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TREE-BASED REPARAMETERIZATION FOR SYMBOL DETECTION IN SPATIALLY MULTIPLEXED MIMO SYSTEMS IN FREQUENCY FLAT FADING

C. M. Vithanage ♠ ◇, *C. Andrieu* ♠, *R. J. Piechocki* ◇ *and J. P. Coon* ♣

♠ Department of Mathematics, University of Bristol, Bristol BS8 1TW, UK.
◇ Centre for Communications Research, University of Bristol, Bristol BS8 1UB, UK.
♣ Toshiba Research Europe Ltd., Bristol BS1 4ND, UK.
Email: {C.Vithanage, C.Andrieu, R.J.Piechocki}@bristol.ac.uk and Justin.Coon@toshiba-trel.com

ABSTRACT

Reduced complexity detection of spatially multiplexed data transmissions under frequency flat fading is considered. Noting that the optimal detection is the generation of marginal distributions of a joint probability distribution, we use the approximate marginal generating algorithm termed the tree-based reparameterization. The resulting system complexity is having an order which is less than quadratic in the number of transmit antennas. Simulation of a 8×8 BPSK system shows a coded bit error rate of 10^{-3} at an SNR level of only 1dB greater than required by the optimal method.

Key words - MIMO, approximate inference, belief propagation, tree-based reparameterization.

1. INTRODUCTION

It is well appreciated that multiple input multiple output (MIMO) wireless channels provide a huge capacity potential [1] and that future wireless communications will invariably resort to the use of such channels. Transmission of spatially multiplexed independent data streams from a transmitter with multiple antennas can lead to the extraction of the huge capacity potential offered, in the presence of optimal detection at the receiver. Unfortunately, optimal detection has a complexity which is exponential in the number of transmit antennas and hence is practically infeasible. Thus reduced complexity symbol detectors such as the V-BLAST (vertical Bell laboratories layered space-time) algorithm [2] are needed to take advantage of the capacity potential of MIMO systems with a large number of transmit antennas. Presently, the state–of–art in near optimal reduced complexity MIMO detectors are the sphere decoders [3]. Other successful approaches include the successive moment matching to Gaussians based approach of [4] and the Gibbs sampling based Monte Carlo sampling approach of [5].

We can consider optimal soft detection in a spatial multiplexing system as the generation of the *a posteriori* marginal probability distributions (which we call as the APPs) of the symbols transmitted by each antenna. This optimal detection can be seen to be a task of marginalization in a joint probability distribution. In this work, we will frequently refer to the representation of probability distributions via undirected graphical models [6]. As will be shown later, the joint distribution on which marginalization needs to be performed in this case correspond to an undirected graphical model with cycles. Had the graphical model been cycle free, the required marginal distributions could have been computed with a complexity which is linear in the number of variables using a message passing algorithm such as the "sum-product algorithm" [7]. Even in the presence of cycles, repeated message passing as if there is no cycles, which is termed "loopy belief propagation" has been shown to produce good approximate marginals in some applications [8].

In this work we apply a generalization of loopy belief propagation termed tree-based reparameterization [9], which has shown to have faster convergence than loopy belief propagation, for the task of approximate marginal generation in MIMO symbol detection.

2. SYSTEM MODEL AND OPTIMAL SYMBOL DETECTION

Let us consider an n_t–transmit antenna, n_r–receive antenna MIMO communication system operating in frequency-flat quasi-static Rayleigh faded channels. Considering an equivalent complex base-band discrete signal model, for each time instant we have

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}. \tag{1}$$

Denoting the matrix transpose operation by $(\bullet)^\dagger$, here $\mathbf{y} = (y^1, ..., y^{n_r})^\dagger$ with y^j being the received signal value on antenna j, $\mathbf{x} = (x^1, ..., x^{n_t})^\dagger$ with x^i being the transmitted modulated symbol by antenna i and $\mathbf{H}$ is an $n_r \times n_t$ matrix with the $(j, i)^{th}$ element being $h^{i,j}$ which is the channel fading coefficient from transmit antenna i to receive antenna j.

