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Per-subcarrier Antenna Selection for OFDMA-based Cognitive Radio Systems

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Talk Outline

- Motivation
- System Model
- Problem Formulation for low-complexity solutions
- Choice of objective function
- Simulation Results
- Summary

Motivation

Cognitive Radio

- Paradigm in wireless spectrum access, where secondary users (SUs) can access the spectrum of primary users (PUs).
- Interference from cognitive users, observed by primary system should be below a defined threshold.
- Efficient algorithms/systems are required that can improve the link quality of SUs while not violating constraints imposed by PUs.

Motivation

Cognitive MIMO can …

- Improve link quality of cognitive users
- Mitigate/reduce interference to primary users, e.g., through beamforming techniques

However,...

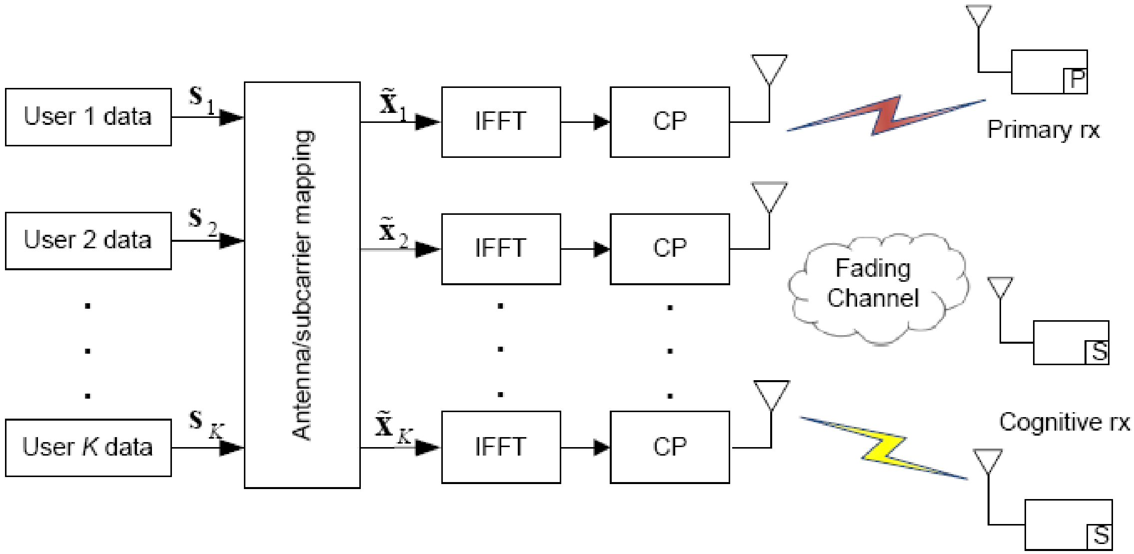
- Requires full channel knowledge of both primary and secondary links
- High hardware and computational complexity

Motivation

- **Cognitive MIMO techniques are impractical for rapidly changing environments**
- **Antenna Selection**
  - Only subset of antenna used for transmission/reception.
  - Does not require full channel state information (CSI)
  - More readily deployed and retains many of the benefits of MIMO systems
- **Per-subcarrier antenna selection can exploit frequency as well as spatial diversity**

System Model

- Downlink multiuser MISO OFDMA-based cognitive network considered
- Total interference power constraint imposed by primary system
  - *K* SUs, 1 PU
  - *N* subcarriers
  - *M* tx antennas, 1 rx



Problem Formulation

Objective: Improve secondary links quality, while limiting interference

- By performing a per-subcarrier antenna selection, i.e., every subcarrier is assigned a single transmit antenna
- Rapidly changing cognitive environments impose low-complexity algorithm

Problem Formulation

- Decision variables are integer in nature
- In general, integer programs are ***NP*-hard** problems
- Except when constraint matrix is **totally unimodular**, in which case, a linear relaxation of the problem is optimal

Linear relaxation is optimal ⇔ **Low-complexity solutions**

Problem Formulation

The concepts of *unimodularity* and *total unimodularity* are related to the determinants of the (sub-) matrices that define the polyhedron.

