Railton, C. J. (2008). Modelling of a microstrip patch antenna using a hybrid FDTD/PEEC method. In IET 7th International Conference on Computation in Electromagnetics, 2008 (CEM 2008). (pp. 156 - 157). Institution of Engineering and Technology (IET). 10.1049/cp:20080246

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Hybridisation of the FDTD technique

by:

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Outline

1. What is the problem? - Structures with both fine detail and large electrical size

2. For example, predicting PCB behaviour can be done by:
   1. Partial Element Equivalent Circuits (PEEC)
   2. Finite Difference Time Domain (FDTD)

3. The best of both worlds - hybridisation

4. Results

5. Onwards

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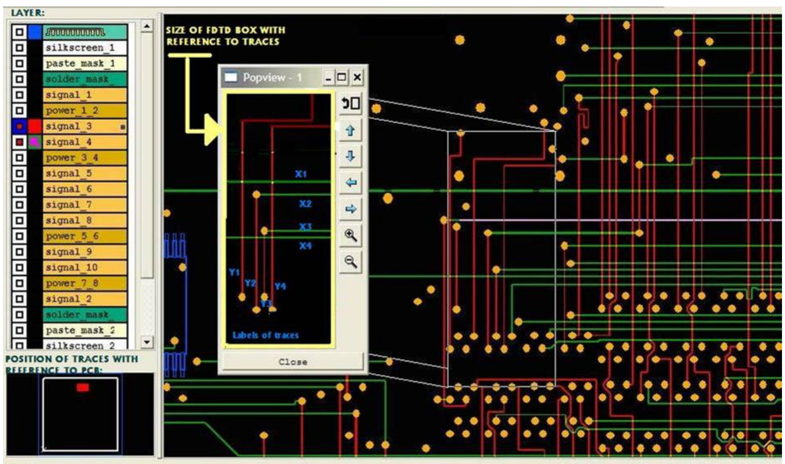
3

An example of the problem?

- Printed Circuits and becoming more complex, dense and fast.
- They operate in complex environments.
- Issues such as signal integrity, interference and crosstalk have become key parts of circuit and system design.
- CAD tools have to keep up with improvements in manufacturing capability.

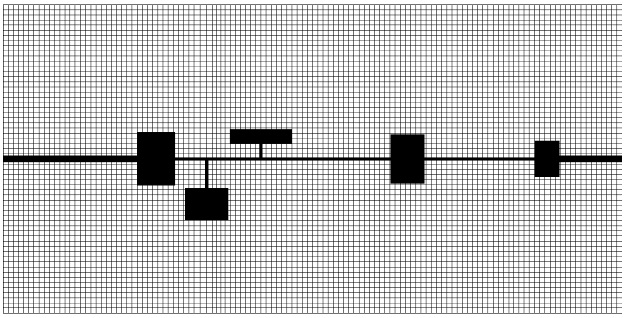
4

A typical modern PCB



Can use standard FDTD but…

- If the structure contains fine detail or boundaries which do not conform to the grid, a very fine mesh is needed.



Strip width ~ λ/100!

Can use PEEC but…

- If the structure size is a significant fraction of a wavelength then retardation effects must be included.

- This seriously complicates the method and can lead to late time instability which is challenging to get rid of.

7

… there is a better way

- Extend existing "thin wire formalisms" to allow for general wire and microstrip circuits.
- Let the formalisms take care of the detail, let the FDTD algorithm take care of the long range interactions.
- The final algorithm can be viewed as a hybrid between FDTD and PEEC

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8

What are thin wire formalisms?

- In standard FDTD, metals are treated by enforcing field boundary conditions. The currents are not explicitly calculated
- With thin wire formalisms, the currents in the wire are explicitly treated using extra differential equations
- This allows the singularities of the fields to be accounted for and allows many wires to be placed within a single FDTD cell.

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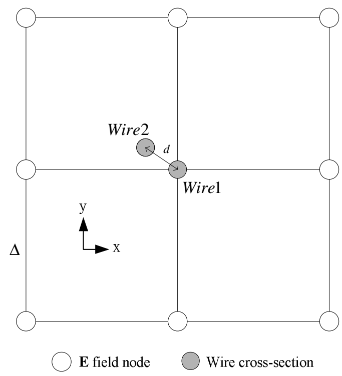
Wire bundles embedded within an FDTD mesh

Consider a bundle of wires in the FDTD mesh. Two wires of the bundle are shown here.



