Railton, C. J., Rozzi, T., & Kot, J. (1986). The efficient calculation of high order shielded microstrip modes for use in discontinuity problems. In 16th European Microwave Conference, 1986. (pp. 529 - 534). Institute of Electrical and Electronics Engineers (IEEE). 10.1109/EUMA.1986.334247

THE EFFICIENT CALCULATION OF HIGH ORDER SHIELDED MICROSTRIP MODES FOR USE IN DISCONTINUITY PROBLEMS

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ABSTRACT

Previously published results for the higher order modes of microstrip have been restricted to few modes whereas for the analysis of discontinuities many modes must be taken into account. A method of obtaining the propagation coefficients and field patterns of a large number of modes with the minimum of computational effort is described. The results have been applied to the microstrip step discontinuity problem using both mode matching and variational formulations.

INTRODUCTION

In order to analyse a microstrip discontinuity it is necessary to calculate the field patterns of a large number of higher order modes, typically in excess of 50. The problem is essentially the location of the zeros of a characteristic equation. Because this equation also contains many poles, sometimes very close to the searched for zeros, care must be taken not to miss solutions on the one hand or to necessitate the performance of prohibitive amounts of computation on the other.

The work described herein uses a discrete space domain formulation to calculate a large number of higher order modes in a way which leads to their fast location and which ensures that no modes are missed out. This includes those pairs of modes with complex conjugate propagation constants of the type which have been recently reported in Finline [1] as well as the normal evanescent modes. It is entirely practicable to implement the computer program to perform these calculations on a "home computer".

FORMULATION

The formulation uses Galerkin's method with the microstrip currents as the unknown functions. By using test functions with the appropriate edge conditions, it is shown that accurate solutions for a very large number of modes are obtained using a basis expansion of only 2 functions in the longitudinal current and the derivative of the transverse current in the strip. In addition the field patterns can be accurately calculated. In all cases where the calculations involve the evaluation of an infinite series, accurate values are obtained by use of asymptotic functions with closed form sums.

The test functions which have been used are:

$$J_{zmr} = J'_{xmr} = \frac{T_m(2X_r/W_r)}{\sqrt{(1 - (2X_r/W_r)^2)}} \tag{1}$$

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where:

X_r is the position of the centre of the rth strip

W_r is the width of the r_{th} strip

$T_m(x)$ are Tchebychev polynomials

These functions are appropriate for strips placed anywhere on the air-dielectric interface and have the correct edge singularity. Their Fourier transforms are easily expressed in terms of Bessel functions and the solution can be just as easily implemented in the spectral domain.

Evaluation of the integral:

$$\int (\underline{E}_i \times \underline{H}_j) \cdot \hat{\underline{z}} \; dS \tag{2}$$

where i and j are different modes and the integral is taken over the guide cross section, shows that the calculated field patterns possess the expected orthogonality properties.

This contrasts with the situation recently reported for Finline [1] where a large number of basis functions are required when using the spectral domain method to calculate accurate field patterns.

The accurate location of the modes is accomplished by making use of the fact that the poles of the characteristic equation occur at the points where there is a slab loaded guide mode in a guide formed by totally removing the strip. Moreover between any two poles there can be zero, two or one roots. In the latter case a simple bisection algorithm will find the single root. In the former cases the minimum of the function is searched for. If roots are present they can be quickly located. In addition between any two roots there can be zero, two or one poles. If during a search of the real axis of the complex plane this requirement is violated, then complex roots are indicated and can be located by means of a quad algorithm. In a similar manner the initial location of the modes of the slab loaded guide is facilitated by the fact that the poles of the characteristic equation coincide with the modes of two different empty guides. By this means the complete mode spectrum can be found with a comparatively small amount of computation. <u>Indeed it is entirely practicable to implement the method on a Sinclair Spectrum computer with an available PASCAL compiler.</u>

The formulation can easily be applied to finline or to multi layer structures.

RESULTS FOR UNIFORM MICROSTRIP

Using the geometry given in Figure 2 at a frequency of 5GHz, modes 18 and 19 have been found to have complex conjugate propagation constants with strip widths in the region of 0.5mm - 2mm. Figure 1. shows the locus of these modes as the strip width varies. Also shown are the adjacent modes, the 17th and 20th, and the modes of a slab loaded guide formed by removing the strip, the latter are the vertical lines. It can

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be seen that the phase of the propagation constant becomes large where the locus crosses the position of a slab guide mode. Higher order complex modes exhibit this same property. Since complex modes occur as low as the 18th, it is necessary to include them in a discontinuity calculation.

VARIATIONAL FORMULATION OF MICROSTRIP STEP DISCONTINUITY

The formulation of the step discontinuity is a generalisation of the method given in [2] for waveguides. The differences arise from the fact that here we have a hybrid mode and we are not able to define a unique wave impedance. We make use of the continuity of E_x and E_y across the discontinuity and deduce that the impedances in the equivalent pi network are stationary with respect to the E field.