Total unimodularity **⇔ polyhedron is integral**

We used this theorem in our work on antenna selection.

- For total unimodularity, entries in the constraint matrix can only take values 0 or 1

Problem Formulation

- Thus, interference constraint not included to avoid increasing the complexity

$$\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{m=1}^{M}\left|\tilde{g}_{n,m}\right|^2 \rho_{k,n,m} p_s \leq \mathrm{I}_{th}$$

- where
  - $\left|\tilde{g}_{n,m}\right|^2$ is the channel gain between the secondary transmitter and primary receiver on the *n*th subcarrier from the *m*th antenna
  - p_s is the transmit power on the *n*th subcarrier
  - I_{th} is the received interference power limit
- Instead constraint included in the objective function

Problem Formulation: Constraints

- **Selection Variable**

$$\rho_{k,n,m} = \begin{cases} 1, & \text{if user } k \text{ is transmitting on the } n \text{ subcarrier from the } m\text{th antenna} \\ 0, & \text{otherwise} \end{cases}$$

- **Exclusive use of subcarrier/antenna pair to one SU**

$$\sum_{k=1}^{K}\sum_{m=1}^{M}\rho_{k,n,m} \le 1, \quad \forall n$$

- **Minimum number of subcarriers to assign to each user**

$$\sum_{n=1}^{N}\sum_{m=1}^{M}\rho_{k,n,m} \ge n_{min}, \quad \forall k$$

Problem Formulation

- Problem Formulated as

$$
\begin{aligned}
\text{maximise} \quad & f(\mathbf{H}_1, \cdots, \mathbf{H}_k, \mathbf{G}, \rho) \\
\text{subject to} \quad & \sum_{n=1}^{N} \sum_{m=1}^{M} \rho_{k,n,m} \geq n_{min}, \quad \forall k \\
& \sum_{k=1}^{K} \sum_{m=1}^{M} \rho_{k,n,m} \leq 1, \quad \forall n \\
& \rho \in \{0, 1\}
\end{aligned}
$$

- where $f\left(\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_K, \mathbf{G}, \boldsymbol{\rho}\right)$ is a function of the channel gain matrix between the SU tx –SU rx, $\mathbf{H}_k$,channel gain matrix between SU tx – PU rx, $\mathbf{G}$, and the decision variables, $\boldsymbol{\rho}$

Choice of Objective Functions

Two possibilities to trade-off interference to PU and sum channel gain of SUs

- **Ratio of channel gains**

$$f(\mathbf{H}_1,\cdots,\mathbf{H}_k,\mathbf{G},\boldsymbol{\rho})=\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{m=1}^{M}\rho_{k,n,m}\frac{|\tilde{h}_{n,m}^{k}|^2}{|\tilde{g}_{n,m}|^2}$$

