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A computationally efficient multi-mode equaliser based on reconfigurable frequency domain processing

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Abstract— **The trend for increased data rates and mobility in broadband wireless communication systems requires that systems be designed to operate with high spectral efficiency in wideband channels. Such requirements make OFDM an attractive transmission technology due to the fact it exploits the computational efficiency associated with frequency domain equalisation. This has lead to the analysis of the applicability of such techniques to existing systems [1][2]. In particular, the 3GPP-UMTS system, where HSDPA offers a significant increase in performance by using multi-code W-CDMA transmission, is considered herein. This paper proposes a computationally efficient solution that performs equalisation and despreading using frequency domain processing, enabling the same equaliser to be used for both OFDM and W-CDMA. The solution is thus particularly applicable to devices that support fixed-mobile convergence where multi-mode support of these two transmission techniques is likely to be required. A complexity analysis is included and performance is verified though simulation.**

Keywords- Frequency domain equalisation, HSDPA, Reconfigurable radio.

I. INTRODUCTION

OFDM is proving to be an attractive solution for radio communication systems that support high data rates through dispersive channels. This is largely due to the fact that the use of frequency domain processing enables computationally efficient equalisation of the wideband channel to be performed.

Significant analysis of the application of computationally efficient frequency domain equalisation for other communication systems based on a wideband single carrier modulation has been performed [1][2]. The analysis suggests that, providing a cyclic prefix is available, equalisation becomes computationally feasible. However, it is typically found in these systems, such as the current 3G mobile based on W-CDMA, that no cyclic prefix exists as, at the time of design, it was assumed time domain processing would be used. Thus techniques that enable frequency domain equalisation without requirement of a cyclic prefix are considered in this paper, building on the previous work.

In particular, the applicability of such techniques to the 3GPP UMTS-HSDPA system is considered as frequency domain equalisation is relevant for two reasons. The first being that in order to realize the peak data rates of 14Mbps enabled by the standard, advanced techniques are required in HSDPA enabled receivers; simple RAKE receivers would significantly restrict the performance due to a combination of the low spreading factors used and their lack of ability to combat the loss of spreading code orthogonality, a product of multi-path propagation. In this case, frequency domain processing would make the implementation of chip level equalisers feasible resulting in a significant performance improvement.

Secondly, the process of fixed mobile convergence that is likely to occur through projects such as BT's bluephone will require the implementation of more than one radio access mode in the terminal [3]. As OFDM is being adopted in many of the standardised systems offering broadband wireless access, such as 802.11/HiperLAN and 802.16/HiperMAN, and 3G systems are likely to exist for several years, the same equaliser based on frequency domain processing can be used for both transmission techniques.

The remainder of the paper is organised as follows: Section II develops the frequency domain equaliser in the case of ideal cyclic prefix insertion in the transmitter; Section III develops a technique for mitigating the effects of beyond cyclic prefix multi-path; Section IV provides a computational complexity comparison of the various algorithms; Section V provides simulation results; and Section VI concludes the work.

II. FREQUENCY DOMAIN RECEIVERS

The use of frequency domain equalisers in OFDM is well documented. This section details the development of an equivalent frequency domain equaliser for W-CDMA, initially assuming that a cyclic prefix is available.

A. Frequency domain equalisation

In the case of a cyclic prefix at least the same length as the maximum delay spread of the channel, the signal at the input to the receiver after segmentation and subsequent removal of the cyclic prefix is given by:

$$\mathbf{r}_i = \mathbf{H}_C \mathbf{s}_i + \mathbf{n}_i \qquad (1)$$

Where $\mathbf{r}_i$ is the ith block of the received signal, $\mathbf{s}_i$ is the ith block of transmitted spread symbols or chips prior to insertion of cyclic prefix, $\mathbf{n}_i$ is the noise and $\mathbf{H}_C$ is the circulant channel matrix of the following form:

$$\mathbf{H}_C = \begin{bmatrix} h[0] & 0 & \cdots & \cdots & \cdots & \cdots & h[2] & h[1] \\ h[1] & h[0] & & & & & h[3] & h[2] \\ \vdots & \vdots & \ddots & & & & \vdots & \vdots \\ h[L-2] & h[L-3] & & \ddots & & 0 & h[L-1] \\ h[L-1] & h[L-2] & & \ddots & & 0 & 0 \\ 0 & h[L-1] & & & \ddots & \vdots & \vdots \\ \vdots & \vdots & & & & h[0] & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & h[1] & h[0] \end{bmatrix} \quad (2)$$