We will consider a normalization such that each $h^{i,j}$ has a circularly symmetric Gaussian distribution with a variance of 1 (i.e. has a distribution $\mathcal{CN}(0,1)$). Let the transmitted symbols be selected from a set $B = \{a_1, ..., a_N\}$ with $|B| = N$. Also, $\mathbf{w} = (w^1, ..., w^{n_r})^\dagger$ where w^j is the spatially uncorrelated additive white noise manifesting at the j^{th} receive antenna with a distribution $\mathcal{CN}(0, N_0)$. We

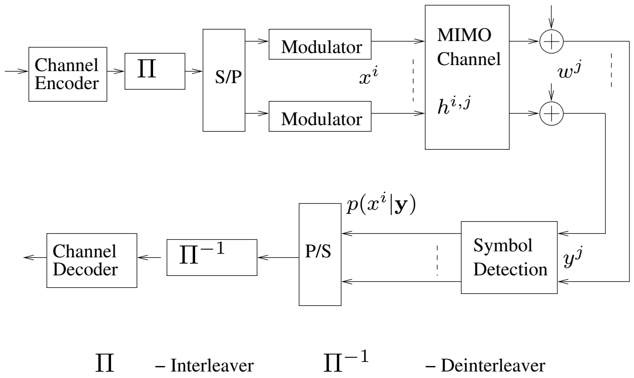

Fig. 1. The System Model

can consider optimal detection considering the subsequent channel decoding operation as the generation of the marginal distributions $p(x^i|\mathbf{y})$ for each $i \in \{1, \cdots, n_t\}$. Considering no prior information on $\mathbf{x}$, with proper normalization, the posterior distribution of the space-time symbols can be computed as

$$p(\mathbf{x}|\mathbf{y}) \propto \exp\left(-\frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{N_0}\right). \tag{2}$$

From this joint posterior distribution, $p(x^i|\mathbf{y})$ for each i can be computed in a brute force manner by a total enumeration. Because of the enumeration over the configuration space of $\mathbf{x}$ (which has a size N^{n_t}), the complexity of the optimal algorithm is exponential in the number of transmit antennas, n_t. For large n_t this represents a prohibitive complexity for practical implementation and reduced complexity symbol detection methods become necessary.

In the following, we will consider undirected graphical models for the representation of probability distributions [6]. We can see that optimal detection is the task of generating marginal distributions from a joint distribution, with a corresponding undirected graphical model which has loops. One popular method of obtaining approximate marginals corresponding to loopy graphs is the application of loopy belief propagation [8], which is basically the repeated execution of the steps of the sum-product algorithm ignoring the loops of the graph. In [9], Wainwright *et. al.* presents a generalization of loopy belief propagation called tree-based reparameterization (TRP), which is shown to have faster convergence. In this work, we will apply the TRP method for the MIMO symbol detection problem. Before describing the algorithm, let us first consider how the sum-product algorithm can be applied to compute the exact marginal distributions of a joint distribution which has a tree (more specifically, chain) structured undirected graphical model.

3. SUM-PRODUCT ALGORITHM FOR A CHAIN STRUCTURED UNDIRECTED GRAPHICAL MODEL

Let us restrict our attention to joint distributions of the set of discrete random variables $\mathbf{x}$ with a corresponding undirected graphical model which is chain structured as shown in Fig. 2. Let V be the set of vertices of the graph and E be the set of edges. Noting that the *cliques* (sets of fully connected nodes) of the graph are the singleton variables and pairs of connected variables, let us assume that the joint probability distribution factorizes according to a set of clique potentials as

$$p(\mathbf{x}) \propto \prod_{s \in \mathsf{V}} \psi_s(x^s) \prod_{(s,t) \in \mathsf{E}} \psi_{s,t}(x^s, x^t). \tag{3}$$