- **Weighted difference of channel gains**

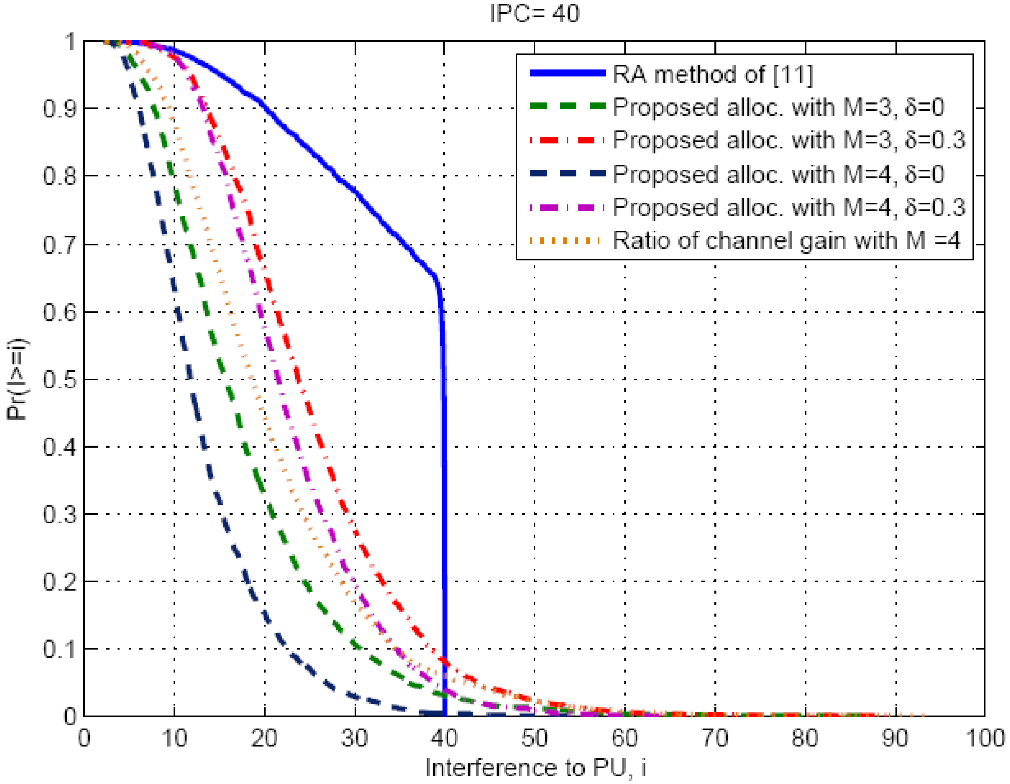
$$f(\mathbf{H}_1,\cdots,\mathbf{H}_K,\mathbf{G},\boldsymbol{\rho})=\delta\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{m=1}^{M}\rho_{k,n,m}|\tilde{h}_{n,m}^{k}|^2-(1-\delta)\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{m=1}^{M}\rho_{k,n,m}|\tilde{g}_{n,m}|^2$$

where $\left|h_{n,m}\right|^2$ is the SU-SU channel gains and δ is the weight variable