Wire bundles embedded within an FDTD mesh

The E field, tangential to the wires, at a point, *r*, can be expressed in terms of the potentials as follows:

$$E_z(r) = -\frac{\partial}{\partial z}\phi(r) - \frac{\partial}{\partial t}A_z(r)$$

Where, the potentials round an *infinite* bundle may be approximated as:

$$A_z(r) = \sum_j \frac{\mu I_j}{2\pi}\ln\left(\left|r - r_j\right|\right) \qquad \phi(r) = \sum_j \frac{\lambda_j}{2\pi\varepsilon}\ln\left(\left|r - r_j\right|\right)$$

11

Wire bundles embedded within an FDTD mesh

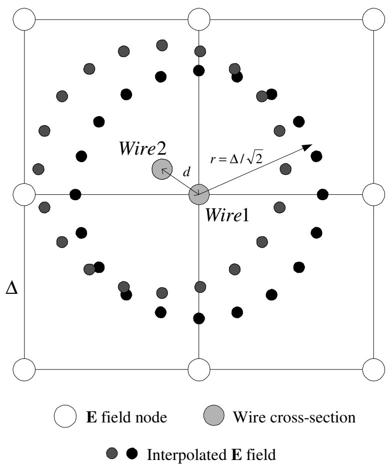
Therefore the E field, tangential to the wires, at a point, *r*, can be expressed in terms of the E field on the i^{th} wire as follows:

$$E_z(r) = \frac{\partial}{\partial z}\left(\phi(r_i) - \phi(r)\right) + \frac{\partial}{\partial t}\left(A_z(r_i) - A_z(r)\right) + E_z(r_i)$$

12

Wire bundles embedded within an FDTD mesh

Following Ledfelt[1] we choose a set of weighting functions, $w_i(r)$, to be non-zero on a circular shell centred on the i^{th} wire and zero elsewhere.



[1] Ledfelt, G: "A stable subcell model for arbitrarily oriented thin wires for the FDTD method", International Journal of Numerical Modelling : Electronic Networks, Devices and Fields, 2002(15), pp. 503-515

13

Wire bundles embedded within an FDTD mesh

Now multiply each side of the equation by each of the weighting functions, $w_i(r)$ in turn and integrate over all space. This leads to a set of equations, one for each wire:

$$\frac{\partial \mathbf{I}}{\partial t} = \mathbf{L}^{-1}\mathbf{X} - c^2 \frac{\partial \boldsymbol{\lambda}}{\partial z}$$

where:

$$L_{ij} = \langle A_j, w_i \rangle - A_j(d_{ij}) \qquad X_i = \langle E, w_i \rangle + V_{si}/\Delta$$

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V_{si} is a voltage source if present

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14

These can be discretised in space using central differences

$$\frac{\partial \mathbf{I}}{\partial t} = -c^2 \mathbf{D}\boldsymbol{\lambda} + \mathbf{L}^{-1}\mathbf{X} \qquad \frac{\partial \boldsymbol{\lambda}}{\partial t} = -\mathbf{D}^T \mathbf{I}^n$$

Where, for a wire: $\mathbf{D} = \frac{1}{\Delta}\mathbf{C} = \frac{1}{\Delta}\begin{pmatrix} 1 & -1 & 0 & \vdots & 0 \\ 0 & 1 & -1 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \\ \cdots & \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & \vdots & 1 \end{pmatrix}$

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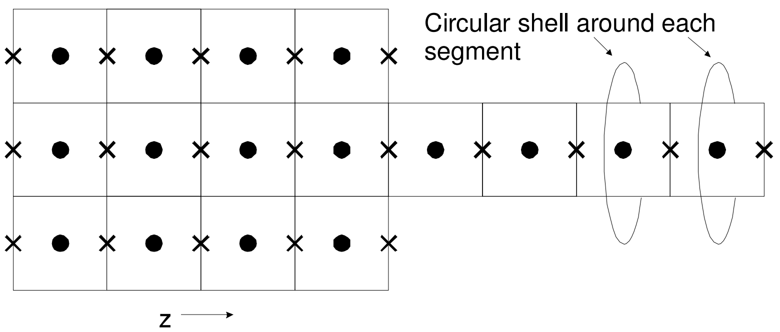
15

