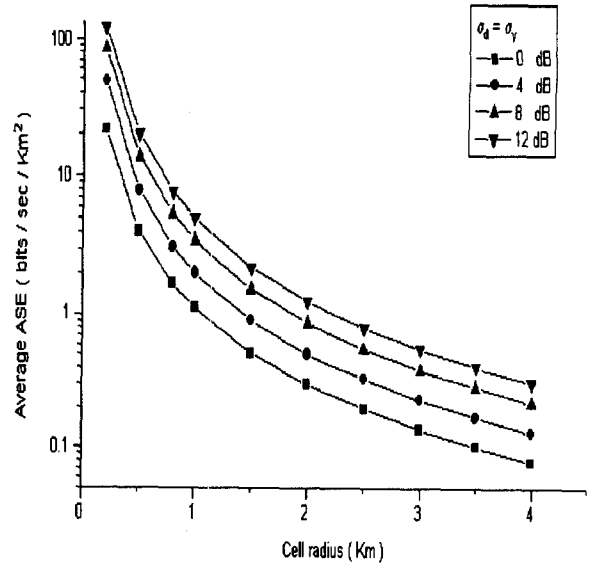

Figure 5: *Average ASE versus cell radius for different values of shadowing standard deviations. (h_b = 10m; h_m = 2 m; f_c = 900 MHz; $\rho_{du} = \rho_{yy}$ = 0.4; D/R = 3.0; N = 6; a = b =2; z = 10)*
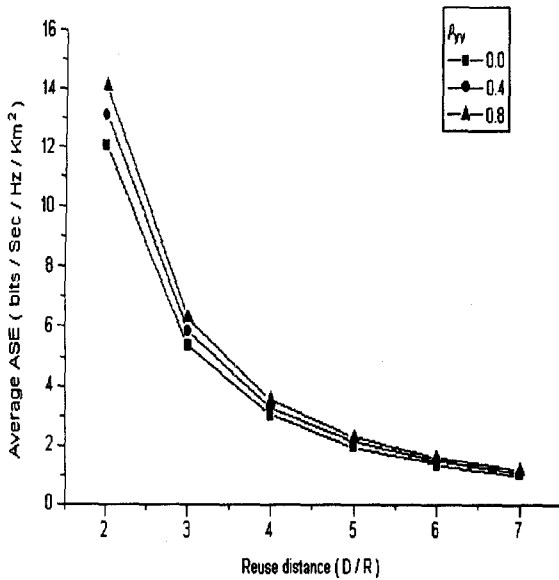

Figure 4: *Average ASE versus normalised reuse distance for different values of correlation coefficients between individual interfering signals. (R = 800 m; h_b = 10m; h_m = 2 m; f_c = 900 MHz; $\sigma_d = \sigma_y$ = 8 dB; ρ_{du} = 0.0; N = 6; a = b =2; z = 10)*
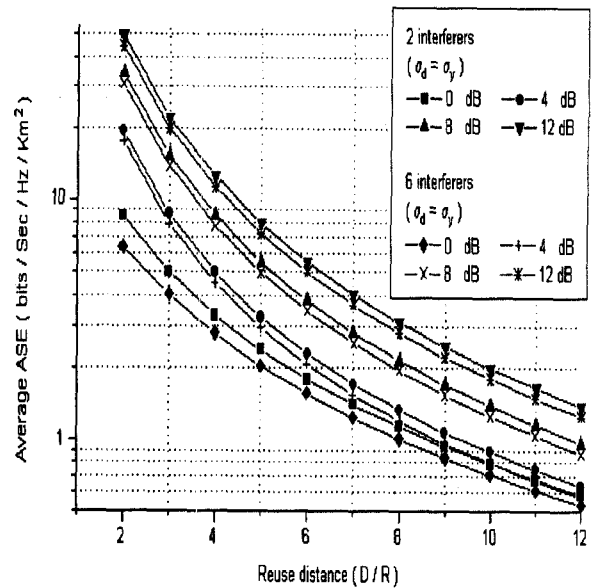

Figure 6: *Average ASE versus normalised reuse distance for different shadowing standard deviations and number of interfering BS. (R = 500 m; h_b = 10m; h_m = 2 m; f_c = 900 MHz; $\rho_{du} = \rho_{yy}$ = 0.0; N = 6; a = b =2; z = 10)*