In particular we get for Z_{11}

$$\frac{1}{Z_{11}} = \frac{< \underline{\xi}(r') | \underline{\underline{G}}(r,r') | \underline{\xi}(r) >}{< \underline{\xi} , \underline{H_1^1} >^2} \tag{3}$$

where the Green's function is given by:

$$\underline{\underline{G}}(r,r') = \sum_{n=2}^{\infty} \frac{\underline{H_n^1}(r)\underline{H_n^1}(r')}{< \underline{E_n^1} , \underline{H_n^1} >} + \frac{\underline{H_n^2}(r)\underline{H_n^2}(r')}{< \underline{E_n^2} , \underline{H_n^2} >} \tag{4}$$

Where the superscripts 1 and 2 refer to the corresponding regions in Fig. 3, the subscript n refers to the mode number in that region and $\xi(r)$ is the trial field function.

Similar expressions are obtained for the other impedances. The equations are solved using Galerkin's method and expanding the unknown E field in a set of basis functions. For a quickly converging solution these basis functions must take account of the singularity of the electric field at the corner.

Once the Z matrix is calculated, it can be transformed into the more useful S matrix using standard techniques.

Figure 4. shows the results of |S11| plotted against frequency for a step discontinuity with the geometry given in Figures 2 and 3. The quasi-static results [3] are plotted for comparison. It can be seen that they agree at the low frequency limit. Fifty modes on each side of the discontinuity were required for good convergence. It can be seen that, at low frequqencies, |S11| decreases with frequency, this is the result which would be given by ignoring all higher order modes. As the frequency rises and the higher order modes begin to exert a significant influence, it can be seen that the value of |S11| rises.

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CONCLUSION

The technique described herein is capable of producing the propagation constants and field patterns of large numbers of higher order microstrip modes and can be extended to other planar waveguide structures. The modes are calculated efficiently and accurately using the minimum of computational effort. The results have been used in a variational formulation of the microstrip step discontinuity with results in agreement with quasi static formulations in the low frequency limit.

