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10.1109/DYSPAN.2008.85

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Noncooperative Equilibrium Solutions for Spectrum Access in Distributed Cognitive Radio Networks

(Short Paper)

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Abstract—This paper considers the problem of channel selection and dynamic spectrum access in distributed cognitive radio networks. The ability of a cognitive radio to adaptively switch between channels offers tremendous scope to optimize performance. In this paper, the dynamic spectrum access in a distributed network is modeled as a noncooperative game and the equilibrium solutions are obtained through a bimatrix game. The cost term of the utility function and the several possible definitions of "price" and how they characterize the equilibrium solutions provides a new perspective on the analysis. In distributed cognitive radio networks, the secondary users are vulnerable to several unexpected events such as primary user arrival or a deep fade or sudden increase in interference which could potentially disrupt or disconnect the transmission link. In such cases, any strategic decision or information that could lead to uninterrupted channel access and facilitate maintaining links could be modeled as a Stackelberg game. Performance characteristics for both the leader and follower nodes for the defined utility functions are given.

I. Introduction

The underutilization of the limited spectrum has sparked the need for flexible spectrum policies and dynamic spectrum access [1]. Cognitive radios are autonomous radios that learn about their environment and adaptively optimize their performance by modifying their transmission parameters, including switching to a different channel. Dynamic Spectrum Access (DSA) is a new paradigm whereby a cognitive radio device opportunistically accesses the unutilized or under-utilized spectrum bands. The ability of a cognitive radio to adaptively switch between channels: *spectrum mobility* - offers tremendous scope to optimize performance. The dynamic spectrum access is challenging in a distributed type network, particularly when the devices lack cooperation. Game theory is a mathematical framework that provides a natural platform to study the effects of players' decision strategies and equilibrium solutions in a competitive environment with limited resource constraints. In game theoretical analysis, the optimizing parameter and the definition of utility function characterizes the resulting equilibrium solution(s), provided that they exist.

Spectrum resource management and dynamic spectrum access have attracted significant research work recently. In [2] Zhao *et al.* studied dynamic spectrum access based on partially observable Markov decision processes and proposed a decentralized cognitive MAC protocol. The optimal spectrum allocation problem is analyzed through a variant of graph coloring problem in [3] which described approximation algorithms for centralized and distributed spectrum allocations. Game theory has also been extensively used in modeling resource allocation in cellular networks and recently in the context of cognitive radio networks due to its effectiveness in modeling dynamic strategic decisions. However, most of the work has been in analyzing power control or spectrum pricing/auctioning. In [5]-[7] the power control is modeled as a noncooperative game; they study the existence and convergence properties of equilibrium solutions. In [7] Nie *et al.* proposed a game theoretic distributed adaptive channel allocation scheme for cognitive radios and formulated to capture selfish and cooperative behaviors of the players. Noncooperative channel allocation and load-balancing algorithms are considered in [8] and spectrum utilization maximization in [9]. In [10] Bloom *et al.* proposed a master-slave approach of updating transmission powers and frequencies and modeled as a Stackelberg game. The channel selection in a distributed noncooperative type network is less explored and the discrete nature of the problem makes it relatively difficult to model.

In this paper, the ability of a distributed cognitive radio to select a channel satisfying its minimum requirement (e.g. data rate or SINR) and the methodology of dynamically accessing available spectrum is considered. The dynamic spectrum access in a distributed network is modeled as a noncooperative game and the equilibrium solutions are obtained through a bimatrix game. The proposed utility functions are based on the signal to interference plus noise ratio (SINR) and the associated cost of accessing a channel.

