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# A Distributed Algorithm for Wireless Resource Allocation Using Coalitions and the Nash Bargaining Solution

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**Abstract**—In this paper a distributed, low-complexity, fast and fair resource allocation algorithm for a multiuser, wireless LTE OFDMA channel is proposed. Based on the game theoretic concept of the Nash Bargaining Solution and by grouping users into coalitions of size 2, a cooperative solution to the problem of subcarrier allocation is achieved. The fairness that our algorithm provides matches that offered by the widely accepted Proportional Fair (PF) scheduler. Our simulation results show that the proposed algorithm achieves a sum rate that is almost equivalent (i.e. 90%) to the sum rate achieved by the PF scheduler, while only requiring minimal exchange of information between nodes. At the same time, efficiency enhancements and its distributed nature render it fast and low-complexity enough to be implemented in a real-time wireless system.

## I. INTRODUCTION

A fairly well known principle in wireless communication systems is that it is possible to achieve a significant received energy gain in a multiuser system if channel resources are divided between different users in a dynamic and intelligent fashion. This opportunity is referred to as multiuser diversity (MUD) and an example is illustrated in [1] for the case of how it may be achieved in a frequency selective wireless communications channel. [1] also shows how multiuser diversity can be exploited to enhance system performance. Results of a numerical, information theoretic analysis of multiuser diversity showing capacity gains as a function of the number of users are given in [2]. In this paper we propose a resource allocation algorithm for a multiuser wireless channel, where the existence of independent communication links between the users and their instantaneous fluctuation both in the time and frequency domains, allows for the benefits of MUD to be harvested.

The majority of work in the existing literature makes the assumption that a master device takes responsibility for dictating the resource allocation process. This simplifies protocol issues, but the various costs associated with this approach cannot be ignored; the central controller needs information about the channel quality of all wireless devices it is responsible for allocating resources to. Furthermore, the allocation decisions have to be sent back to wireless devices, thus altogether generating an overhead that impedes system performance.

Additionally, this centralized approach does not lend itself very well to the emerging ad-hoc/mesh/distributed access systems.

The aforementioned costs suggest that there are potential benefits to be achieved if the resource allocation process takes place in a more distributed fashion. Coalition formation is a game theoretic concept that can be used to achieve this. Using this approach, entities (in this case, network devices) can cooperate with other entities on a task that is either too difficult for a single entity or, as is the case in the resource allocation process, on a task that could be performed more efficiently by several entities [3]. Coalition formation can either be centrally- or self-organized. Expensive algorithms exist for optimal solutions for the former; the latter is robust, scales well, and uses simple heuristics to form beneficial coalitions at low complexity.

In this paper we present a simple, low-complexity distributed algorithm for subcarrier allocation in an LTE wireless channel. Our approach utilizes the Nash Bargaining Solution and the concept of coalition formation to propose a fast and efficient algorithm to form self-organized coalitions.

The rest of the paper is organized as follows: section II presents related work, section III outlines our system model, section IV briefly describes the Nash Bargaining Solution and section V presents our algorithm. Finally, results from simulations are presented in section VI and conclusions are drawn in section VII.

## II. RELATED WORK

A thorough presentation of game theory application in wireless channel resource allocation can be found in [4]. [5] argues that cooperative game theory is not well suited to the distributed nature of wireless networking; something that the work presented in this paper contradicts. Nash Bargaining Solution (NBS) and coalitions are used in [6] to achieve fair resource allocation for multiuser, OFDMA-based wireless networks. A partially distributed scheme for the allocation of subcarriers is proposed, where the network's base station simply acts like a market place where the bargaining of subcarriers among the users takes place. A scheme similar

to the above but with reduced complexity, is proposed in [7]. Finally, a distributed approach for fair resource allocation in wireless networks using NBS is presented in [8], as well as in [9].

The novelty of our work lies in the fact that it considers low complexity as a key element. Most of the relevant literature in this research area discusses the resource allocation problem mainly in terms of optimal power, rate and fairness performance and largely neglects complexity issues. Our work proposes an efficient and low-complexity algorithm that is suitable for implementation in a real-time system.

