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# An Investigation of Pattern Correlation and Mutual Coupling in MIMO Arrays

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## Introduction

- MIMO systems employ antenna arrays for increased capacity.
- The increased capacity is a result of diversity.
- All antenna systems exhibit mutual coupling between elements.
- The role of this coupling in diversity performance is often controversial.
- The coupling is especially important when elements are closely spaced (e.g. on a user terminal).
- Does the coupling improve diversity performance? or degrade it?



## Definition of Pattern Correlation

Following Vaughan and Bach Anderson, the envelope correlation between two radiation patterns  $\mathbf{F}_1$  and  $\mathbf{F}_2$  is given by:

$$|\rho|^2 = \frac{\left| \oint_{\Omega} \mathbf{F}_2 \cdot \mathbf{F}_1^* d\Omega \right|^2}{\left( \oint_{\Omega} |\mathbf{F}_1|^2 d\Omega \right) \left( \oint_{\Omega} |\mathbf{F}_2|^2 d\Omega \right)}$$

- This is the measure of similarity between two radiation patterns.
- Considering correlation in 3D,  $\Omega$  is the solid angle and the limits of integration are a sphere.
- a rule-of-thumb is that good diversity operation is possible when  $|\rho|^2$  is  $< 0.5$
- provided also that the SNR at each antenna is approximately equal (Vaughan & Bach Anderson).*

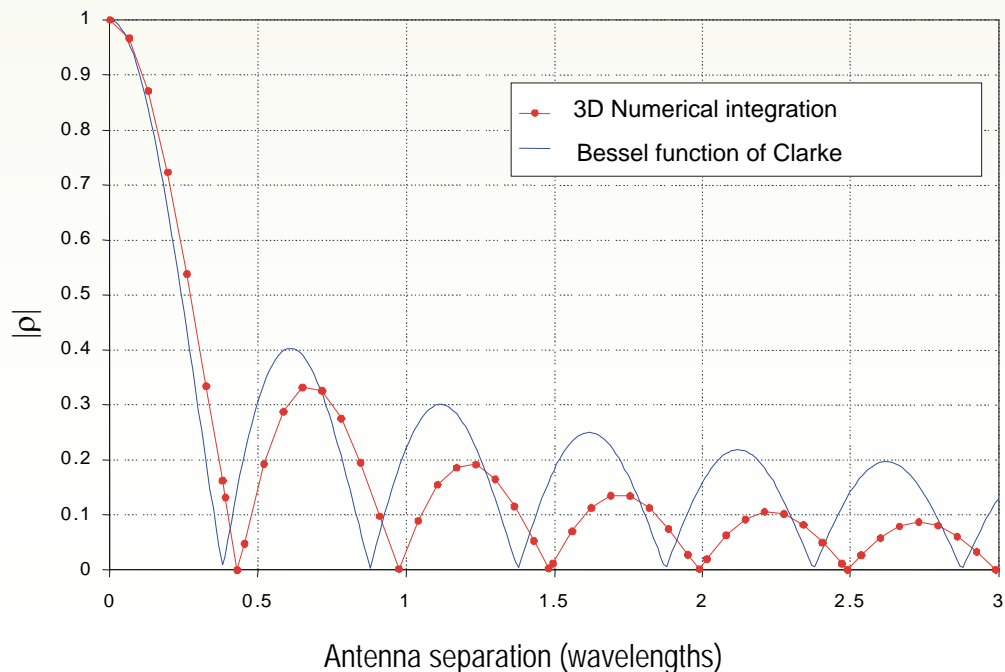


# Analytical Pattern Correlation

The correlation between fields at two points (Clarke 1968), separated by a distance  $d$ :

$$|\rho|^2 = \left[ J_0\left(\frac{2\pi d}{\lambda}\right) \right]^2$$

This result is also valid for the correlation between identical antenna patterns. It was derived assuming correlation around a circle in the far-field – not a sphere.