The remainder of the paper is organized as follows: Section II describes the system model and assumptions. Section III introduces the utility functions and formulates the problem as a noncooperative game. Section IV shows the existence of an equilibrium solution and discusses its analysis. Section V outlines some of the unexpected events and models the channel access as a Stackelberg game. Section VI provides some simulation results, with conclusion and future works presented in Section VII.
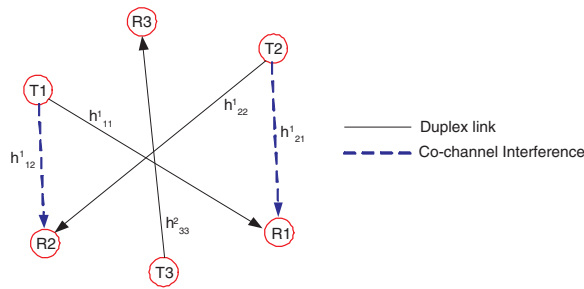
Fig. 1. Wireless Network with 3 user-pairs: User-pairs 1 and 2 in channel 1 and user-pair 3 in channel 2; Subscript denotes user index and superscript denotes channel index.

II. SYSTEM MODEL

Let $\mathcal{N} = \{1, 2, \ldots, N\}$ be the set of cognitive radio user-pairs distributed randomly in the network. The terms "user" and "pair" are used interchangeably and user i means transmitter i and receiver i. Let $\mathcal{M} = \{1, 2, \ldots, M\}$ be the set of channels available for dynamic spectrum access. The number of users in a channel at any point in time is given by b^k, $k \in \mathcal{M}$. The channel selected by a user i is denoted by $\varphi(i) \in \mathcal{M}$. The SINR of a user i on selecting a channel $\varphi(i)$ is given by

$$\gamma_i = \gamma_i^{\varphi(i)} = \frac{h_{ii}^{\varphi(i)} p_i}{\sum_{\forall j \in b^{\varphi(i)}, j \neq i} h_{ji}^{\varphi(i)} p_j + N_0} \tag{1}$$

where $h_{ii}^{\varphi(i)}$ is the channel gain of the selected channel $\varphi(i)$, p_i is transmission power of user i, N_0 is the background noise power at the receiver which is assumed to be same for all the users. The channel gain is defined as $h_{ii}^{\varphi(i)} = d_{ii}^{-4} \alpha^{\varphi(i)}$, where d_{ii} is the Euclidean distance between the transmitter i and receiver i, with path loss exponent 4 and $\alpha^{\varphi(i)}$'s are independent, unit mean random variables that model frequency selective fading across the channels.

It is assumed that each user has a maximum transmit power constraint, $p_i \leq \bar{p}$, to ensure fairness among users in the shared channel. It is also assumed that there is a constraint on the total power in each channel, $\sum_{i=1}^{b^{\varphi(i)}} p_i \leq \bar{P}$ i.e. the total power of all users in channel $\varphi(i)$ does not exceed $\bar{P}$. It is assumed that the adjacent channel interference (ACI) is negligibly small or zero and only co-channel interference (CCI) is considered in the analysis. Fig 1 shows a model network with 3 user-pairs and illustrates the co-channel interference.

III. UTILITY FUNCTIONS

The channel selection and dynamic spectrum access is modeled as a noncooperative game. Let S_i denote the set of strategies associated with player i. In this case, the players' strategies are the choices of a transmitting channel, $s_i = 1, 2, \ldots, M$. The utility function U_i for user i is a function of its SINR, and through the SINR a function of its strategy s_i, and strategies of all other users, s_{-i}; to explicitly show this dependence, we introduce the notation $\tilde{U}_i(s_i, s_{-i})$ to denote the utility function of user i as a function of the strategies.