### III. SYSTEM MODEL AND DESCRIPTION

We focus on the downlink scenario of the SCM LTE channel [10]. The simulated network consists of a single Base Station (BS) with users (Mobile Stations) randomly scattered around it. Table I presents the channel and system parameters in more detail. The channel quality each individual user experiences is strongly dependent on its distance from the BS and therefore users' SNR values vary accordingly. In our simulations we utilize all 1024 channel subcarriers and no subcarrier sharing between users is allowed; at any time a single subcarrier is only occupied by a single user. Finally, we assume that there exists a reliable and fast feedback channel, so that users can communicate with each other and exchange the information required for the subcarrier allocation process. This feedback channel is also used for the choice of an appropriate coalition structure to be fed back from the mobile devices to the BS. Our performance metric is the rate achievable by each user for the whole duration of the channel simulation. We calculate the theoretical rate of user  $k$  in subcarrier  $s$ , using the following equation:

$$R_{k,s} = \frac{W}{S} \times \log_2 (1 + SNR \times \|H_{k,s}\|^2), \text{ bits/s}, \quad (1)$$

where  $W$  is the channel's bandwidth,  $S$  is the number of subcarriers in the channel and  $H_{k,s}$  is the channel gain for user  $k$  in subcarrier  $s$ . Finally, the sum rate achieved by user  $k$  for the whole channel simulation is calculated by adding up the rates for all the subcarriers the user was allocated:

$$R_k = \sum_{i=1}^y \sum_{S_n} R_{k,s}, \quad (2)$$

where  $S_n$  is the vector of subcarriers allocated to user  $k$  and  $y$  is the number of simulated channel realizations (i.e. 2000).

### IV. NASH BARGAINING SOLUTION

In this section, we briefly present the basics of the Nash Bargaining Solution [11]. As stated in detail in [6], the *game* in this case is the subcarrier allocation problem, the *players* are the users participating in the wireless network and the *goal* is to maximize the chosen utility function for all users simultaneously. Similarly to the aforementioned reference and to the vast majority of the relevant literature, we choose the

TABLE I  
SYSTEM & CHANNEL PARAMETERS

Channel & System Parameters	
Carrier Frequency	2 GHz
Transmission Bandwidth	10 MHz
Time Slot / Sub-frame duration	0.5 ms
Number of subcarriers	1024
Base Station transmit power	43 dBm (20W)
Propagation Model	SCM Urban Macro
Noise Power	-104 dBm
User Equipment Noise Figure	6 dB
Cell radius	150 m
MS velocity	10 m/s

data rate (eq. 3) achieved by the user in a single channel realization to serve as the utility function:

$$U = \sum_{S_n} R_{k,s} \quad (3)$$

Therefore, in order to determine the NBS, we need to find the subcarrier allocation matrix that maximizes the product of data rates (i.e. the Nash function) for the users-members of each coalition:

$$\prod_{i=1}^n (R_i - R_{min}^i), \quad (4)$$

where  $n$  is the number of users in each coalition,  $R_i$  is the sum rate achieved by user  $i$  over the allocated subcarrier groups (i.e. subchannels) and  $R_{min}^i$  is the minimal rate requirement of user  $i$ . As described in [6], the unique Nash Bargaining Solution (i.e. the subcarrier allocation matrix in our system) that maximizes the above equation satisfies the following axioms:

- 1) *Individual Rationality*
- 2) *Feasibility*
- 3) *Pareto Optimality*
- 4) *Independence of Irrelevant Alternatives*
- 5) *Independence of Linear Transformations*
- 6) *Symmetry*

These axioms ensure that the NBS maximizes all  $R_i$  simultaneously and, at the same time, provides a fair operation point for all participating *players*.

### V. COALITION FORMATION AND SUBCARRIER ALLOCATION

#### A. Coalition Formation

Aiming to avoid the computational complexity of finding the optimal coalition size and structure, we pursue a simple approach and, similar to [6], we use a standard size of 2 for all formed coalitions. Therefore, all coalitions comprise of two members, apart from the case where the number of users is odd. In this case, we select to leave the single remaining user on its own and allocate the subcarriers that were not allocated to any coalition member to this user. The fact that all users get the same number of subcarriers (see section V-B '*Bargaining*

for Subcarriers'), guarantees that our algorithm remains fair to all users and is still able to deliver the cooperative game theory benefits.

Our approach to coalition formation is straightforward; all possible combinations of 2-sized groups among the users participating in the channel are formed and then, after the bargaining within each coalition has completed, the appropriate coalition structure (i.e. combination of coalitions) is selected and subcarriers are allocated according to the bargaining outcome (i.e the subcarrier allocation matrix that yields the highest product of data rates for the members of each coalition). In order to keep the simulation time requirements at a reasonable level we chose to use a relatively small number of users (i.e. 5) in our model. Moreover, this number was chosen to illustrate the algorithm's fair behavior when the number of users is odd and, therefore, not all users (in this case only one user) are part of a coalition.

