Linear Equalizers for Quasi-Synchronous Block Spread CDMA Systems

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IEEE Globecom Conference
December 2009, Hawaii, USA

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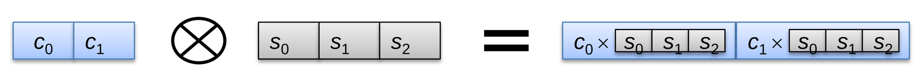
Overview

Block spread CDMA (BS-CDMA) [1] is a multi-access technique whereby user-specific precoding along with orthogonal spreading codes are used to achieve multiuser interference (MUI) free reception when signals of all users arrive at the base station synchronously.

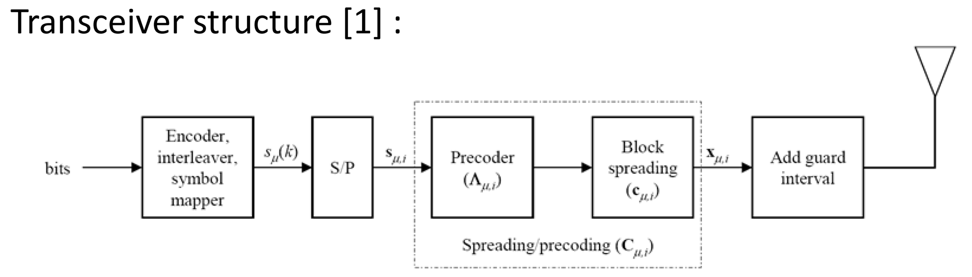
In practice however, imperfect synchronization destroys the orthogonality among users and MUI occurs. This paper investigates the design of linear frequency domain equalizers to reduce the MUI for a quasi-synchronous BS-CDMA system. An optimal frequency domain linear minimum-mean squared error (LMMSE) equalizer is derived. Further simplification leads to a novel sub-optimal equalizer with reduced computational complexity. It is shown through simulation that the proposed equalizers effectively suppress the error floor due to quasi-synchronous reception when channel coding is used.

BS-CDMA System Model

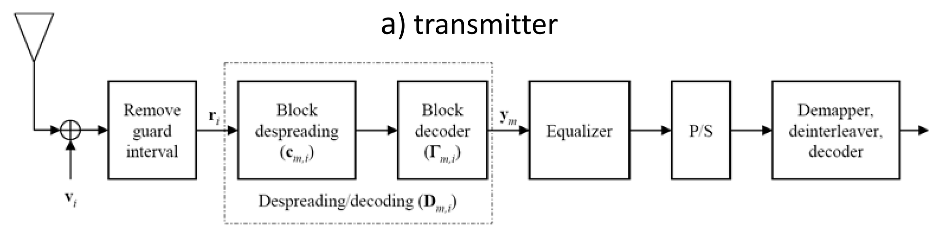
BS-CDMA spreads a block of symbols with user specific spreading codes.



Transceiver structure [1] :



a) transmitter

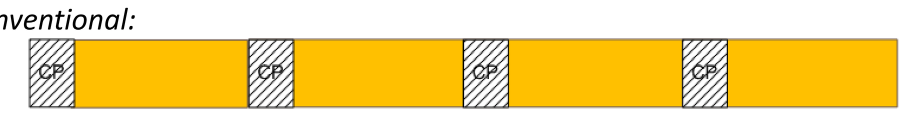


b) receiver

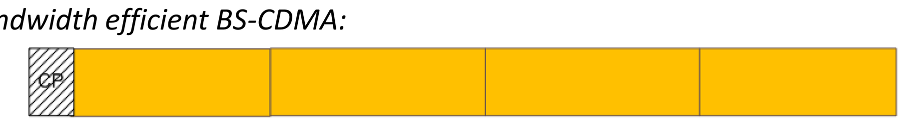
Features of synchronous BS-CDMA

- Achieves MUI-free reception by using mutually shift-orthogonal spreading codes such as the discrete Fourier transform (DFT) codes.
- Achieves higher bandwidth efficiency by inserting cyclic prefix *after* block spreading

Conventional:



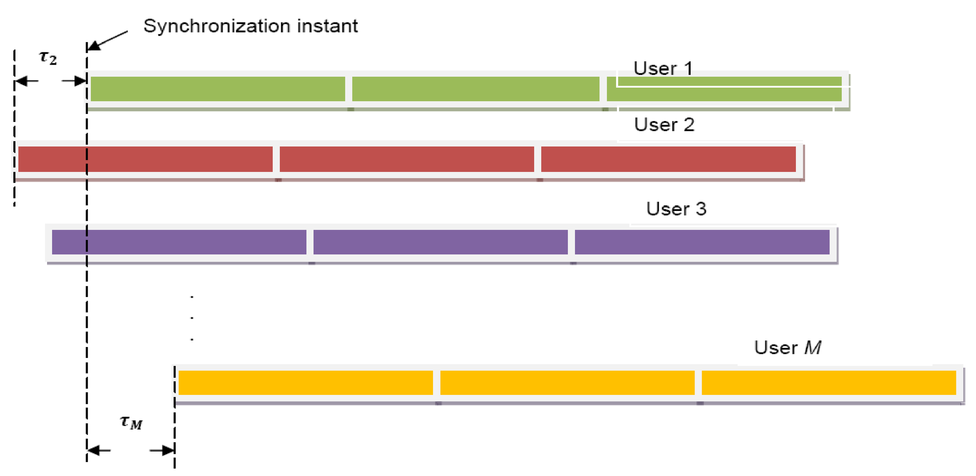
Bandwidth efficient BS-CDMA:



Quasi-synchronous reception

Quasi-synchronous reception assumption:

The timing control at the base station is good enough such that signals from different users are approximately synchronized to within a few chips.



Interference users can be categorised into:

- Users whose signals arrive τ_a chips *after* the synchronization instant
- Users whose signals arrive τ_b chips *before* the synchronization instant

Received signal after despreading

$$\mathbf{z}_{m,i} = \underbrace{\mathbf{S}_{m,i}}_{\text{desired signal}} + \underbrace{\mathbf{D}_m^H \sum_{a=1}^{N_a}\left(-\boldsymbol{\Delta}_a^U \mathbf{C}_{L_{CP}}^d \mathbf{x}_{a,i} + \boldsymbol{\Delta}_a^U \mathbf{x}_{a,i-1}\right)}_{\text{interference signals arriving after synchronization instant}} + \underbrace{\mathbf{D}_m^H \sum_{b=1}^{N_b}\left(\boldsymbol{\Delta}_b^L \mathbf{x}_{b,i} - \boldsymbol{\Delta}_b^L \mathbf{C}_{L_{CP}}^d \mathbf{x}_{b,i+1}\right)}_{\text{interference signals arriving before synchronization instant}} + \underbrace{\mathbf{v}_i}_{\text{noise}}$$

where

- $\mathbf{C}_{L_{CP}}^d$ is a circulant matrix obtained by circularly shifting the identity matrix down by L_{CP}
- $\boldsymbol{\Delta}_a^U$ and $\boldsymbol{\Delta}_b^L$ are upper and lower triangular matrices with structures

The triangular areas contain the only non-zero entries in the matrices.

- N_a and N_b are the number of users whose signals arrive after and before the synchronization instant

Important observations

- Interfering signals *arriving after* the synchronization instant are caused by the *last* $L - L_{CP} + \tau_a - 1$ channel taps.
- Interfering signals *arriving before* the synchronization instant are caused by the *first* τ_b channel taps.

For practical channels such as the exponential decay channel, the energy of the first few taps dominates. Users whose signals arrive earlier than the synchronization instant contribute majority of the interference power.

Novel Frequency Domain Equalizers

Optimal LMMSE Equalizer

- *Objective:* to minimise the mean square error (MSE) between the transmitted and recovered signal, given by

$$\grave{o} = \mathrm{Tr}\left\{\mathrm{E}\left[\left(\mathbf{s}_{m,i} - \mathbf{w}_{m,i}\right)\left(\mathbf{s}_{m,i} - \mathbf{w}_{m,i}\right)^H\right]\right\}$$

where $\mathbf{w}_{m,i}$ is the data block after equalization.

- Obtained by solving $\partial\grave{o} / \partial \mathbf{G}_{opt}^* = 0$

Calculation of the optimal LMMSE equalizer requires matrix inverse -- **high computational complexity!**

Suboptimal Equalizer

- *Objective:* To reduce the computational complexity of the optimal LMMSE equalizer
- Obtained by taking expectation over the channel taps as well
- Can be computed recursively by using the matrix inverse lemma

Matrix inverse lemma:

If a matrix is defined as: $\mathbf{C} = \mathbf{A} + \mathbf{x}\mathbf{y}^H$

Then its inverse can be computed as: $\mathbf{C}^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1}\mathbf{x}\mathbf{y}^H\mathbf{A}^{-1}}{1 + \mathbf{y}^H\mathbf{A}^{-1}\mathbf{x}}$

$$\mathbf{G}_{subopt}^* = M\sigma_{m,i}^2 \boldsymbol{\Xi}_m^H \left[\boldsymbol{\Pi} + \sum_{k=1}^{K} \alpha_k \mathbf{f}_k \mathbf{f}_k^H\right]^{-1}$$

M	The number of users
$\sigma_{m,i}^2$	Signal energy
$\boldsymbol{\Xi}_m, \boldsymbol{\Pi}$	$P \times P$ Diagonal matrices determined by the channel impulse response
α_k	Random variables determined by channel statistics
$\mathbf{f}_k$	kth column of the DFT matrix

Recursive computation of the matrix inverse:

Initialization: $\mathbf{A} = \boldsymbol{\Pi}$

For $k = 1{:}K$

- $\mathbf{C} = \mathbf{A} + \alpha_k \mathbf{f}_k \mathbf{f}_k^H$
- $\mathbf{C}^{-1} = \mathbf{A}^{-1} - \alpha_k \frac{\mathbf{A}^{-1}\mathbf{f}_k\mathbf{f}_k^H\mathbf{A}^{-1}}{1 + \mathbf{f}_k^H\mathbf{A}^{-1}\mathbf{f}_k}$
- $\mathbf{A} = \mathbf{C}^{-1}$

End For

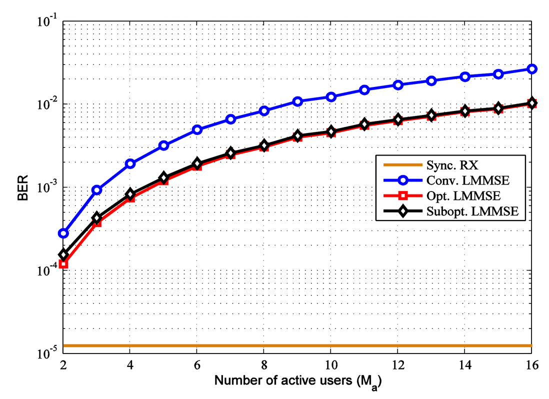
Complexity decreases from $O(P^3)$ to $O(P^2)$

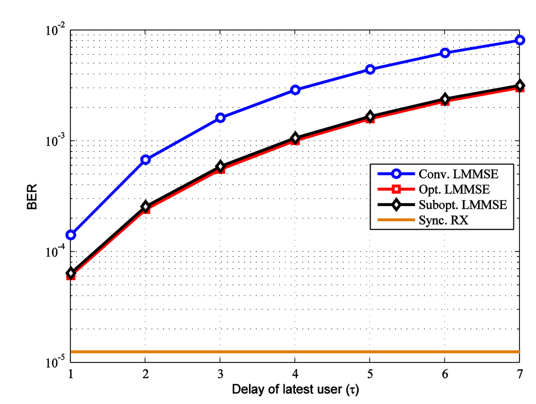
Simulation Results

Simulation parameters:

- QPSK modulation
- Exponential decaying Rayleigh fading channel
- Block length of 16
- Channel length of 9, and cyclic prefix length of 8
- Systems with M_a active users
- Signals of the only asynchronous user arrive later than the sync. instant, with a delay of τ

Effects of system parameters on the performance of BS-CDMA:



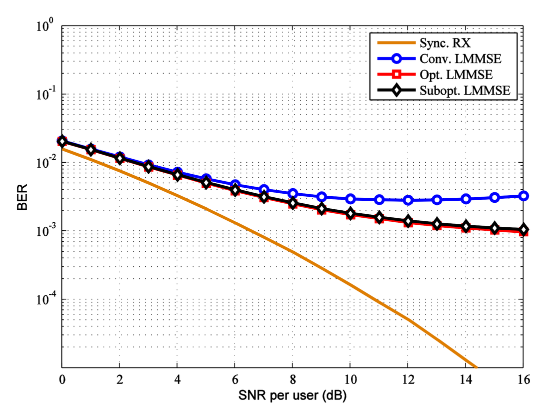


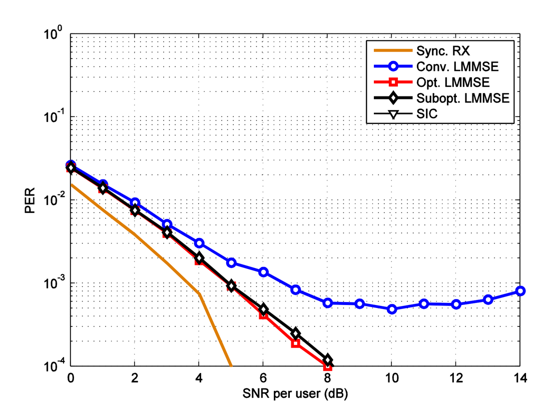
Observations:

- Systems using the proposed optimal and sub-optimal LMMSE equalizers achieve very close performance
- Compared to the BER of a system using the conventional LMMSE equalizer, systems using the proposed equalizers provide improved performance
- Performance of systems with quasi-synchronous reception degrades with longer delays or larger number of users

Performance for uncoded and coded BS-CDMA:

- 8 active users
- The only asynchronous user has a delay of 7
- Convolutional codes with constraint length 7 and code rate ½ for the coded systems





Observations:

- The proposed sub-optimal equalizer achieves a close performance to that of the optimal equalizer, both outperform systems using the conventional LMMSE equalizer
- Performance degradation compared to the ideal system with synchronous reception can be mitigated by using effective channel coding

References

[1]. J.P. Coon, "Precoded block-spread CDMA with maximum user support and frequency domain equalization," in *Proc. IEEE Inter. Conf. On Comm. (ICC), Glasgow 2007* .