Where $\mathbf{h}$ is the complex channel impulse response vector. Due to the fact $\mathbf{H}_C$ is a circulant matrix, it is diagonalised by the DFT and IDFT matrix such that [4]:

$$\mathbf{H}_f = \mathbf{F}^H \mathbf{H}_C \mathbf{F} \quad (3)$$

Where $\mathbf{F}$ is the symmetric matrix given by $\mathbf{F}(m,n) = N^{-1/2} \exp(j2\pi(m-1)(n-1)/N)$. The diagonal elements of $\mathbf{H}_f$ are given by the DFT of the channel impulse response, $\mathbf{h}$:

$$\mathbf{h}_f = N^{1/2} \mathbf{F}^H \mathbf{h} \quad (4)$$

The output of the time domain MMSE equaliser and hence estimation of the transmitted chips is given by:

$$
\begin{aligned}
\hat{\mathbf{s}}_i &= \mathbf{R}_{rr}^{-1}\mathbf{R}_{sr}\mathbf{r} \\
&= \left(E\big[(\mathbf{H}_C\mathbf{s}+\mathbf{n})(\mathbf{H}_C\mathbf{s}+\mathbf{n})^H\big]\right)^{-1} E\big[\mathbf{s}(\mathbf{H}_C\mathbf{s}+\mathbf{n})^H\big]\mathbf{r} \\
&= \left(\mathbf{H}_C E[\mathbf{ss}^H]\mathbf{H}_C^H + E[\mathbf{nn}^H]\right)^{-1} E[\mathbf{ss}^H]\mathbf{H}_C^H\mathbf{r} \\
&= \left(\sigma_s^2 \mathbf{H}_C\mathbf{H}_C^H + \sigma_n^2\mathbf{I}\right)^{-1}\sigma_s^2\mathbf{H}_C^H\mathbf{r} \\
&= \left(\frac{\mathbf{H}_C^H}{\mathbf{H}_C\mathbf{H}_C^H + \delta^2\mathbf{I}}\right)\mathbf{r} = \mathbf{W}_{le}\mathbf{r}
\end{aligned}
\quad (5)
$$

Where σ_s^2 and σ_n^2 are the signal and noise power respectively and $\delta^2 = \sigma_n^2/\sigma_s^2$. By substituting (3) into (5), it is possible to arrive at a description of the frequency domain MMSE equaliser:

$$
\begin{aligned}
\hat{\mathbf{s}}_i &= \left[(\mathbf{F}\mathbf{H}_f\mathbf{F}^H)^H(\mathbf{F}\mathbf{H}_f\mathbf{F}^H) + \mathbf{F}\delta\mathbf{I}_N\mathbf{F}^H\right]^{-1}(\mathbf{F}\mathbf{H}_f\mathbf{F}^H)^H\mathbf{r}_i \\
&= \left[\mathbf{F}(\mathbf{H}_f^H\mathbf{H}_f)\mathbf{F}^H + \mathbf{F}(\delta\mathbf{I}_N)\mathbf{F}^H\right]^{-1}\mathbf{F}(\mathbf{H}_f^*)\mathbf{F}^H\mathbf{r}_i \\
&= \mathbf{F}\left[\frac{\mathbf{H}_f^*}{\left(|\mathbf{H}_f|^2 + \delta\mathbf{I}_N\right)}\right]\mathbf{F}^H\mathbf{r}_i
\end{aligned}
\quad (6)
$$

Where the block subscript is dropped and $\mathbf{s}_f$ and $\mathbf{n}_f$ are the frequency domain versions of the transmitted spread signal and noise and $\hat{\mathbf{s}}_f$ is the estimated frequency domain version of the transmitted spread signal. Due to the fact $\mathbf{H}_f$ is diagonal it follows that the denominator in the frequency domain MMSE estimator matrix $\mathbf{W}_f$ is also diagonal and therefore the inverse of the denominator is given simply by:

$$\left(|\mathbf{H}_f|^2 + \delta^2\mathbf{I}_N\right)^{-1} = \begin{bmatrix} \left(|h_f(1)|^2 + \delta^2\right)^{-1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \left(|h_f(N)|^2 + \delta^2\right)^{-1} \end{bmatrix} \quad (7)$$

Hence, matrix inversion is not required in the case of frequency domain equalisation.