### B. Bargaining for Subcarriers

Following the formation of all possible coalition structures, the outcome that each structure can yield needs to be determined. Towards this aim, bargaining within each coalition's members using the NBS takes place. An array of all possible subcarrier permutations for the two users comprising each coalition is generated; we examine all the different subcarrier-user permutations to determine which one generates the highest product of user rates (i.e. among the members of each coalition). These computations take place within each coalition, hence the distributed nature of the algorithm. A limited signalling between coalition members is required in order to exchange the necessary information (i.e. achievable rates for each subcarrier) used during the calculations. However, the exhaustive search for the optimal subcarrier-user permutation is simply infeasible, due to the huge complexity of this operation. Therefore, we choose to make some major modifications to this approach, so as to achieve a significantly improved efficiency for our algorithm.

1) *Subcarrier Grouping*: Allocating subcarriers one by one is a very complex process, requiring the testing of  $n^k$  subcarrier permutations, where  $n$  is the number of users in each coalition and  $k$  is the number of channel subcarriers. Additionally, most practical systems allocate resources in physical blocks that are usually larger than a single unit. Therefore, our approach is to group consecutive subcarriers into groups and allocate them as a single unit. This results in a slight loss of sum rate performance, as a degree of fine-grained control of the allocation process is lost. However, our experiments show that using 10-sized subcarrier groups is an excellent compromise between sum rate performance and algorithm efficiency, as now  $2^{102}$  instead of  $2^{1024}$  permutations need to be tested. Generally, forming subcarrier groups of size  $s$  provides a  $n^{k \times \frac{(s-1)}{s}}$  - fold increase in speed. An additional benefit is that memory requirements for the algorithm execution are significantly reduced, thus allowing it to run on less powerful devices, where memory might be limited.

TABLE II  
THE PROPOSED ALLOCATION ALGORITHM

<b>The subcarrier allocation process</b>	
1. Initiation	
- Create array of all possible coalition structures	
- Create array of all possible subcarrier group permutations for each coalition	
- Create array that holds achievable rates for all users	
2. For each coalition structure	
- For each subcarrier group permutation	
- For each coalition of the coalition structure	
- perform Nash Bargaining among the coalition members	
- after bargaining, update array that holds users' rates	
3. Choose appropriate coalition structure	
- calculate Jain's fairness index for each coalition structure	
- for PF fairness, choose coalition structure with lowest Jain's index	
4. Go to 1. and repeat allocation process as necessary	

2) *Equal number of subcarriers*: Extensive experimentation with the PF scheduler revealed that all users in the wireless channel get roughly the same number of subcarriers per channel simulation. Similarly, testing the proposed algorithm showed that the best strategy is to allocate an equal number of subcarriers to each user in order to maintain a level of fairness similar to proportional fairness. The difference in individual user rate performance stems from the fact that subcarriers differ in 'quality' for each user. Thus, by allocating equal number of subcarrier groups to each user we make our algorithm much faster, as the number of subcarrier permutations that need to be tested is significantly reduced. By making this choice the number of permutations is further reduced by a factor of 5, for the case of a network with 5 users, 1024 subcarriers and 10-sized subcarrier groups.

3) *No minimum rate requirement*: The NBS solution requires that eq. 4 for the members of each coalition is maximized. However, depending on the minimal rate requirement of each user, the aforementioned equation might not converge to a solution and thus render the allocation process incomplete. To avoid this complication, and based on the fact that the allocation of equal number of subcarriers to each user guarantees a minimum level of service, we set  $R_{min}^i$  equal to zero. This choice also improves the execution speed of the algorithm and provides [6] a proportionally fair behavior to our allocation process.