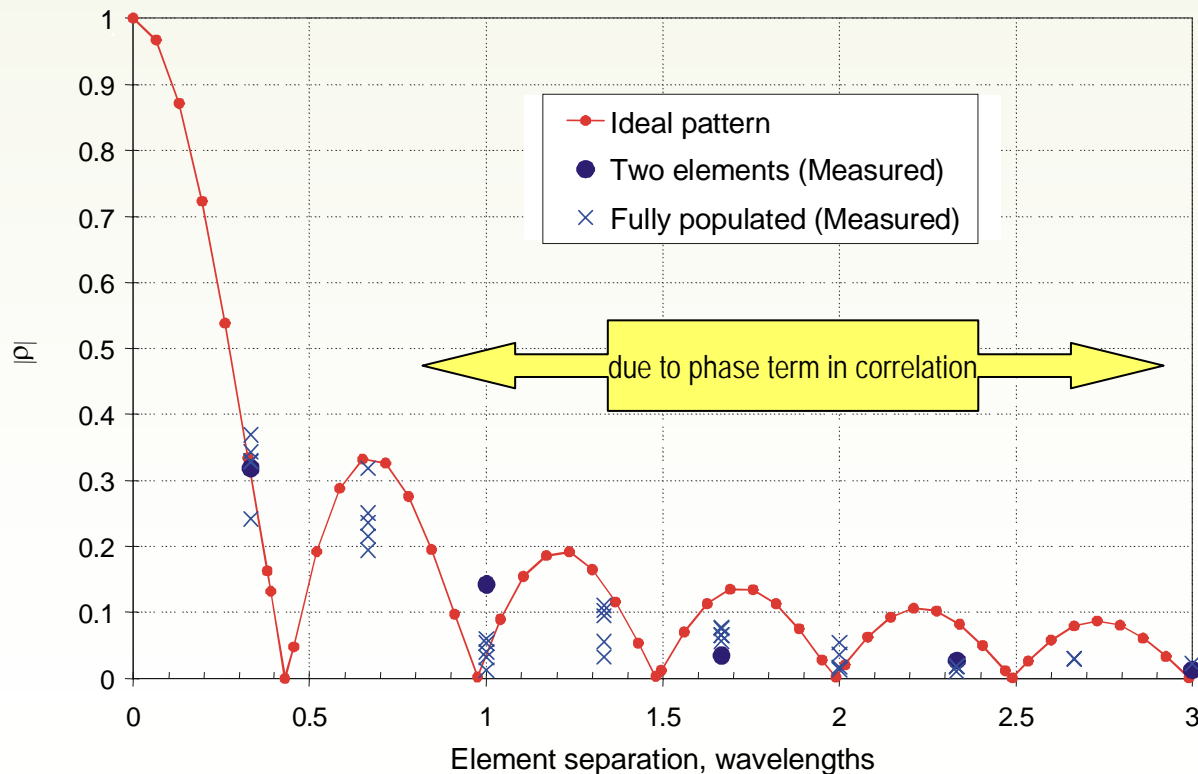


Note the more rapid decay  
Note the null close to half-wavelength



# Experimental Pattern Correlation (1)

Experimental results using an array (or just 2 elements) of monopoles:

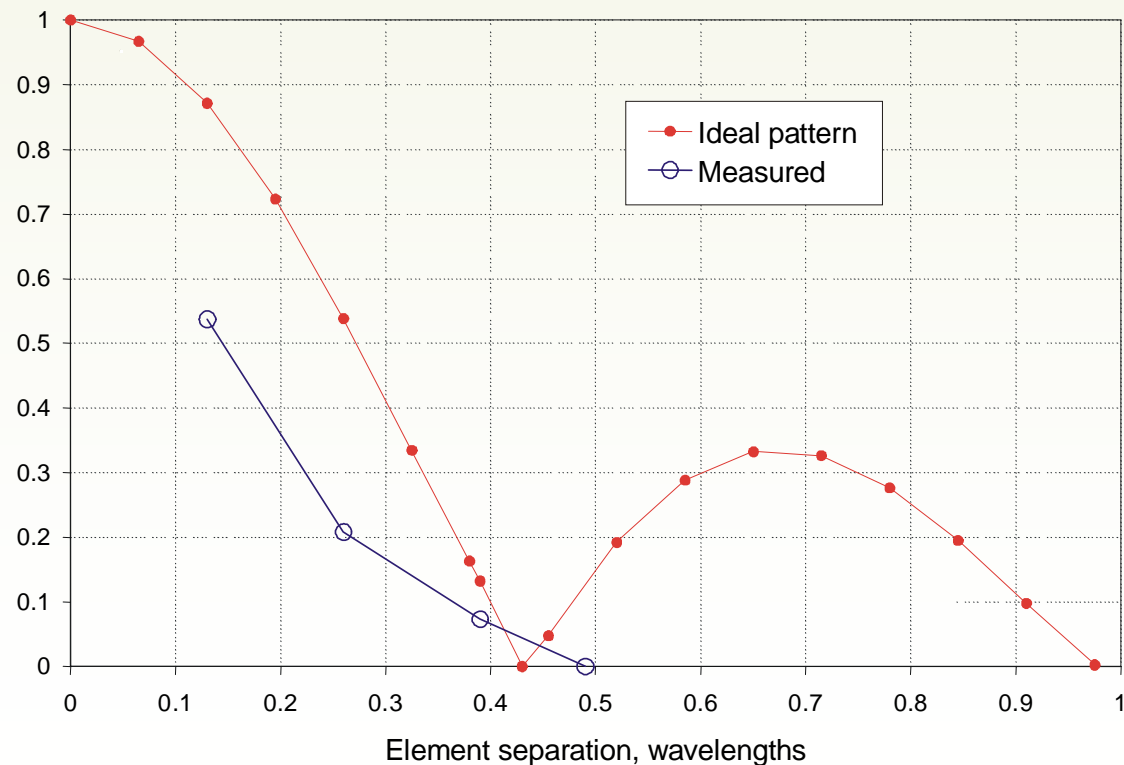


Patterns are not identical.  
Ground plane is finite.  
Note again a more rapid decay.  
Note some "scatter" on results.  
Note correlation at 0.35 similar to 0.7!



# Experimental Pattern Correlation (2)

Experimental results using a redesigned array of 2 monopole elements:



Low correlation down to very small spacings  
Due purely to mutual coupling distorting the pattern



## Alternative Calculation of Pattern Correlation

The pattern correlation result can also be found from the input ports.

Derived by Clarke in 1968 in terms of the Z-matrix, and restated by Blanch, Romeu and Corbella in terms of S-parameters (El. Letts. vol 39, May 2003).

$$|\rho|^2 = \frac{\left| \oint_{\Omega} \mathbf{F}_2 \cdot \mathbf{F}_1^* d\Omega \right|^2}{\left( \oint_{\Omega} |\mathbf{F}_1|^2 d\Omega \right) \left( \oint_{\Omega} |\mathbf{F}_2|^2 d\Omega \right)} = \frac{|S_{11}^* S_{12} + S_{21}^* S_{22}|}{\left(1 - (|S_{11}|^2 + |S_{21}|^2)\right) \left(1 - (|S_{22}|^2 + |S_{12}|^2)\right)}$$

An important result =>

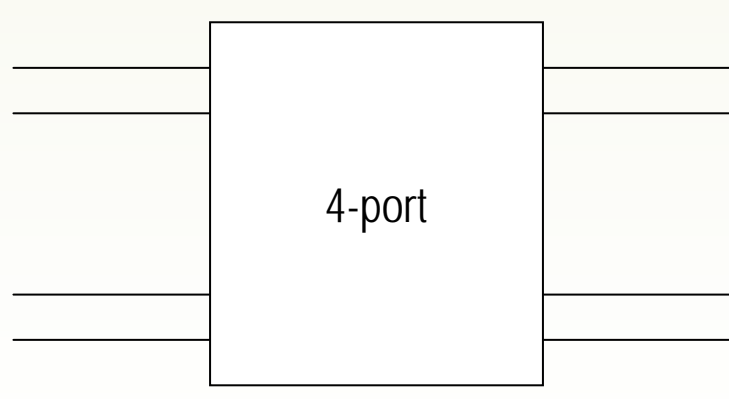
- The S-parameters, and hence the correlation, may be (almost) arbitrarily altered by the addition of a network at the antenna terminals, regardless of spacing.
- Matching is important (indeed, a perfect match gives zero pattern correlation).





## Introducing a 4-Port (1)

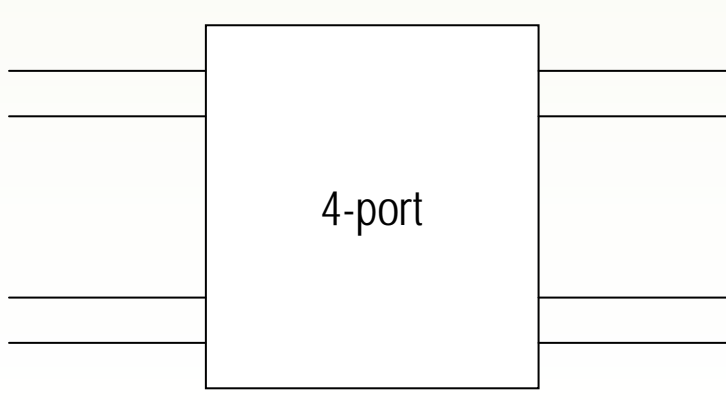
- The S-parameters, and hence the correlation, may be (almost) arbitrarily altered by the addition of a network at the antenna terminals, regardless of spacing.
- For a two element array, the network is a 4-port.