Now, if s_i^* is a Nash equilibrium solution, then for all i and all s_i,

$$\tilde{U}_i(s_i^*, s_{-i}^*) \geq \tilde{U}_i(s_i, s_{-i}^*) \tag{2}$$

A. Objective and utility function

The objective for each user i is to maximize γ_i. In terms of the utility function of user i, this is formulated as the maximization problem:

$$\max \quad U_i(\gamma_i^{\varphi(i)}) \tag{3}$$

$$\text{subject to} \quad p_i \leq \bar{p}$$

The channel selection for user i is then considered as the integer-maximization of its utility functions, with $p_i = \bar{p}$ and $p_j, j \neq i$ fixed

$$\varphi(i) = \arg \max_{\varphi(i) \in \mathcal{M}} U_i \left(\frac{h_{ii}^{\varphi(i)} \bar{p}}{\sum_{j \neq i} h_{ij}^{\varphi(i)} p_j + N_0} \right) \tag{4}$$

The power control scheme for wireless networks has been extensively studied. Here we assume that the dynamic range $[0 \leq p_i \leq \bar{p}]$ is limited and hence $p_i = \bar{p}$. We therefore look into the method of selecting the "best channel". From (4) this could be broken into a problem of finding a better channel in terms of high channel gain $h_{ii}^{\varphi(i)}$ and finding a channel with least interference component $\sum_{j \neq i} h_{ji}^{\varphi(i)} p_j$. However, the overall ratio SINR determines the nature of the channel quality.

To obtain explicit results, we pick U_i as a logarithmic plus a linear one:

$$U_i(\gamma_i^{\varphi(i)}) = a_i \log(1 + \gamma_i^{\varphi(i)}) - b_i \lambda^{\varphi(i)} \gamma_i^{\varphi(i)} \tag{5}$$

where a_i and b_i are the user preference parameters, and $\lambda^{\varphi(i)}$ is the unit price set for channel $\varphi(i)$. Thus, the term $\lambda^{\varphi(i)} \gamma_i^{\varphi(i)}$ denotes the cost for the desired SINR. The log utility function approximates the Shannon capacity or the maximum rate achievable by user i in the selected channel $\varphi(i)$. The objective of maximizing γ_i and the greedy approach of finding the best channel could be neutralized by the 'cost' function (second term in the utility) which is a function of SINR.

IV. EQUILIBRIUM SOLUTIONS

A. Existence of pure strategy Nash Equilibrium

Proposition 1: *Nash equilibrium exists in pure strategies for the utility function defined in (5).*

Proof: To show the existence of a pure strategy Nash equilibrium, a simple case of a 2-user and 2-channel bimatrix game is investigated, before developing a general theory for N players and M channels. Let us consider a simple bimatrix game comprised of two (2 x 2)-dimensional matrices, $A = \{a_{ij}\}$ and $B = \{b_{ij}\}$, with each pair of entries (a_{ij}, b_{ij}) denoting the outcome of the game. If player $P1$ adopts the strategy (in this case, channel selection) "row i" and player $P2$ adopts the strategy "column j", then a_{ij} (respectively, b_{ij}) denotes the payoff for player $P1$ (respectively, player $P2$). If U_1 and U_2 are user utility matrices then,

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$$U_1 = \begin{array}{|c|c|} \hline \log(1+\frac{h_{11}^1 p_1}{h_{21}^1 p_2+n_0})-\lambda^1 \frac{h_{11}^1 p_1}{h_{21}^1 p_2+n_0} & \log(1+\frac{h_{11}^1 p_1}{n_0})-\lambda^1 \frac{h_{11}^1 p_1}{n_0} \\ \hline \log(1+\frac{h_{11}^1 p_1}{n_0})-\lambda^2 \frac{h_{11}^2 p_1}{n_0} & \log(1+\frac{h_{11}^2 p_1}{h_{21}^2 p_2+n_0})-\lambda^2 \frac{h_{11}^2 p_1}{h_{21}^2 p_2+n_0} \\ \hline \end{array}$$

$$U_2 = \begin{array}{|c|c|} \hline \log(1+\frac{h_{22}^2 p_2}{h_{12}^1 p_1+n_0})-\lambda^1 \frac{h_{22}^1 p_2}{h_{12}^1 p_1+n_0} & \log(1+\frac{h_{22}^2 p_2}{n_0})-\lambda^2 \frac{h_{22}^2 p_2}{n_0} \\ \hline \log(1+\frac{h_{22}^2 p_2}{n_0})-\lambda^1 \frac{h_{22}^1 p_2}{n_0} & \log(1+\frac{h_{22}^2 p_2}{h_{12}^2 p_1+n_0})-\lambda^2 \frac{h_{22}^2 p_2}{h_{12}^2 p_1+n_0} \\ \hline \end{array}$$