Simulation Results

Simulation Results

ccdf of interference to PU

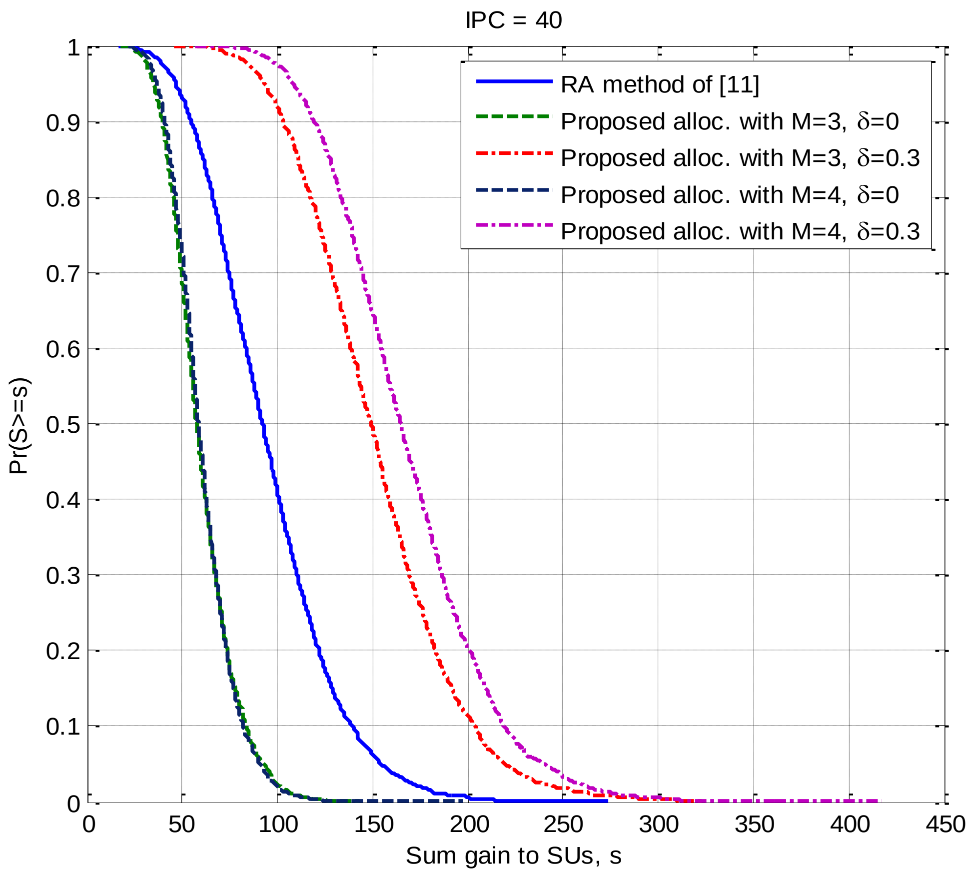


- N=128 subcarriers
- K=4 SUs
- Interference power constraint = 40 units
- **Observations**:
  - Low probability of exceeding interference constraint
  - Ratio of channel gains offers less control

Resource allocation in [11] explicitly includes IPC in formulation – *NP-hard*

Simulation Results

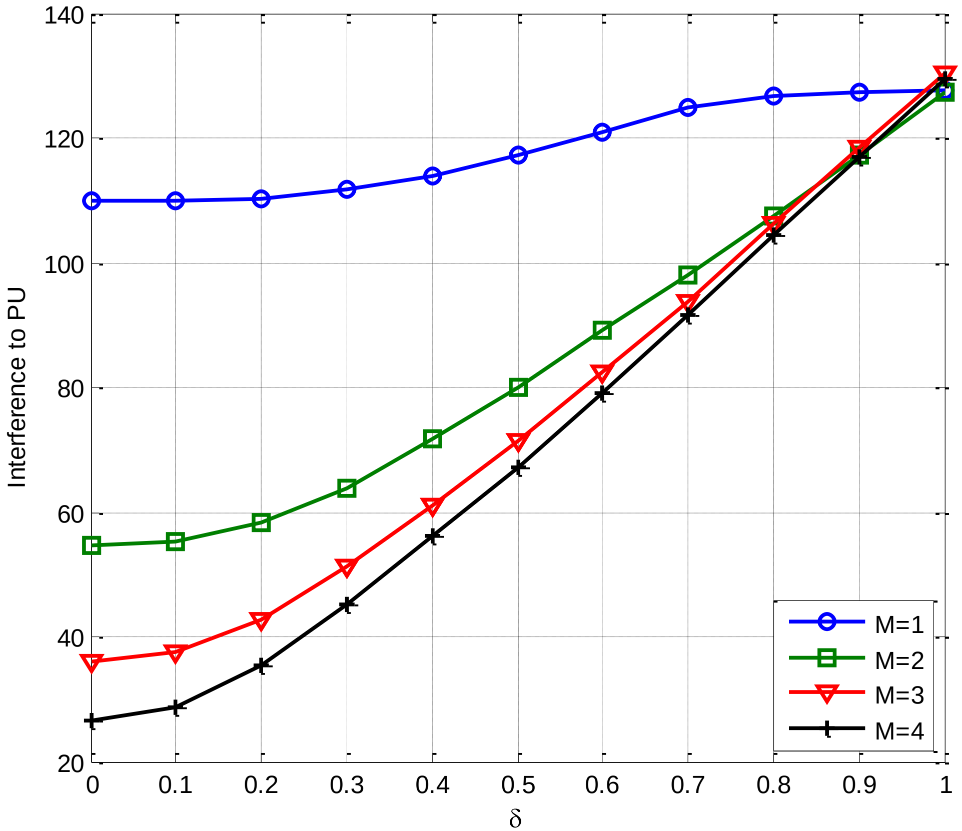
Ccdf of sum channel gains of SU



- N=128 subcarriers
- K=4 Sus
- Interference power constraint = 40 units

Simulation Results

Interference to PU for different δ



- N=128 subcarriers
- K=4 Sus
- Interference power constraint = 40 units

Summary

- Antenna selection provides a good trade-off between benefits of cognitive MIMO and hardware/computational complexity
- Proposed algorithm can be solved using linear programs, leading to low complexity solutions
- Weighted difference of channel gains provides more control through parameter δ