### C. Choice of Coalition structure

The last stage of the allocation process is to select the 'winning' coalition structure. Our simulations indicate that the proposed algorithm generates almost identical fairness and sum rates across all coalition structures and, therefore, the choice of a specific coalition structure only marginally changes the outcome of the allocation process. This result offers the opportunity to further reduce the allocation algorithm's

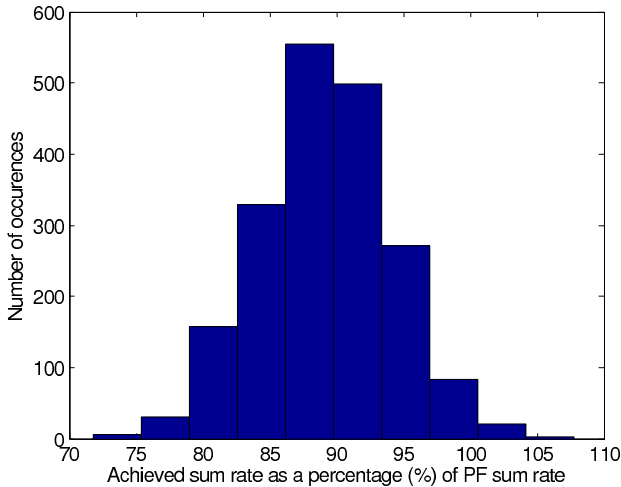


Fig. 1. Sum rate comparison

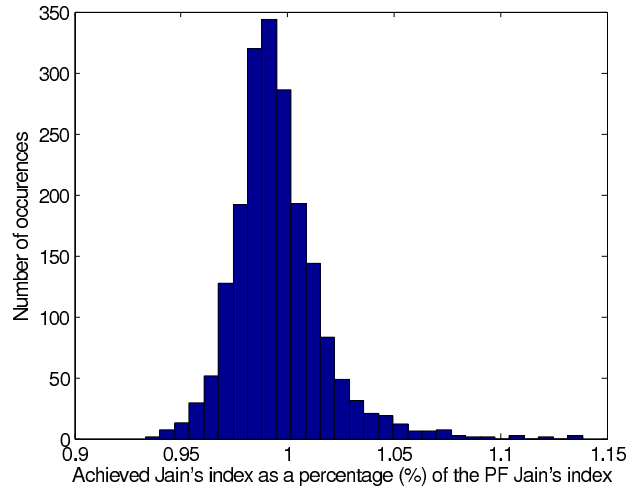


Fig. 2. Jain's index comparison

complexity, by sacrificing only a minimal percentage of the sum rate, and will be explored in future work.

#### D. Efficiency Enhancements

1) *Permutations sampling*: Despite eliminating the subcarrier group permutations that do not provide an equal number of subcarrier groups to all users, there is still a very large number of permutations to be tested. Based on the observation that no significant differences from permutation to permutation exist, we only test a sample of them. Sampling at a ‘rate’ of  $1/m$  (i.e. test 1 out of  $m$  permutations) increases the algorithm execution speed by a factor of  $m$ . According to our simulations, for values of  $m$  up to  $10^3$  the loss of sum rate performance does not exceed 7% and for  $m$  up to  $10^4$  loss does not exceed 15%.

2) *Realizations step*: By repeating the allocation process less often than every channel realization we are able to yield a great increase in algorithm efficiency, maintain fairness and only reduce the sum rate by an insignificant amount, as proved during our simulations. Repetition of the allocation process every  $l$  realizations provides an  $l$ -fold increase in algorithm speed. For example, setting  $l$  equal to 10 produces a 10-fold increase in speed, while only a 4% loss in sum rate is incurred for the mobility scenario simulated. Further enhancement is possible, but repeating the process even less often can delay users’ access to channel resources and thus degrade user performance in time-sensitive applications.

## VI. RESULTS

The results of the proposed algorithm are compared against results generated by the widely accepted Proportional Fair scheduler [12]. The time window we used for the PF scheduler is 500-subcarriers long. According to our experiments, this is a value that provides a good balance between sum rate and short-term fairness among users. Simulation was performed for 2000 different channels (i.e. different users’ locations), with each channel being simulated for 2000 realizations (i.e. uncorrelated

instances of small scale fading effects). The results presented in this section are averaged over the 2000 different channels.

#### A. Sum rate

We use the sum rate achieved by the users in the channel as a basic performance metric. The proposed algorithm yields a sum rate that is equivalent to 90% of the PF sum rate on average, with this number varying from about 75% to 105%. The reason for this variance is the fact that users’ long-term channel qualities vary significantly (i.e. due to the random location of the users around the Base Station) between the different channels that are simulated. This is an excellent result, given the speed improvement over the PF scheduler that our algorithm achieves, and is illustrated in Fig. 1, where a histogram of the sum rate as a percentage of the PF sum rate is presented.