Given this set of potential functions, the sum-product al-

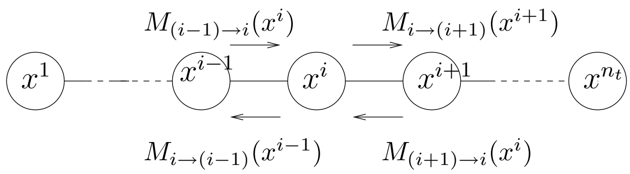

Fig. 2. Message passing of the sum-product algorithm

gorithm can be executed as a set of forward and backward message passing recursions through the chain. For example, the message from the variable x^i to the variable x^{i+1}: $M_{i\to(i+1)}$ is generated as

$$M_{i\to(i+1)}\left(x^{i+1}\right) \propto \sum_{x^i \in B} \left[\psi_{i,i+1}\left(x^i, x^{i+1}\right) \psi_i\left(x^i\right) M_{(i-1)\to i}\left(x^i\right)\right]. \tag{4}$$

This recursive forward/backward message passing gives, for example, the exact marginal probability mass functions of the singleton variables as

$$q_i\left(x^i\right) \;\propto\; M_{(i-1)\to i}\left(x^i\right) \psi_i\left(x^i\right) M_{(i+1)\to i}\left(x^i\right). \tag{5}$$

One can note that the expression for a joint distribution in terms of potential functions as in (3) is not unique. For a tree connected set of variables, given the marginals of variables corresponding to the nodes and edges of the tree, another *reparameterization* (as termed by Wainwright [9]) of the joint distribution $p(\mathbf{x})$ is as

$$p\left(\mathbf{x}\right)=\prod_{s\in\mathsf{V}}q_{s}\left(x^{s}\right)\prod_{(s,t)\in\mathsf{E}}\frac{q_{s,t}\left(x^{s},x^{t}\right)}{q_{s}\left(x^{s}\right)q_{t}\left(x^{t}\right)} \tag{6}$$

4. THE TREE-BASED REPARAMETERIZATION

Now let us relax the condition that the graphical model is a tree, and let the set E of (3) to possibly include cycles, with the restriction that the joint distribution can be represented by a set of potentials involving singleton and pairwise variables. A resulting example of an undirected graphical model is shown in Fig. 3, which is actually the graph corresponding to $p(\mathbf{x})$ in MIMO symbol detection. The graph in Fig. 3 is called *complete* since all the nodes are pairwise connected. Still, we can always identify a collection of edges of the graph which will correspond to a spanning tree (which we will denote as the set E_k). By a spanning tree, we refer to a tree which covers all the nodes of the graph. Then, we can decompose (3) as

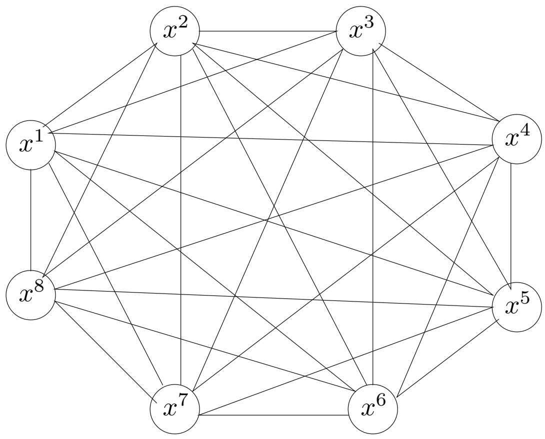

Fig. 3. The undirected complete graphical model corresponding to $n_t = 8$.

$$p\left(\mathbf{x}\right)\propto\underbrace{\prod_{s\in\mathsf{V}}\psi_{s}\left(x^{s}\right)\prod_{(s,t)\in\mathsf{E}_k}\psi_{s,t}\left(x^{s},x^{t}\right)}_{\propto\, p^{k}(\mathbf{x})}\cdot \underbrace{\prod_{(s,t)\in\mathsf{E}\backslash\mathsf{E}_k}\psi_{s,t}\left(x^{s},x^{t}\right)}_{\propto\, p^{\backslash k}(\mathbf{x})}. \tag{7}$$

Now, we can choose to run the sum product algorithm to find the "pseudo marginals" or "beliefs" of $p^k(\mathbf{x})$. Let us assume that the computed "beliefs" in this manner are given as $q_s(x^s)$ and $q_{s,t}(x^s,x^t)$ for $s \in \mathsf{V}$ and $(s,t) \in \mathsf{E}_k$. Given these beliefs, we can give a reparameterization of $p^k(\mathbf{x})$ as

$$p^{k}\left(\mathbf{x}\right)=\prod_{s\in\mathsf{V}}q_{s}\left(x^{s}\right)\prod_{(s,t)\in\mathsf{E}_k}\frac{q_{s,t}\left(x^{s},x^{t}\right)}{q_{s}\left(x^{s}\right)q_{t}\left(x^{t}\right)} \tag{8}$$