From the above matrices, the Nash equilibrium (NE) are the points where both users share the same channel and have their minimum SINR requirements satisfied. It is clear that since the above indices in U_1 and U_2 involve multiple parameters, the NE is parameter dependent. If $\lambda^1 < \lambda^2 \frac{\gamma^2}{\gamma^1}$ then (row 1, column 1) is the Nash equilibrium. Similarly, if $\lambda^2 < \lambda^1 \frac{\gamma^1}{\gamma^2}$ then (row 2, column 2) is the Nash equilibrium. If NE is denoted by $\varphi^* = (I, J)$, then user 1 selects channel I and user 2 selects channel J. If φ^* is the Nash equilibrium channel selection, then

$$\varphi^* = \begin{cases} (1,1) & \text{if } \lambda^1 < \lambda^2 \frac{\gamma^2}{\gamma^1} \\ (2,2) & \text{if } \lambda^2 < \lambda^1 \frac{\gamma^1}{\gamma^2} \end{cases}$$

The study is extended for a 3-user x 2-channel case. The entries of matrix A corresponds to possible outcomes of all 3 players if player P3's strategy is fixed at channel 1, and the matrix B provides possible outcomes if P3's strategy is fixed at channel 2.

$$A = \begin{array}{|c|c|} \hline U_1(\gamma_1^1), U_2(\gamma_2^1), U_3(\gamma_3^1) & U_1(\gamma_1^1), U_2(\gamma_2^2), U_3(\gamma_3^1) \\ \hline U_1(\gamma_1^2), U_2(\gamma_2^1), U_3(\gamma_3^1) & U_1(\gamma_1^2), U_2(\gamma_2^2), U_3(\gamma_3^1) \\ \hline \end{array}$$

$$B = \begin{array}{|c|c|} \hline U_1(\gamma_1^1), U_2(\gamma_2^1), U_3(\gamma_3^2) & U_1(\gamma_1^1), U_2(\gamma_2^2), U_3(\gamma_3^2) \\ \hline U_1(\gamma_1^2), U_2(\gamma_2^1), U_3(\gamma_3^2) & U_1(\gamma_1^2), U_2(\gamma_2^2), U_3(\gamma_3^2) \\ \hline \end{array}$$

If $\frac{h_{ii}^{\varphi(i)} p_i}{\sum_{j \neq i} h_{ji}^{\varphi(i)} p_j + N_0} \geq \gamma_{i,min}$, then the Nash equilibria channel selection is given by

$$\varphi^* = \begin{cases} (1,1,2), (1,2,1), (2,1,1) \text{ and } (1,1,1) & \text{if } \lambda^1 < \lambda^2 \frac{\gamma^2}{\gamma^1} \\ (2,2,1), (2,1,2), (1,2,2) \text{ and } (2,2,2) & \text{if } \lambda^2 < \lambda^1 \frac{\gamma^1}{\gamma^2} \end{cases}$$

On the other hand, if $\frac{h_{ii}^{\varphi(i)} p_i}{\sum_{j=1, j \neq i}^{3} h_{ji}^{\varphi(i)} p_j + N_0} < \gamma_{i,min}$ and if $\frac{h_{ii}^{\varphi(i)} p_i}{\sum_{j=1, j \neq i}^{2} h_{ji}^{\varphi(i)} p_j + N_0} \geq \gamma_{i,min}$, i.e. $b^1 = b^2 = 2$, then

$$\varphi^* = \begin{cases} (1,1,2), (1,2,1), (2,1,1) & \text{if } \lambda^1 < \lambda^2 \frac{\gamma^2}{\gamma^1} \\ (2,2,1), (2,1,2), (1,2,2) & \text{if } \lambda^2 < \lambda^1 \frac{\gamma^1}{\gamma^2} \end{cases}$$