B. Frequency domain despreading

After equalisation it is possible to use the spreading code matched filter to despread the equalised signal yielding the estimate of the transmitted symbol sequence [5]. Due to the fact the channel is equalised, only a single output of the filter is required per symbol.

Alternatively, it is possible to perform joint equalisation and despreading in the frequency domain, where the estimate of the transmitted symbol at the output of the combined detector is given by:

$$y[n] = IFFT\{c_f[k]w_f[k]r_f[k]\} \quad (8)$$

Where c_f is the FFT of the spreading code, $w_f = diag\{\mathbf{W}_f\}$ and r_f is the FFT of $\mathbf{r}_i$.

The estimate of the N transmitted symbols in the length NC block with a spreading factor of C are then given by $\hat{d}[i] = y[iC]$, where $i = 0,\ldots,N-1$ is the symbol number. Thus it is only necessary to calculate N/C points of the IFFT and hence a reduction in computational complexity can be afforded by performing despreading in the frequency domain.

III. INTER-BLOCK INTERFERENCE SUPPRESSION

If the cyclic prefix is no longer available, or is insufficient in length, then interference will be experienced from the tail of the previous block and energy from the current block will be lost in to the following block. It follows that the received signal in the case of no cyclic prefix is given by:

$$\mathbf{r}_i = \mathbf{H}_C\mathbf{s}_i + \mathbf{H}_{IBI}\mathbf{s}_{i-1} - \mathbf{H}_{IBI}\mathbf{s}_i + \mathbf{n}_i \quad (9)$$

Where the inter-block interference matrix, $\mathbf{H}_{IBI}$, is defined as:

$$\mathbf{H}_{IBI} = \begin{bmatrix} 0 & 0 & \cdots & \cdots & \cdots & h[2] & h[1] \\ 0 & 0 & & & & h[3] & h[2] \\ \vdots & \vdots & \ddots & & & \vdots & \\ 0 & 0 & & \ddots & & 0 & h[L-1] \\ \vdots & \vdots & & & \ddots & \vdots & \vdots \\ 0 & 0 & & & & 0 & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & 0 & 0 \end{bmatrix} \quad (10)$$

Such that $\mathbf{H} = \mathbf{H}_C - \mathbf{H}_{IBI}$.

The impact this has on the bit error rate across the received block is illustrated in Fig. 1 where the blue line indicates that the error is greatest at the block edges.

The technique proposed in this paper to mitigate the effects is to extract the middle section of the block at the output of the equaliser, where the BER is at its lowest. In order to prevent samples from being lost as a result of discarding the edges of the block, overlapping of consecutive blocks is performed at the input of the equaliser. Thus by adopting this technique, as illustrated in Fig. 2, it will be possible to reduce the average BER experienced in the case of insufficient cyclic prefix whilst maintaining a continuous stream of received symbols.

This process is referred to as "overlap and extract" (OAE) due to the nature of the processing performed.

IV. COMPUTATIONAL COMPLEXITY

Due to the impracticalities associated with optimal time domain linear MMSE equalisation, it is assumed that a sub-optimal finite length linear transversal filter is used to perform time domain equalisation where the number of taps are set equal to the channel delay spread. Thus the time domain equaliser requires a K point convolution per chip comprising K multiplications, where K must be at least the same as the number of chips that constitute the channel delay spread. Whereas the frequency domain equaliser requires an NC-point FFT followed by NC frequency domain multiplications and finally an NC-point IFFT to process NC chips, where N and C are respectively the number of symbols per block and number of chips per symbol.

In the case of time domain processing, despreading can be performed following equalisation by a single correlator and accumulator, adding an additional complexity of C multiplies per symbol. Thus the total complexity for N symbols is given by the equation in Table I.