This reparameterization can be plugged back into (7) to determine the new potential functions for the graph. In its execution, tree-based reparameterization algorithm consists of considering a series of spanning trees. For each spanning tree, the sum product algorithm is executed to recompute the "beliefs" of the constituent variables and pairs of variables which in turn keeps reparameterizing parts of (3).

5. THE TRP MIMO DETECTOR

5.1. Initial potential functions for the MIMO symbol detection

For the idea of tree-based reparameterization as presented in section 4 to be extended for the problem of symbol detection in flat faded MIMO channels, we need to find the initial potential functions associated with the singleton and pairwise variables. From (2) we can see that

$$p\left(\mathbf{x}\left|\mathbf{y}\right.\right)\propto\exp\left(\frac{2}{N_0}\mathrm{Re}\left(\mathbf{x}^{\ddagger}\mathbf{H}^{\ddagger}\mathbf{y}\right)-\frac{1}{N_0}\mathbf{x}^{\ddagger}\mathbf{H}^{\ddagger}\mathbf{H}\mathbf{x}\right),$$

where $(\bullet)^{\ddagger}$ denotes the conjugate transpose operation. Therefore the posterior distribution can be decomposed into the form of (3) with a corresponding complete undirected graphical model to produce the initial potential functions as

$$\psi_s^0\left(x^s\right)=\exp\left(\frac{1}{N_0}\Big[-\left|x^s\right|^2\left(\mathbf{H}^{\ddagger}\mathbf{H}\right)_{s,s}+ 2\mathrm{Re}\left\{\left(x^s\right)^{*}\left(\mathbf{H}^{\ddagger}\mathbf{y}\right)_s\right\}\Big]\right) \tag{9}$$

$$\psi_{s,t}^0\left(x^s,x^t\right)=\exp\left(-\frac{2}{N_0}\mathrm{Re}\left\{\left(x^s\right)^{*}\left(\mathbf{H}^{\ddagger}\mathbf{H}\right)_{s,t}x^t\right\}\right). \tag{10}$$

Here, $(\bullet)^{*}$ denotes the conjugation operation, for a matrix $\mathbf{A}$, $(\mathbf{A})_{s,t}$ denotes the $(s,t)^{th}$ element and for a row or column vector $\mathbf{b}$, $(\mathbf{b})_s$ denotes the s^{th} element. Thus, armed with these potential functions to start with, we can execute the tree-based reparameterization algorithm (TRP method) to approximately evaluate the marginal distributions of the symbols transmitted by each antenna.

For a given n_t, we know that the undirected graphical model is a complete graph similar to Fig. 3. For the execution of the algorithm, the first thing to be done is to select a sequence of spanning trees $\mathsf{T}_1,...,\mathsf{T}_k,...,\mathsf{T}_K$ with corresponding edge sets $\mathsf{E}_1,...,\mathsf{E}_k,...,\mathsf{E}_K$. Instead of arbitrary trees, we select a set of chains $\mathsf{C}_1,...,\mathsf{C}_k,...,\mathsf{C}_K$ with each chain spanning all the variables. This standardizes the execution of the sum-product algorithm and will lead to a reduction in implementation complexity. For even n_t, the minimum number of chains to cover all the edges of the graph is $K = \binom{n_t}{2}/(n_t - 1) = n_t/2$. A method to select $n_t/2$ chains to cover all the edges of the graph will be presented in section 5.2. Also, by a consideration of the simulation results, we perform no more than $K = n_t/2$ iterations of the TRP method, which is seen to capture the available information. Considering $n_t/2$ non–overlapping trees only, also obviates the need to update the edge potentials of the graph which reduces the implementation complexity.