Generalizing for N users and M channels,

Let $\varphi(i) = \{k_i; k_i \in \mathcal{M}, i \in \mathcal{N}\}$ be the channel selected by user i. Let $\frac{h_{ii}^{k_i} p_i}{\sum_{\forall j \in B^{k_i}} h_{ji}^{k_i} p_j + N_0} \geq \gamma_{i,min}$ where B^{k_i} is the set of users in channel k_i for which the inequality holds for user i, i.e. the set of users sharing the channel and satisfying user i's criterion. Let b^{k_i} be the number of users in set B^{k_i}.

The channels are arranged according to their cost, say, $\lambda^{k_1} < \lambda^{k_2} < \lambda^{k_3} < \dots \lambda^{k_m}, \forall k \in \mathcal{M}$. If there exists $\lambda^{k_1} < \lambda^{k_2} \frac{\gamma^{k_2}}{\gamma^{k_1}} < \lambda^{k_3} \frac{\gamma^{k_3}}{\gamma^{k_2}} < \dots \lambda^{k_m} \frac{\gamma^{k_m}}{\gamma^{k_{m-1}}}$ then channel k_1

could take maximum of b^{k_1} users and channel k_2 takes b^{k_2} user and so on. The Nash equilibrium is then given by

$$\varphi* = \{(k_1)_1, \dots, (k_1)_{b^{k_1}}, (k_2)_1, \dots, (k_2)_{b^{k_2}}, \dots k_x\}_N \quad (6)$$

and the repeated permutations of this combination. The number of such equilibrium combinations is given by $\frac{m!}{b^{k_1}! b^{k_2}! \dots b_x!}$

For example, let $N = 4$, $M = 3$. If $\lambda^2 < \lambda^3 < \lambda^1$ and if $b^2 = b^3 = 2$, then the channel selection is given by $\varphi* = \{2, 2, 3, 3\}$ and the repeated permutations of the set. The number of NE through repeated permutations for the given example is $\frac{4!}{2!2!} = 6$ solutions.

V. CHANNEL SELECTION AS STACKELBERG GAME

In distributed cognitive radio networks, the secondary users are vulnerable to unexpected events such as primary user arrival or a deep fade or sudden increase in interference. In such cases, any strategic decision or information that could facilitate or lead to an uninterrupted channel access and link maintenance could be modeled. The strategy enforcing node(s) take the role of leader and the rationally following (benefiting) nodes take the role of followers and the arrangement is modeled as Stackelberg game. So far noncooperative games have been considered where the roles of the players (users) are symmetric, that is to say, no single player dominates the decision process. The channel access of the disconnecting secondary user due to the other user's strategy is modeled as a Stackelberg game [11]. Before looking into modeling the channel access as a Stackelberg game, some of the scenarios and unexpected events which could potentially disconnect or disrupt the channel access are listed. Unexpected events are changes in the network which happen faster than the channel scanning time.

A. Unexpected Events

1) Primary user arrival: In the event of a primary user's arrival one or more secondary users have to evacuate the channel with minimal or no interference. In a distributed network, the secondary user's response to such an unexpected event is a reactive one. The channel switching has to be immediate.

2) Interference: In a dynamic access regime, multiple SUs attempting to access a channel is a real possibility despite power constraints, and that could lead to a sudden increase in interference. Another source of interference is adjacent channel interference.

3) Fading: Fading is another source of disconnection. This is an intrinsic characteristic of a channel and depends on the location and mobility of the user.

B. Channel access as a leader-follower game

So far the dynamic spectrum access has been analyzed as a noncooperative game where the cognitive radios autonomously decide the channels to access based on the assessed local information. In the event of unexpected scenarios, any strategic decision or information by a user could facilitate uninterrupted channel access. Such a user is referred to as a 'leader'. The

user(s) who do not have information on accessing channel and rationally decide whether to follow or not are referred to as 'followers'.