Circuits embedded in the FDTD mesh



The approach used in the thin wire formalism can be readily extended to deal with this situation

16

The “in-cell” mutual inductances

For example, the “in-cell” mutual inductance between two segments in the x-z plane and orientated in the z direction can be calculated by direct integration like this:

$$A_1(x,y,z) = \frac{\mu}{4\pi} \iint_{segment1} \frac{1}{\sqrt{(x-x')^2 + (y-y_1)^2 + (z-z')^2}} dx'dz'$$

$$L_{21} = \frac{1}{2\pi r_o} \oint_{circle2} A_1(x_2 + r_o\cos(\phi), y_2 + r_o\sin(\phi), z_2)d\phi - A_1(x_2, y_2, z_2)$$

17

The wire update equations

The “in-cell” mutual capacitances can be calculated similarly and the update equations are given by:

$$\frac{\partial \mathbf{I}}{\partial t} = \mathbf{L}^{-1}\left(\mathbf{X} - \mathbf{CP}\boldsymbol{\lambda}\right) \qquad \frac{\partial \boldsymbol{\lambda}}{\partial t} = \mathbf{C}^{T}\mathbf{I}^{n}$$

where, *C*, is the connection matrix and
P is the inverse capacitance matrix

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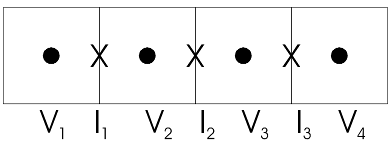
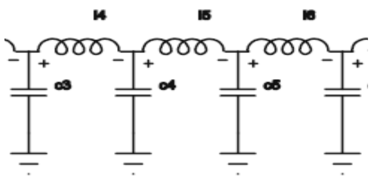
18

Comparison with PEEC methods





In the PEEC method the self and mutual inductances between segments are used in an equivalent circuit

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19

The PEEC mutual inductances

The mutual inductance between two segments in the x-z plane can be calculated like this:

$$L_{21} = \iint_{segment\,2} A_1(x_2, y_2, z_2)\,dxdz$$

compared with the “in-cell” mutual inductance:

$$L_{21} = \frac{1}{2\pi r_o} \oint_{circle\,2} A_1(x_2 + r_o\cos(\phi), y_2 + r_o\sin(\phi), z_2)\,d\phi - A_1(x_2, y_2, z_2)$$

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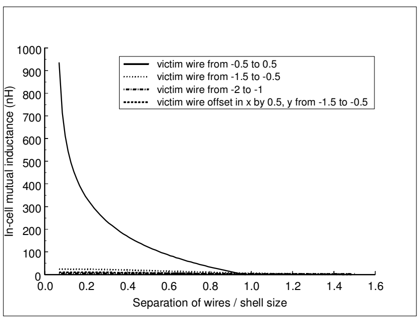
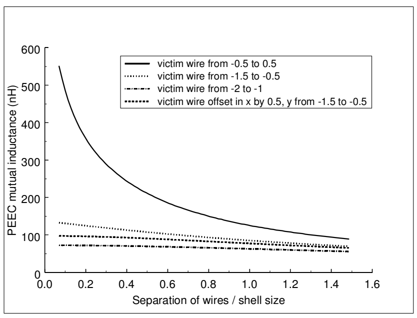
20

Comparison of mutual coupling

Hybrid

PEEC





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21

Comparison of mutual coupling

- In PEEC, it has been shown [1] that mutual coupling effects are significant at distances of up to 5λ.
- Retardation effects seriously complicate the method [2].
- In the hybrid approach mutual coupling is very low at distances greater than the size of the FDTD cell
- Long range interactions are dealt with by FDTD