For a given time instant, the potentials associated with the singleton and pairwise variables are initialized using (9) and (10). Thereafter, the TRP detector at the k^{th} iteration, selects the chain C_k and the corresponding edge set E_k. Let us assume the chosen chain leads to an ordering of the indices of the variables as $\{k_1 \rightarrow ... \rightarrow k_{i-1} \rightarrow k_i \rightarrow k_{i+1} \rightarrow ... \rightarrow k_{n_t}\}$. The sum product algorithm is executed to compute the new forward-backward messages ($M^k_{k_{i-1}\rightarrow k_i}\left(x^{k_i}\right)$ and $M^k_{k_{i+1}\rightarrow k_i}\left(x^{k_i}\right)$) along the chain as well as the new "beliefs" of the variables associated with the vertices ($q^k_{k_i}\left(x^{k_i}\right)$) of this chain using (5).

From the reparameterization provided by (8), this enables the computation of the new potentials associated with these same variables corresponding to the vertices V of the chain as

$$\psi^k_{k_i}\left(x^{k_i}\right) \propto \psi^{k-1}_{k_i}\left(x^{k_i}\right) M^k_{k_{i-1}\rightarrow k_i}\left(x^{k_i}\right) M^k_{k_{i+1}\rightarrow k_i}\left(x^{k_i}\right) \quad (11)$$

Finally, with the observation that the initial edge potentials $\psi^0_{s,t}\left(x^s, x^t\right)$ needs to be computed only once per frame leads to the TRP detection algorithm given in Table 1.

Table 1. Pseudo code of the TRP detector for MIMO symbol detection

$\mathcal{A}$ Select the sequence of chains $\mathsf{C}_1, ..., \mathsf{C}_k, ..., \mathsf{C}_K$.
$\mathcal{B}$ Initialize the edge potentials $\psi^{init}_{s,t}\left(x^s, x^t\right)$ for $(s,t) \in \mathsf{E}$ using (10).
$\mathcal{C}$ For each received signal vector $\mathbf{y}$
- $\mathcal{C}.1$ Set $k = 1$. Initialize $\psi^0_s\left(x^s\right)$ for $s \in \mathsf{V}$ using (9) and $\psi^0_{s,t}\left(x^s, x^t\right) = \psi^{init}_{s,t}\left(x^s, x^t\right)$ for $(s,t) \in \mathsf{E}$.
- $\mathcal{C}.2$ Perform message passing along the chain C_k and compute the new potentials of the variables associated with the vertices V using (11).
- $\mathcal{C}.3$ Increase k by one. If $k > K$ go to $\mathcal{C}.4$, other wise go to $\mathcal{C}.2$.
- $\mathcal{C}.4$ Output the properly normalized $\psi^K_s\left(x^s\right)$ for $s \in \mathsf{V}$ as the computed posterior marginal distributions.

5.2. A sequence of spanning chains

We can see that there are many possibilities of $n_t/2$ spanning chains which will cover all the edges of the graph once and only once (the number of transmit antennas, n_t is assumed to be even, which is practically the usual case.). The identification of one particular sequence of chains is as follows.

First, let us define the operation $\lfloor k \rfloor_{n_t}$ acting on some $k \in \{-(n_t - 1), \cdots, 0, \cdots, n_t - 1\}$ as $\lfloor k \rfloor_{n_t} = k$ if $k > 0$ and $\lfloor k \rfloor_{n_t} = (k + n_t)$ if $k \leq 0$. Then the sequence of chains $\mathsf{C}_1, ..., \mathsf{C}_k, ..., \mathsf{C}_K$ can be described by the k^{th} chain containing the sequence of variables with the indices ordered as $\{\lfloor k \rfloor_{n_t} \rightarrow \lfloor k+1 \rfloor_{n_t} \rightarrow \lfloor k-1 \rfloor_{n_t} \rightarrow \lfloor k+2 \rfloor_{n_t} \rightarrow \cdots \rightarrow \lfloor k + n_t/2 \rfloor_{n_t}\}$.

6. COMPLEXITY COMPARISON

The optimal method of obtaining the posterior probability mass function with its enumeration over all the possible configurations of $\mathbf{x}$ can be seen to have a complexity per time instant, in the order of $\mathcal{O}(N^{n_t} {n_r}^2)$.