THANK YOU

Simulation Results

Sum Channel gains of SUs

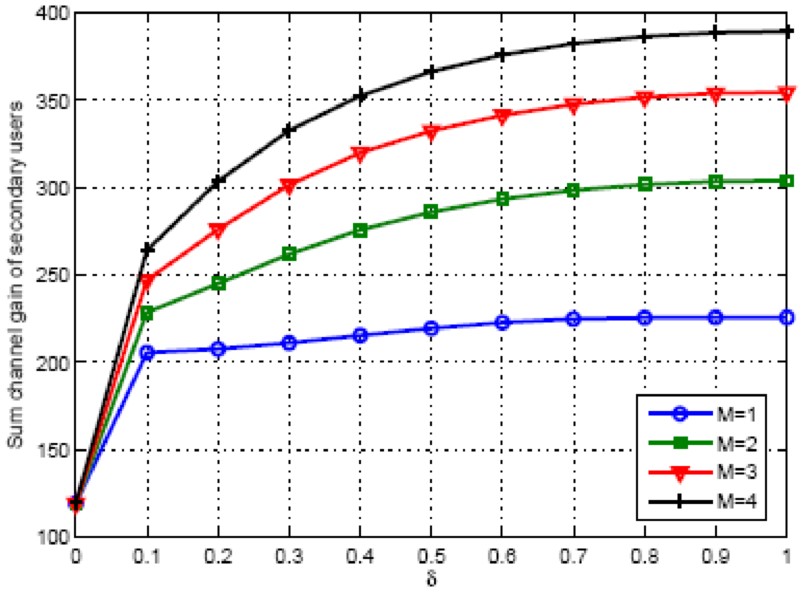
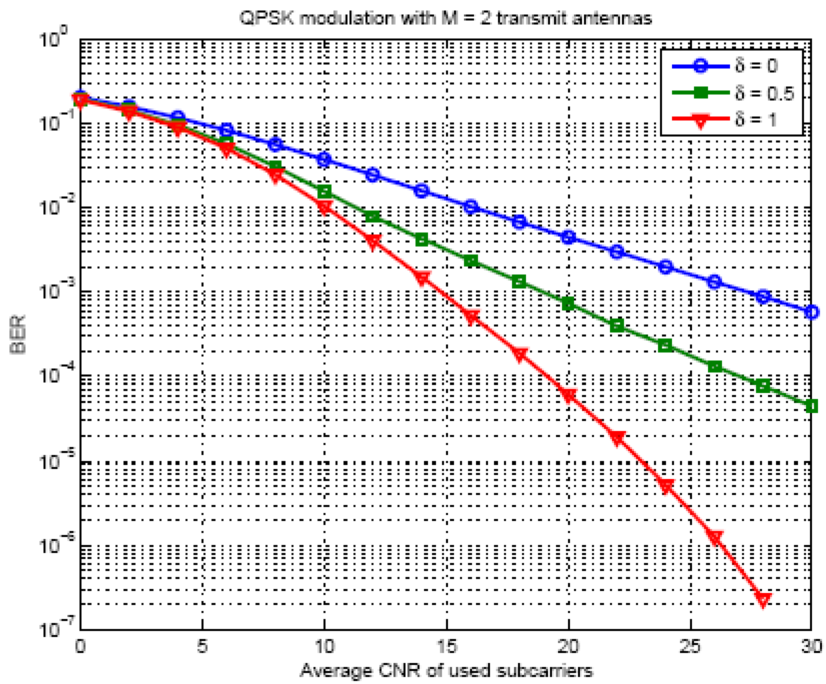