Since this is a distributed type network and the users access channels in noncooperative fashion, this leader-follower scenario is only temporary. To declare a node as a leader, the node finds a channel for access and declares or broadcasts the 'new' channel information. In the cases of multiple nodes satisfying the above criteria in the same channel only one node is declared as a leader. Hence one channel corresponds to one leader which forms the strategy profile for the follower. The leader node does not have any other control over other users in addition to strategic information. The leader-follower relationship lasts only until the follower's initial access in the newly switched channel.

C. Utility function

The utility function defined in (5) is used here with the modification by including a virtual cost (Λ) in the price term for the leader(s) and the follower(s).

$$U_L = a(L) \log(1+\gamma_L^{\varphi(L)}) - b(L)\lambda^{\varphi(L)}\gamma_L^{\varphi(L)} + \sum_{j \in b^k} {}_{j \neq L} \Lambda_L \gamma_j^{\varphi(L)}$$ (7)

$$U_F = a(F) \log(1+\gamma_F^{\varphi(L)}) - b(F)\lambda^{\varphi(L)}\gamma_F^{\varphi(L)} - \Lambda_L \gamma_F^{\varphi(L)}$$ (8)

where λ is the price per unit for the channel and Λ is the virtual price that the leader sets for the switching channel.

VI. SIMULATION RESULTS

This section presents some of the simulation results investigating the defined utility function, channel access performance and a brief study on the performance of the Stackelberg game. In this simulation, a varying number of users-pairs (1 to 15) are considered, accessing M = 5 available channels. The user SINR threshold values are randomly selected from 15dB to 30dB. The noise power spectrum density at each user-pair is $N_0 = -100dBm$. The maximum transmit signal power on each channel is $\bar{P} = 50mW$. The shared channels are assumed to be equal in the start of simulation. For utility function (5) the user preference parameter is set $a(i) = 0.5$ and $b(i) = 0.5$ unless changed. The results are averaged over 100 simulation runs.

The simulation results compare the performance of the defined utility function with functions considering only SINR or cost. The users arrive randomly and access the channel based on their utility. The users recalculate their utility in the event of any changes in channel conditions.

A. Characteristics of utility function and channel access

The utility function defined in (5) is compared with the functions that consider only SINR and the function considering only cost. The cost (utility) is the product of unit price of the selected channel and SINR. In Figure 2 the average interference perceived is plotted against the number of users. Figure 3 shows the average utility (cost). In Fig 2 and Fig 3 there are three regions that characterize the utility function. In
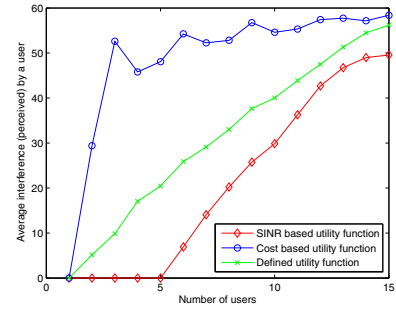

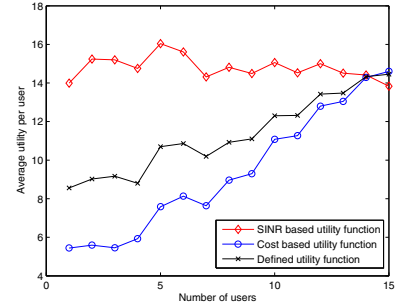Fig. 2. Average interference perceived by an user ; M = 5; Avg SINR per user = 30dB;

Fig. 3. Average utility(cost) of an user ; M = 5; Avg SINR per user = 30dB;

Fig 2, in the initial stage as the users arrive, the interference in SINR based function is nil since as the users arrive each user finds a channel with least or no interference irrespective of the price. On the other hand, for the price based function the interference is additive since the least priced channel attracts the users. With the user preference parameter of $a(i) = 0.5$ and $b(i) = 0.5$ the defined utility function balances between SINR and price function. In the mid region, the difference between the functions demonstrates their characteristics, with the SINR based functions interference averse and the price based function interference prone. In the end, when the channels are saturated with users, all users face relatively equal interference. With the balanced user preference parameter, the defined utility function accesses the channels based on merit of utility and balances between the two without unduly compromising either of them.