For the TRP algorithm, the complexity of the algorithm is governed by the message passing operation on each tree decoding and has an order $\mathcal{O}(KN^2(n_t - 1))$. From a consideration of the simulation results for fully connected systems, the number of TRP iterations is actually set to $n_t/2$. Hence the complexity of the algorithm is $\mathcal{O}(N^2 n_t(n_t - 1))$.

Thus, the TRP decoder reduces the system complexity from $\mathcal{O}(N^{n_t} {n_r}^2)$ to $\mathcal{O}(N^2 n_t(n_t - 1))$ with a bit error rate performance as given in the next section. Therefore, in terms of the transmit antennas, the complexity is $\mathcal{O}(n_t(n_t - 1))$ which is less than the $\mathcal{O}(n_t^3)$ complexity of low complexity decoders such as the V-BLAST decoder (fast version of [10]).

7. SIMULATION RESULTS

In the following, each frame transmission contained 1152 data bits which were encoded by a rate half turbo coder and iterleaved using a random interleaver. The turbo coder consisted of two constituent $(5,7)_8$ convolutional codes. The transmitted symbols were chosen from a BPSK alphabet. The sequence of chains were selected as described in section 5.2. The turbo decoder performed 4 iterations.

Fig. 4 and Fig. 5 show the decodings in 4×4 and 8×8 systems respectively. Here, E_s denotes the average energy per transmitted symbol $\mathbf{x}$.

8. CONCLUSIONS

We have used Wainwright's tree-based reparameterization for the reduced complexity symbol detection in spatially multiplexed MIMO systems. The resulting algorithm has an $\mathcal{O}(n_t(n_t-1))$ complexity which is much less than the $\mathcal{O}(n_t^3)$ complexity of other popular reduced complexity symbol detection algorithms. Simulation results for the 8×8 BPSK system show that at a coded bit error rate of 10^{-3}, the TRP method is only 1dB away from the optimal performance.

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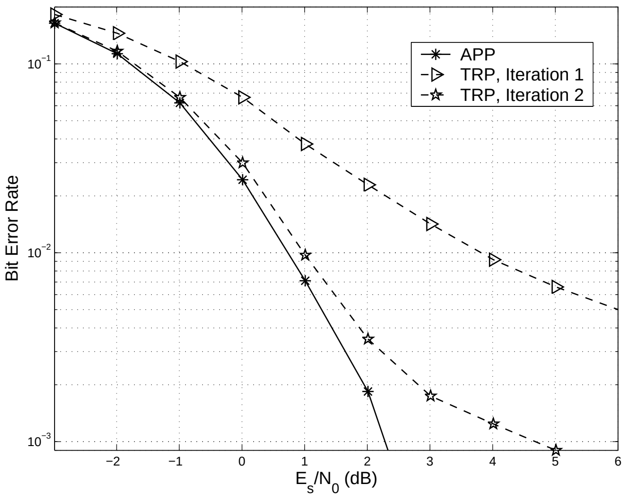

Fig. 4. Bit error rate performance in $n_t = 4$, $n_r = 4$ system.

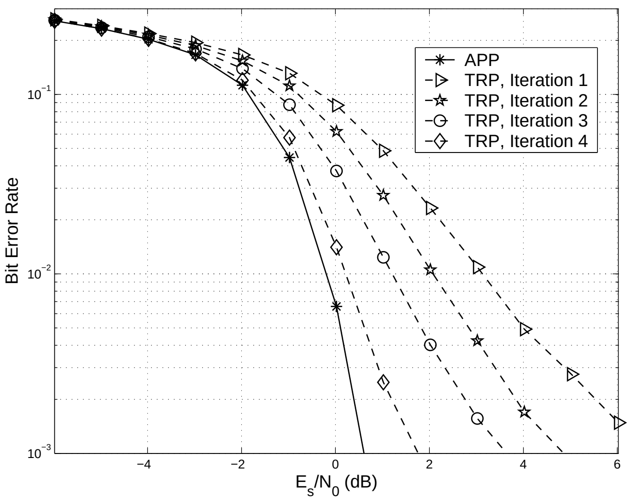

Fig. 5. Bit error rate performance in $n_t = n_r = 8$ system.

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