## Introducing a 4-Port (2)

- For these initial investigations, assume infinitesimally thin dipoles, Richmond's formula is therefore valid (assuming a sinusoidal current distribution).
- Radiation patterns and mutual couplings may be calculated from simple integral expressions.
- The input parameters to the left of the 4 port may then be found.
- These parameters then give the pattern correlation (equivalently the (loaded) patterns may be calculated – the result is the same, but more time-consuming to evaluate).
- 4-port parameters are then optimised to give zero correlation.

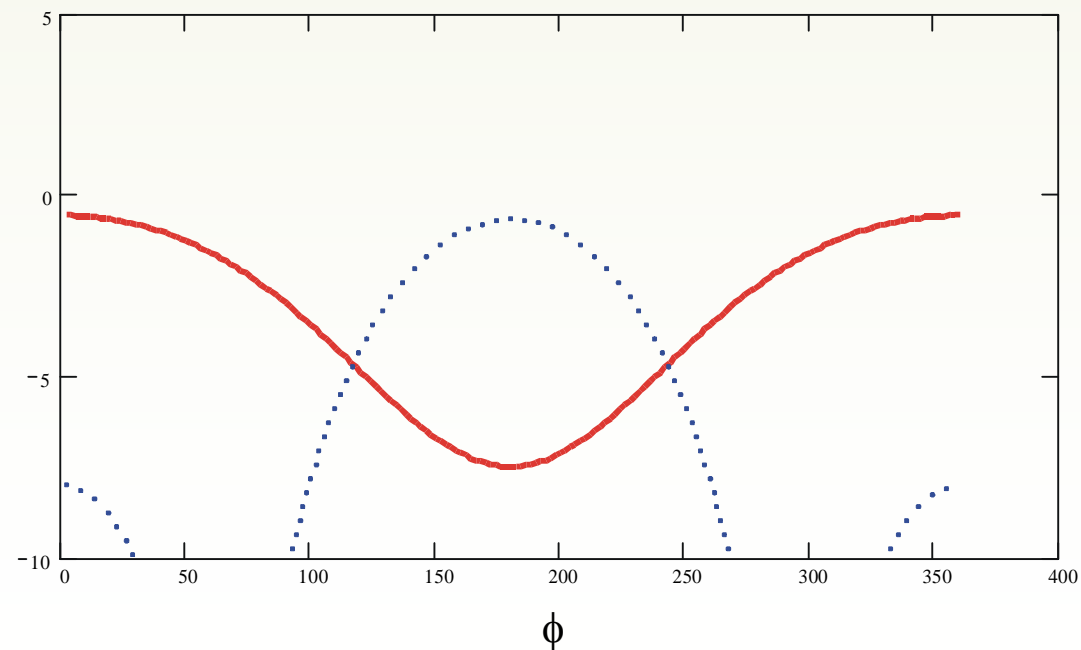




## Introducing a 4-Port (3): Results

- There is more than one 4-port that will give zero correlation.
- Dipole separations of only  $\lambda/20$