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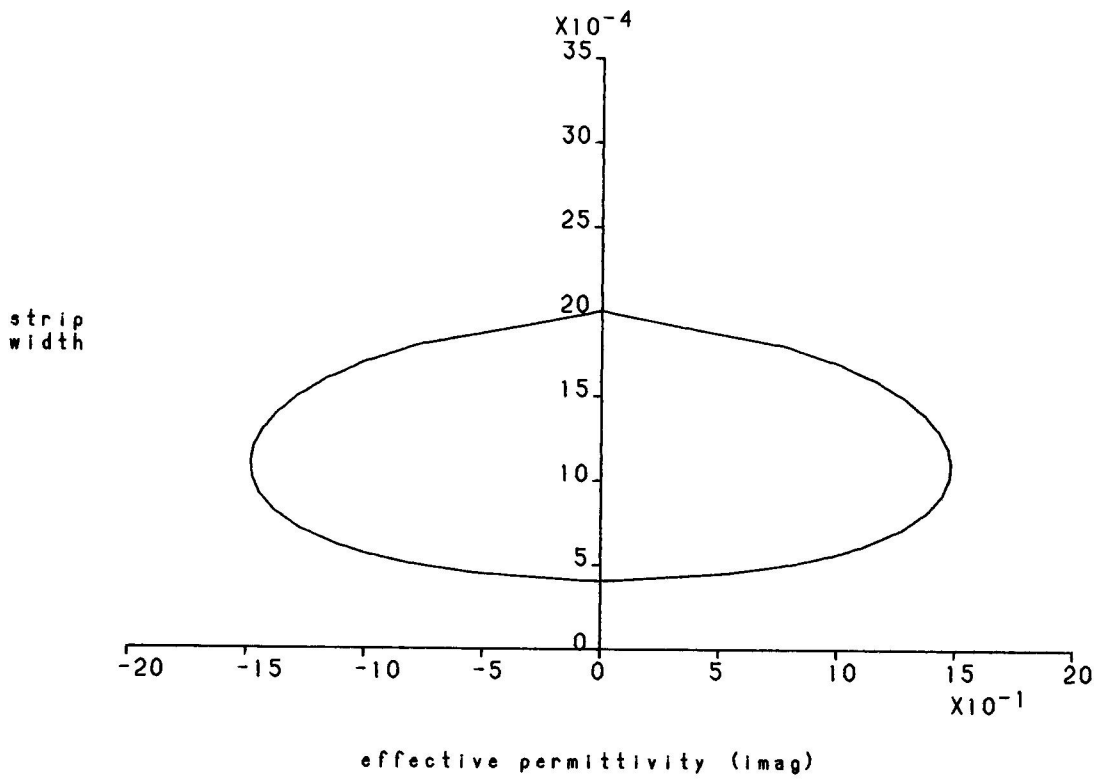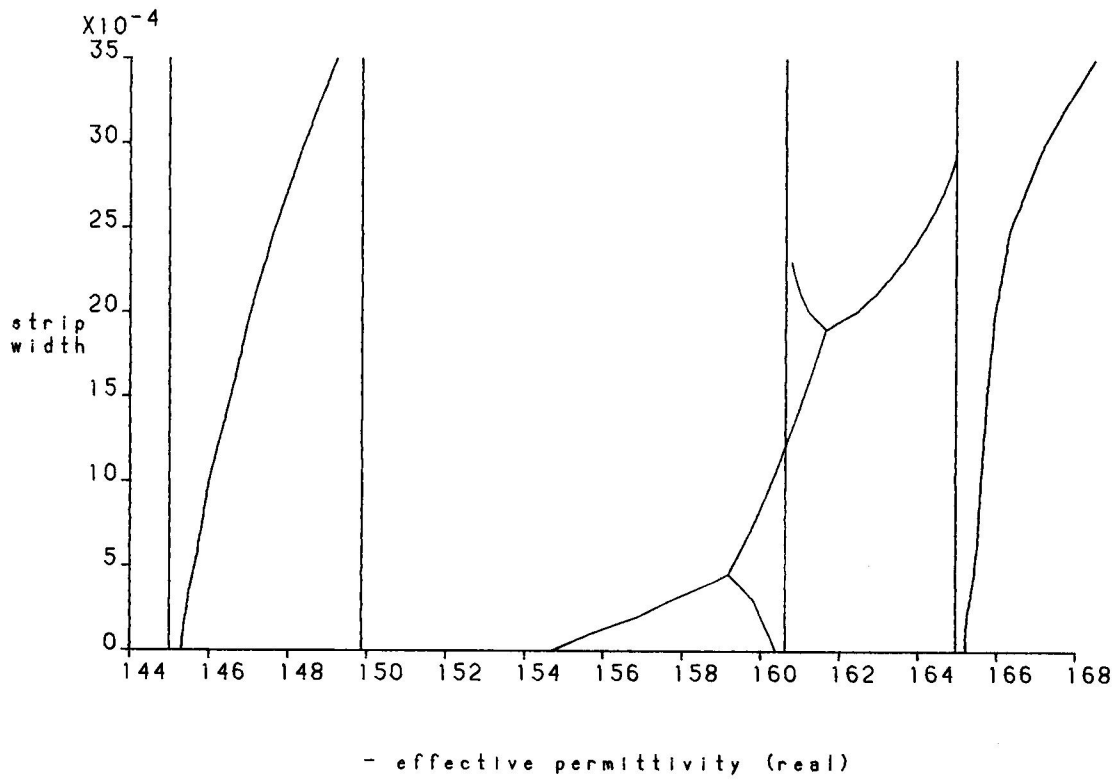Radio and Electronic Engineer Vol. 46 No. 7 page 360 1976

X10⁻⁴

strip
width

— effective permittivity (real)

X10⁻⁴

strip
width

effective permittivity (imag)

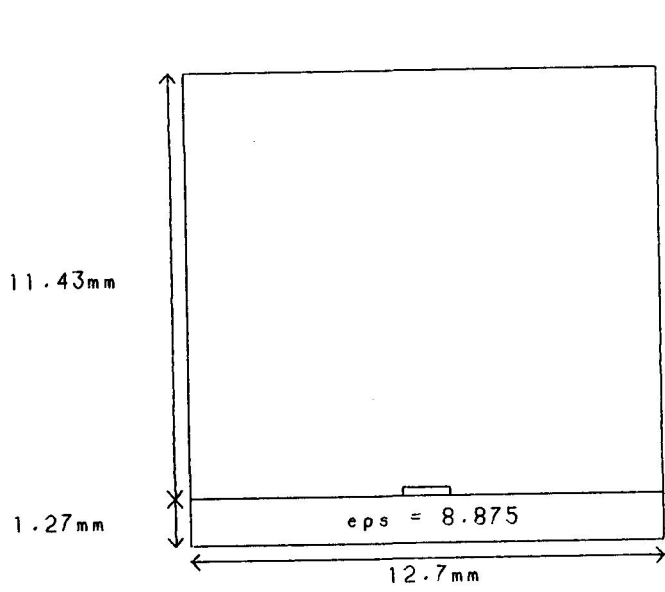Fig. 1. Effective Permittivity v.strip width
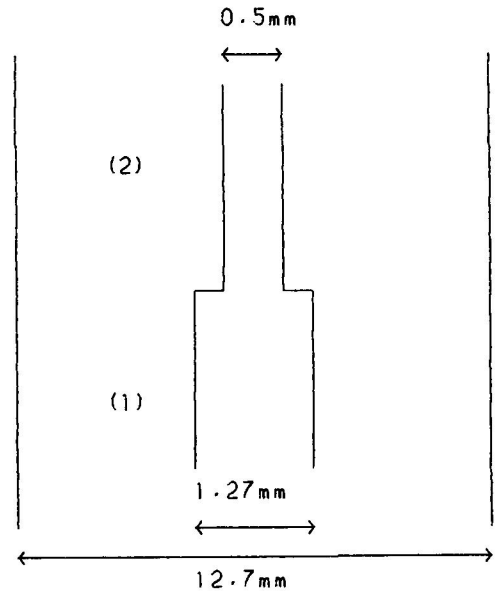
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Fig.2. Microstrip Cross Section