B. Spectrum mobility: Adaptive nature

The adaptive nature of the defined utility function is shown in Fig 4 and Fig 5. Since the utility function is SINR based, any change in user arrival or channel switching results in a change in interference and the user begins to search for the least interfered channel. The change in interference and its adaptability is illustrated in Fig 4. The channel switching and resulting change in utility is shown in Fig 5.

C. Performance of Stackelberg game

In this section, the utilities and interference changes due to the unexpected events are illustrated for the leader node
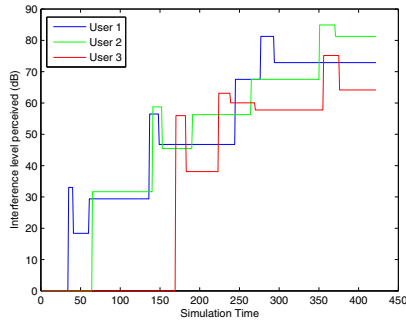
4

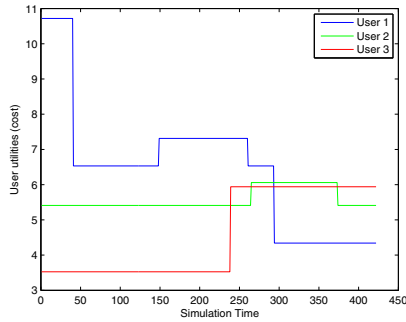Fig. 4. Variation in interference perceived vs user arrival

Fig. 5. Variation in user utility(cost) vs user arrival

and the follower node respectively. In the example simulation, the unexpected event happens at 63rd simulation time unit. Fig 6 illustrates the utilities before and after the unexpected event. The dip in the leader's utility cost after the event is due to the virtual price (Λ) from followers. Fig 7 shows the corresponding interference variations.

VII. CONCLUSION AND FUTURE WORK

This paper has analyzed channel selection and dynamic spectrum access as a noncooperative game. The existence of a pure strategy Nash equilibrium and the condition for equilibrium is shown through a bimatrix game. The importance of the definition of the utility function and how cost factor influences the equilibrium were demonstrated with simulation
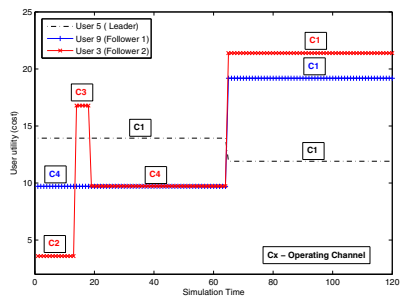

Fig. 6. Utility (cost) of leader and follower nodes; Avg SINR per user = 25dB;
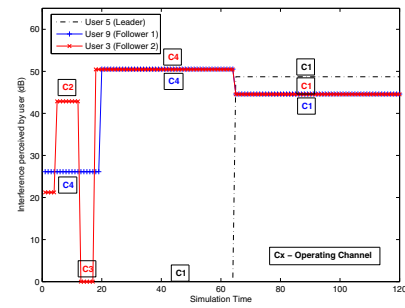

Fig. 7. Interference perceived by leader and follower nodes; Avg SINR per user = 25dB;

results. The three regions for interference and utility illustrated the characteristics of the utility functions. The adapting nature in the event of interference and the user controllable nature of the utility functions were illustrated. In the event of unexpected scenarios, the users that share some information to avoid the link disconnection is modeled as a leader-follower game and analysed for existence of an equilibrium solution. A brief discussion on the performance of the Stackelberg game was also provided.

ACKNOWLEDGMENT

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