Fig. 3. Aggregate gain of SUs with $N = 128$, $K = 4$, $n_{min} = 30$ and p_s=1

- N=128 subcarriers
- K=4 SUs
- Interference power constraint = 40 units

Simulation Results

BER Analysis for Different delta



- N=128 subcarriers
- K=4 SUs
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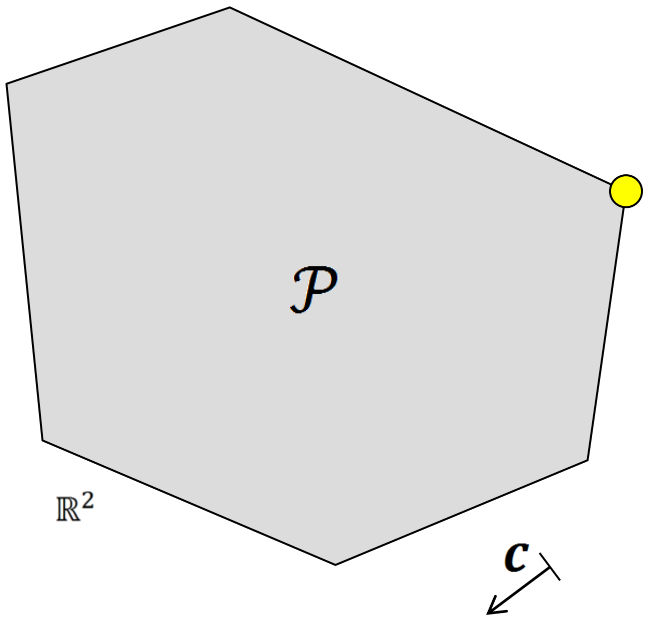
Fig. 4. BER plot for a system with 2 transmit antenna and QPSK modulation

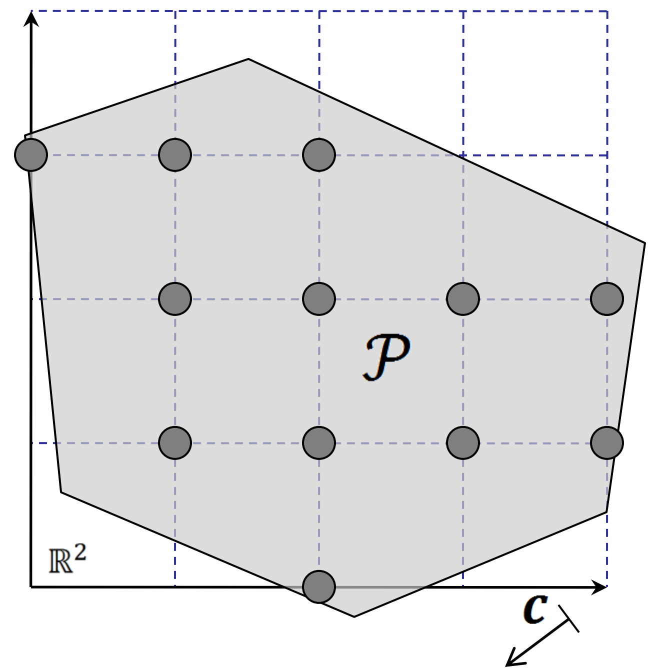
What is Integer Optimization?

Linear optimisation



$$\text{minimise} \quad \boldsymbol{c}'\boldsymbol{x}$$
$$\text{subject to} \quad \boldsymbol{x} \in \mathcal{P}$$

Integer optimisation



$$\text{minimise} \quad \boldsymbol{c}'\boldsymbol{x}$$
$$\text{subject to} \quad \boldsymbol{x} \in \mathcal{P} \cap \mathbb{Z}^2_+$$

Totally Unimodular Matrices

- Definition: *A matrix is totally unimodular if all its square submatrices have determinant +/-1 or 0.*
- Theorem: *If* C *is totally unimodular and* u *is an integer vector, the integer optimisation problem is solved by linear relaxation*
- Corollary*: The constrained antenna selection problem can be solved by linear relaxation, which means simpler solutions*