dB



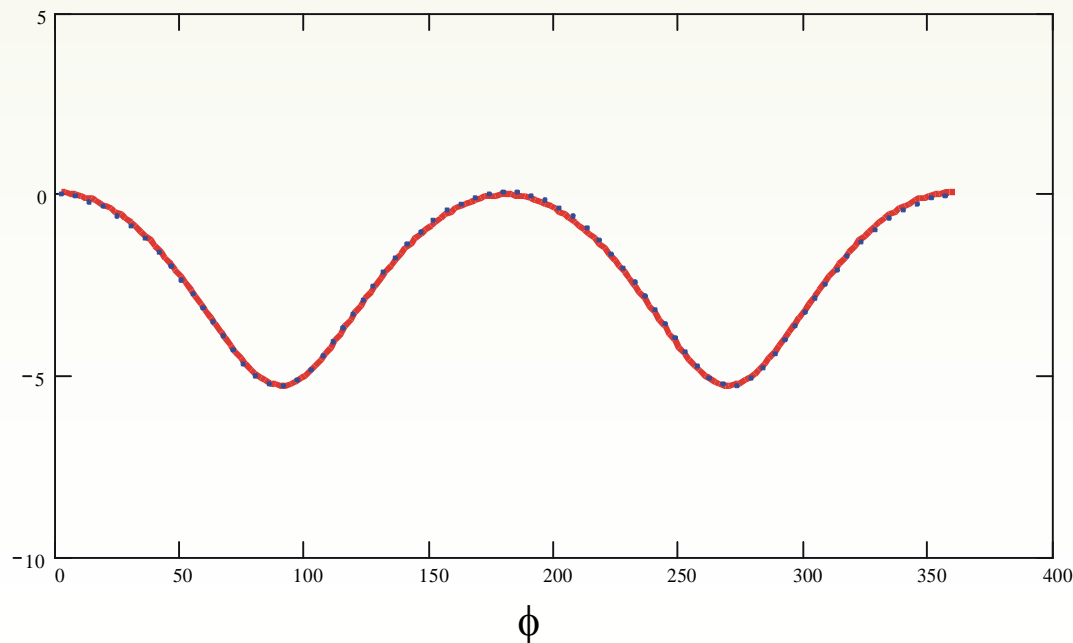
Zero correlation  
Nulls in patterns



## Introducing a 4-Port (4): Results

- There is more than one 4-port that will give zero correlation.
- Dipole separations of only  $\lambda/20$

dB



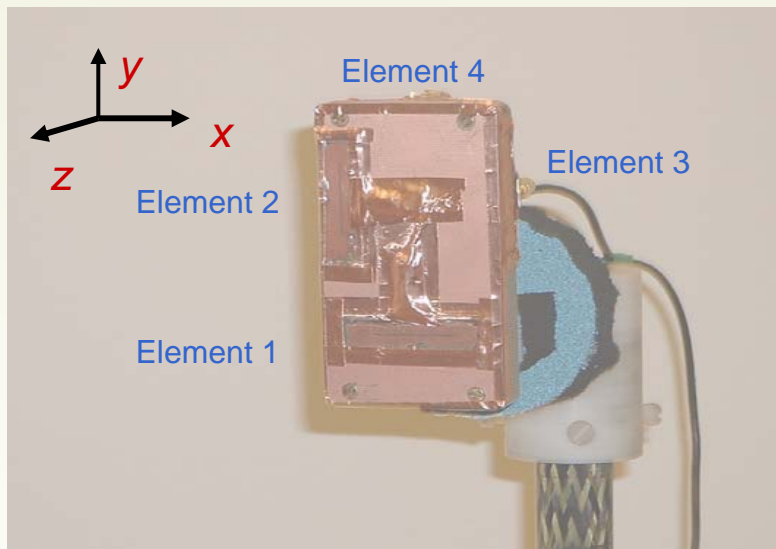
Zero correlation

*Relatively* omni-directional patterns

Note: amplitudes are correlated but phases not shown



# Closely-spaced elements



A PDA-sized box  
4 identical slot antenna elements ( $\sim \lambda/2$  spacing)  
Different orientations.

$$\frac{\left| \oint_{\Omega} \mathbf{F}_2 \cdot \mathbf{F}_1^* d\Omega \right|^2}{\left( \oint_{\Omega} |\mathbf{F}_1|^2 d\Omega \right) \left( \oint_{\Omega} |\mathbf{F}_2|^2 d\Omega \right)}$$

$$\frac{\left| \oint_{\Omega} |\mathbf{F}_2| \cdot |\mathbf{F}_1| d\Omega \right|^2}{\left( \oint_{\Omega} |\mathbf{F}_1|^2 d\Omega \right) \left( \oint_{\Omega} |\mathbf{F}_2|^2 d\Omega \right)}$$

	Element 1	Element 2	Element 3	Element 4
Element 1	<b>1.00 (1.00)</b>	0.024 (0.6356)	0.005 (0.5225)	0.024 (0.733)
Element 2		<b>1.00 (1.00)</b>	0.022 (0.7879)	0.020 (0.5428)
Element 3			<b>1.00 (1.00)</b>	0.015 (0.6063)
Element 4				<b>1.00 (1.00)</b>

NB: Table (like graphs, gives square root of correlation)



## Conclusions

- Pattern correlation gives an indication of diversity performance.
- Element spacing gives low correlation, but this can also be obtained at close spacings.
- Elements spaced at  $\lambda/2$  – or even much less - can give decorrelated patterns through:
  - Distortion of embedded patterns due to coupling.
  - Optimisation of S-parameters (see also Morris & Jensen, 2004 URSI EMTS, vol. 1).
  - Dissimilar antenna elements (or differently-orientated but otherwise identical elements)
- While these techniques can give nearly- or exactly-zero correlation (even with  $\lambda/20$  spacings) , diversity performance will only be maintained if pattern coverage is good (i.e. if the SNR is similar).