#### B. Fairness

Our observations during testing indicated that the fairness achieved is almost identical to proportional fairness. To verify this, we calculate Jain’s fairness index (eq. 5) of both schedulers and compare them:

$$Fairness = \frac{(\sum_{i=1}^n R_i)^2}{(n \times \sum_{i=1}^n R_i^2)} \quad (5)$$

$R_i$  represents the data rate achieved by user  $i$  over the whole simulation (i.e. 2000 realizations or 2 seconds) of the channel and  $n$  is the number of users in the channel. The value for *Fairness* ranges between 0 and 1, with a result equal to 1 indicating that all users achieve the same sum data rate. Our results show that Jain’s index achieved by the proposed algorithm (when the coalition structure with the lowest Jain’s index value is selected) is consistently between 90% and 105% of Jain’s index of the PF scheduler, with the average value being 99.5%. The histogram presented in Fig. 2 illustrates this result.

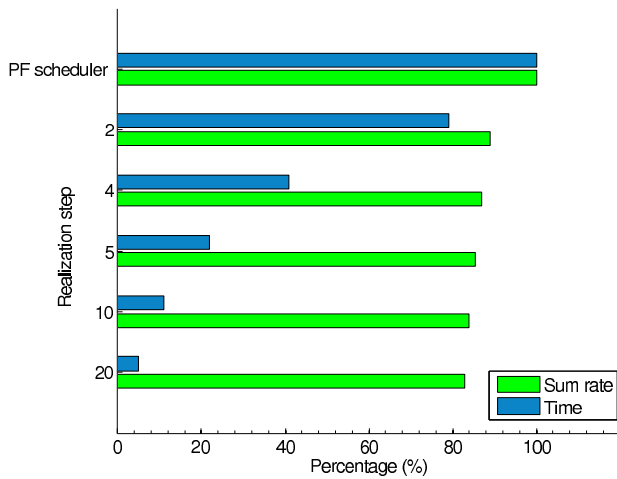


Fig. 3. Rate and Time comparison

### C. Algorithm Efficiency

The efficiency of the proposed algorithm is a very important aspect of its performance. There exist optimal game theoretic solutions to the subcarrier allocation problem, that are however impractical for use in a real-time system. Our algorithm achieves a very good balance between sum rate, fairness and speed. As shown in Fig. 3, where sum rate and execution time for different values of the realization step of our algorithm (i.e. how often the allocation process is repeated) are illustrated, the algorithm manages to retain a great amount of the PF scheduler's sum rate and also execute at a significantly greater speed. These results benefit from the distributed nature of our algorithm, as calculations are off-loaded from the central controller to the mobile devices and thus each device has an easier task to perform. Even though a central controller might have greater computational capabilities than a mobile device, the constantly and rapidly advancing computational power of mobile devices (e.g. smartphones, laptops) guarantees that the performance of the distributed algorithm will not suffer significantly. Moreover, it is possible to make the algorithm even faster by reducing the frequency at which the allocation process is repeated. However, this could degrade performance in terms of delay, as users might have to wait longer to get access to the channel's resources.

## VII. CONCLUSIONS & FUTURE WORK

A low-complexity, fast and efficient distributed algorithm for resource allocation was proposed in this paper. The algorithm is based on the game theoretic concepts of the Nash Bargaining Solution and coalition formation, where wireless users form coalitions and bargain with each other for access to the channel's resources (i.e. subcarriers groups). Additionally, a very significant aspect of the proposed algorithm is the reduced time requirement when compared to the PF scheduler. By retaining 85% of the PF scheduler's sum rate and achieving almost identical fairness, our algorithm is almost 10 times faster. In order for these results to be achieved, a fairly limited signalling between the wireless users is necessary. The amount

of signalling required is less than in a centralized allocation scenario and can be further reduced if necessary by repeating the allocation process less often. While sum rate will not suffer significantly, this could mean that users might have to wait longer for access to the channel's resources. Therefore, an approach like that would be suitable for less delay-sensitive applications.

In the future we plan to further investigate the performance of the proposed algorithm, especially in terms of the trade-off between sum rate and delay. Additionally, the trade-off between algorithm complexity, sum rate and execution speed will attract a large part of our research efforts. A detailed investigation of the overheads involved and the corresponding comparison with the overhead requirements of the PF scheduler will also follow. In addition, we intend to improve speed even more, mainly by smartly reducing the number of different coalition structures our algorithm has to test before allocating the subcarriers. The algorithm's performance with different coalition sizes and a larger number of participating users will also be investigated. Finally, it is our intention to further explore the protocol issues regarding the use of the proposed algorithm and also present its distributed nature and specifics of its operation in more detail.

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