Fig.3. Plan of step discontinuity
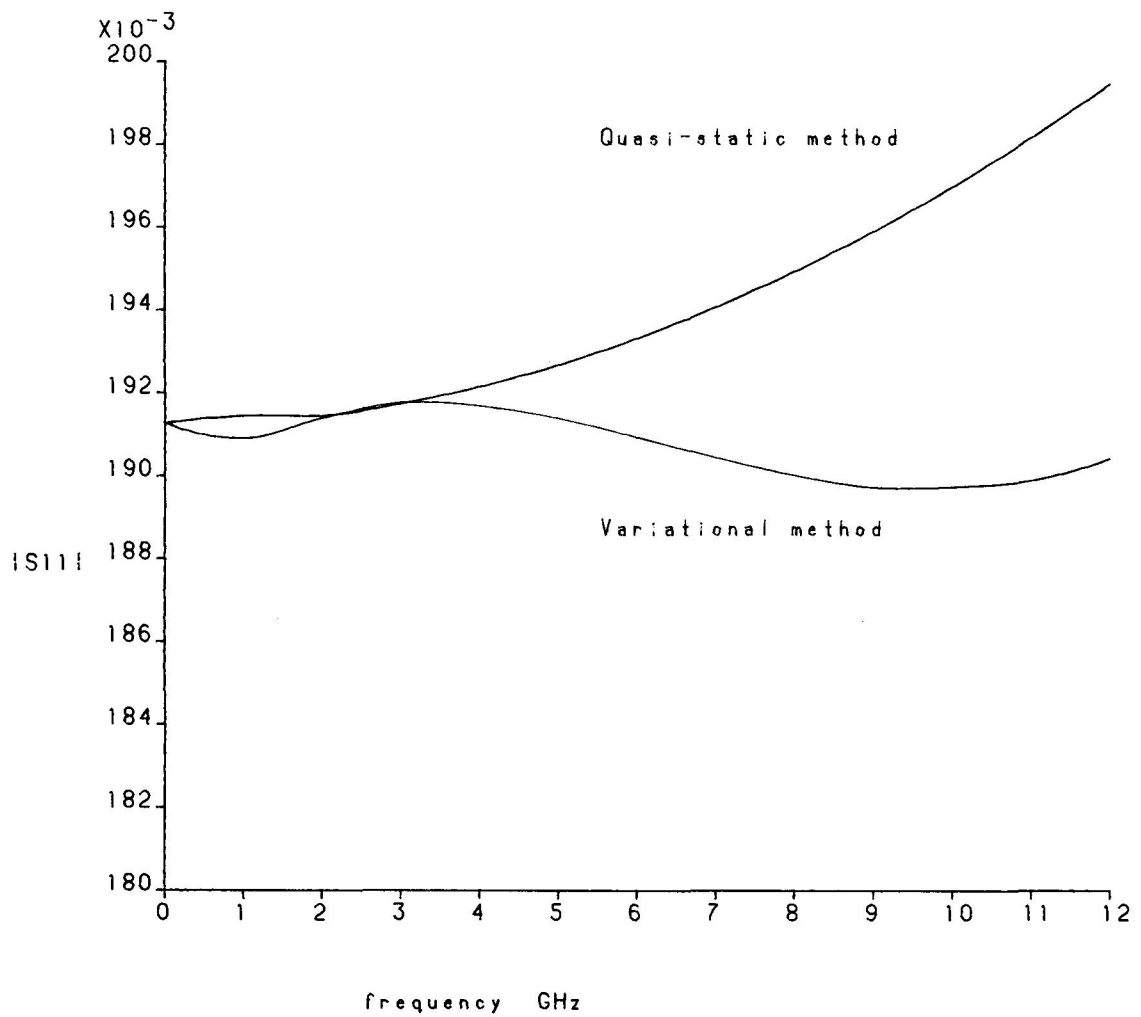
Fig.4. $|S_{11}|$ for a step discontinuity

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