For frequency domain despreading, only N bins of the NC point IFFT require computation. The effect this has is that the number of butterflies in the IFFT requiring computation is reduced, as illustrated in Fig. 3. It is found that in general the required number of butterflies is given by:

$$B = \begin{cases} (NC-1) & N \le 2 \\ (NC-1) + \sum_{i=1}^{(\log_2 N)-1} i2^{i-1} & N \ge 4 \end{cases} \quad (11)$$

Whilst the overlap and extract algorithm does not require any further computational elements, the fact that a proportion of the output samples from any one block are discarded results in the computational resource used to compute these being wasted. The extra computational load is therefore due to the fact that processing P samples only results in Q results. Defining the number of discarded samples, or total region of overlap, as $O = P - Q$, the increase in computational load is given by the factor $(O + NC)/NC$. The same also applies to the case of where frequency domain despreading is incorporated into the equalisation process, where O must be set as a multiple of the

spreading factor and may consequently compound the increase in computational load.

In summary, Table I presents equations for calculating the multiplicative computational complexity of the three algorithms along with that of a RAKE receiver that tracks L multi-paths [6].

TABLE I. COMPUTATIONAL COMPLEXITY EQUATIONS.

Algorithm	Number of multiplications
TD-RAKE	$N(LC + L)$
TD-MMSE	$KNC + NC$
FD-MMSE	$\dfrac{NC}{2}\log_2(NC) + 2NC + B$
FD-MMSE-OAE	$\left(\dfrac{NC}{2}\log_2(NC) + 2NC + B\right)\dfrac{O + NC}{NC}$

Table II compares the total multiplicative computational complexity per symbol of the four receivers for the case of a block size of 1024 with $C = 16$ (as used in HSDPA [7]) and an ITU vehicular A (VA) and B (VB) channel [8] at the sample rate used in UMTS of 3.84Msps which results in 6 multipaths with a delay spread of 10 and 77 samples respectively. The overlap used was 32 and 160 for the VA and VB channels respectively.

TABLE II. COMPUTATIONAL COMPLEXITY COMPARISON.

Receiver	Multiplications per symbol	
	ITU VA	ITU VB
RAKE	102	102
TD-MMSE	176	1248
FD-MMSE	130	130
FD-MMSE-OAE	134	150

Thus the computational complexity of the FD-MMSE-OAE algorithm is between 1.3 and 1.5 times that of the RAKE and between 0.76 and 0.12 of a time domain equaliser, depending on the multi-path delay spread.

V. SIMULATION RESULTS

Simulation of multi-code W-CDMA transmission through an ITU VA VB multi-path channel were conducted for QPSK modulation and a spreading factor of 16. Fig. 4 shows the performance of a RAKE, FD-MMSE receiver with and without CP insertion and FD-MMSE receiver using OAE to mitigate inter-block interference for a block size of 1024 and a VA channel and Fig. 5 shows results for the case of a VB channel.

Comparing the performance of the FD-MMSE with cyclic prefix and the FD-MMSE-OAE it is apparent that whilst similar performance was achievable in the case of a VA channel, for the case of multi-code transmission through a VB channel performance degradation was experienced. However, the FD-MMSE-OAE algorithm improved the BER that was

achieved by the FD-MMSE with no cyclic prefix. Further, in all cases FD-MMSE was shown to significantly outperform the RAKE receiver.

VI. CONCLUSION

In conclusion, a computationally efficient receiver based on frequency domain equalisation and despreading has been proposed that has similar performance in the absence of cyclic prefix the MMSE equaliser with cyclic prefix for a suitable block size after transmission through a VA channel.

Whilst its performance is shown to degrade in more dispersive channels, it always offers significant performance increase over the RAKE receiver for no more that 1.5 time the complexity of the RAKE and significantly less than that required for a time domain equaliser.

Furthermore, as the architecture is based on frequency domain processing it would be possible to implement it in such a way that only a limited amount of reconfiguration would be required for a switch between reception of W-CDMA and OFDM signal, as the FD-MMSE-OAE equaliser contains all the processing components required for OFDM frequency domain based equalisation.