1. M. Verbeek, "Partial Element Equivalent Circuit (PEEC) models for on-chip passives and interconnects", RANA report 02-27, 2002
2. A. Ruehli and E. Chiprout, "The importance of retardation in PEEC models for electrical interconnect and package (EIP) applications", Electrical Performance of Electronic Packaging, 1995, pp 232-234

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22

Example results

1. Microstrip low pass filter
2. Microstrip band pass filter

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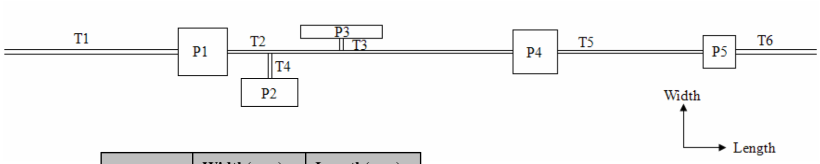
23

The low pass filter geometry



	Width(mm)	Length(mm)
Track 1	0.58	13.00
Track 2	0.26	20.95
Track 3	1.39	0.26
Track 4	2.72	0.26
Track 5	0.26	10.7
Track 6	0.58	6.12
Patch 1	5.11	3.65
Patch 2	3.09	4.14
Patch 3	1.33	5.94
Patch 4	4.72	3.23
Patch 5	3.46	2.35

Substrate height 0.635mm

Box size: 30x60x6mm

Substrate ε_r = 10.5

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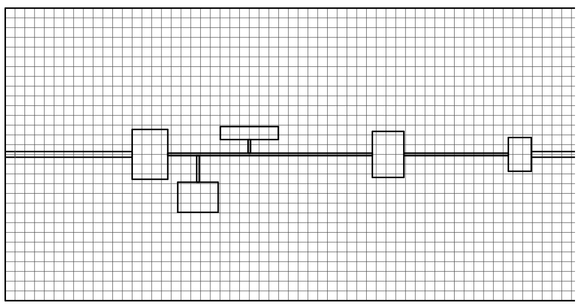
24

The low pass filter and the FDTD mesh



Segment size: 1mm;

FDTD mesh size: 1mm*0.635mm*1mm (x*y*z);

Width of excitation pulse: 200 picoseconds;

Number of iterations: 8200

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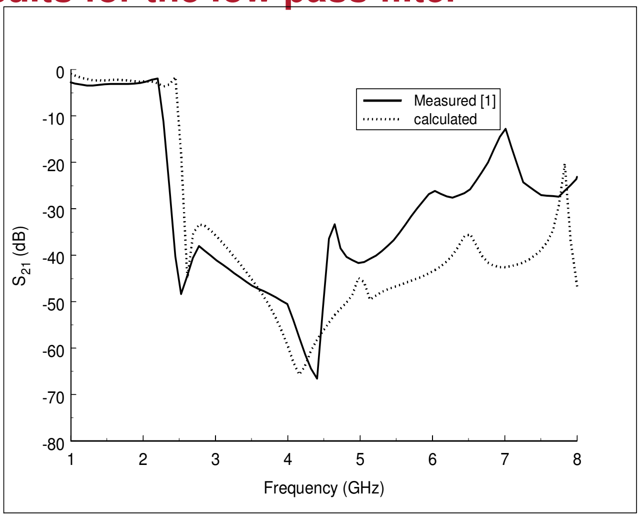
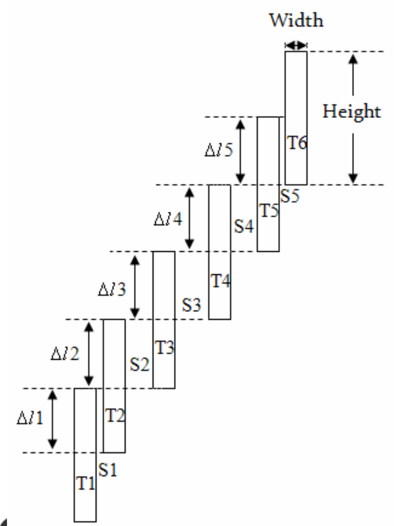
Results for the low pass filter