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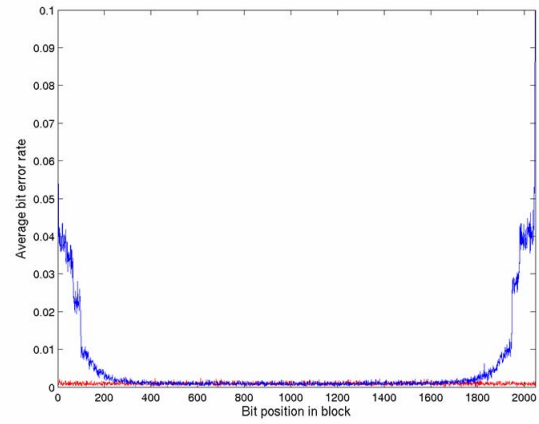
Fig. 1. Uncoded BER across the block for QPSK, SNR=20dB, spreading factor 16, block size of 1024 chips through a Vehicular B channel for (blue) no cyclic prefix, (red) OAE with overlap of 80 samples at either end of the block.
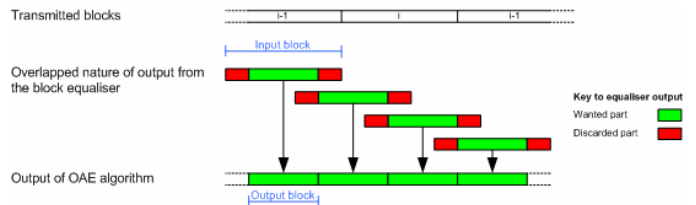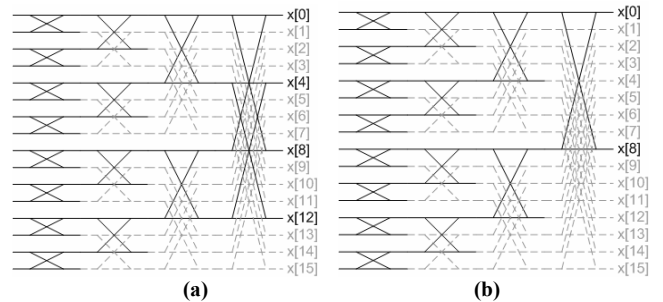

Fig. 2. OAE algorithm.

Fig. 3. Complexity analysis of modified IFFT for frequency domain despreading for the case of a block size of 16 for spreading factor of (a) 4 and (b) 8.
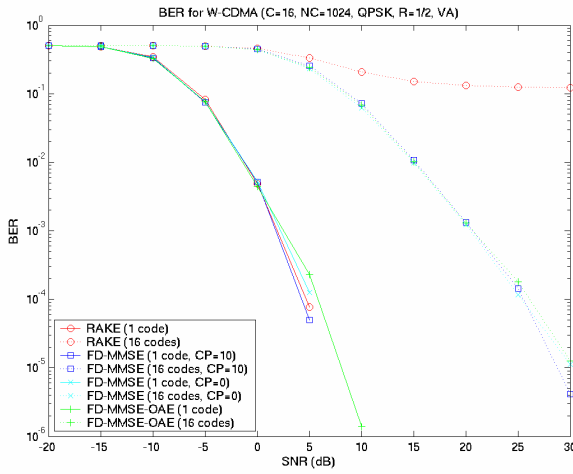
Fig. 4. BER as a function of SNR at the output of the receiver performing frequency domain equalisation and despreading using (i) RAKE, (ii) FD-MMSE with cyclic prefix, (iii) FD-MMSE without CP, (iv) FD-MMSE with OAE for block size of 1024 and VA channel for 1 and 16 multi-code transmission.
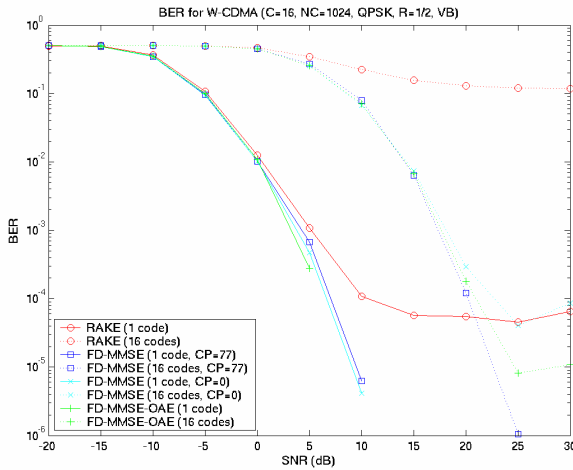

Fig. 5. BER as a function of SNR at the output of the receiver performing frequency domain equalisation and despreading using (i) RAKE, (ii) FD-MMSE with cyclic prefix, (iii) FD-MMSE without CP, (iv) FD-MMSE with OAE for block size of 1024 and VB channel for 1 and 16 multi-code transmission.