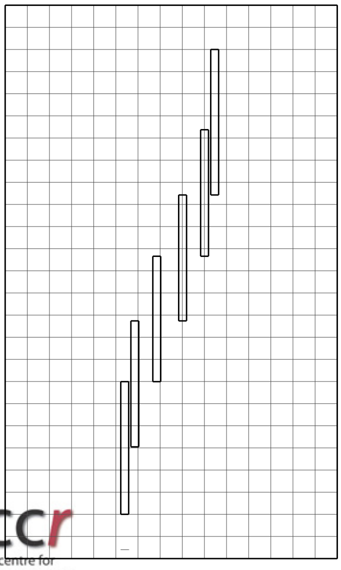


The bandpass filter geometry



Box	Length: 25.0mm	Height 4.0mm
Substrate Relative Permittivity: 9.9		Height 0.4mm
Substrate	Length: 23.0mm	Width: 15.0mm
Track 1	Length: 5.994mm	Width: 0.356mm
Track 2	Length: 5.690mm	
Track 3	Length: 5.665mm	
Track 4	Length: 5.689mm	
Track 5	Length: 5.715mm	
Track 6	Length: 6.573mm	
Δ/1: 0.2946mm	S1: 0.101mm	
Δ/2: 0.2744mm	S2: 0.635mm	
Δ/3: 0.2921mm	S3: 0.812mm	
Δ/4: 0.2768mm	S4: 0.635mm	
Δ/5: 0.2947mm	S5: 0.101mm	

The bandpass filter and the FDTD mesh



Segment size: 0.45mm;

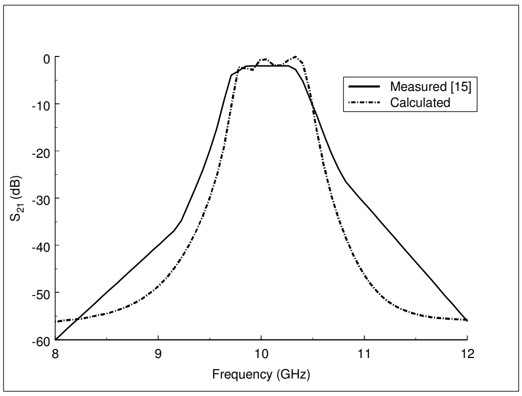
FDTD mesh size:
1mm*0.4mm*1mm (x*y*z);

Width of excitation pulse: 20 picoseconds;

Number of iterations: 8100.

Results using the hybrid method

shell radius = 3mm

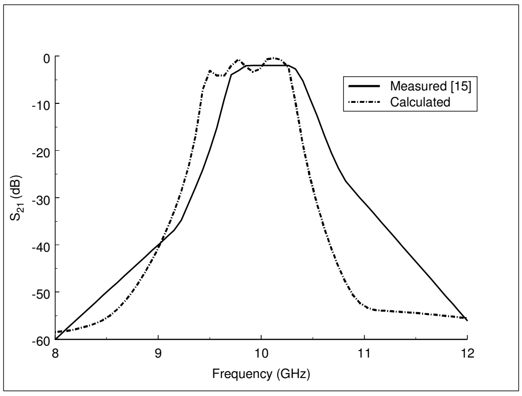


[1] A fast integral equation technique for shielded planar circuits defined on nonuniform meshes Eleftheriades, G.V.; Mosig, J.R.; Guglielmi, M.; Microwave Theory and Techniques, IEEE Transactions on Volume 44, Issue 12, Part 1, Dec. 1996 Page(s):2293 - 2296

29

Results using the hybrid method

shell radius = 1.5mm



[1] A fast integral equation technique for shielded planar circuits defined on nonuniform meshes Eleftheriades, G.V.; Mosig, J.R.; Guglielmi, M.; Microwave Theory and Techniques, IEEE Transactions on Volume 44, Issue 12, Part 1, Dec. 1996 Page(s):2293 - 2296

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30

Conclusions

- It has been shown that an extended wire formalism allows treatment of complex circuits within the FDTD mesh.
- Because the mutual inductance becomes very small when the wire separation is equal to the circle radius, long range effects are not a problem. Retardation is not necessary to be included.
- FDTD takes account of long range interactions
  PEEC takes account of the fine detail.
- Can be extended to include active components and networks.

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