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A Near-Far Resistant Sliding Window Decorrelating Algorithm for Multi-User Detectors in DS-CDMA Systems

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Abstract

The limitations and inherent shortcomings of conventional multi-user detectors in DS-CDMA networks have been recognised [1, 2, 3]. The conventional detector has been acknowledged to be particularly susceptible to multi-user interference and hence the near-far problem. We detail the development of a near-far resistant decorrelating algorithm for application in practical networks. We pay consideration to a mobile radio environment where received user energies can be dissimilar and time varying (A near-far environment). In particular the algorithm is targeted at multiple-access (MA) limited, operating conditions which are often found in systems with a large number of users.

The performance of the algorithm is simulated in AWGN, fading, and mobile radio channels.

1 Introduction

The conventional multi-user detector (Figure 1) recovers the information from a DS-CDMA spread spectrum signal by correlating the incoming signal with synchronised replica codes, followed by a thresholding operation on the result of the correlation. This strategy gives satisfactory performance provided :

1. The assigned signature waveforms have low cross-correlations for all possible relative delays between the transmissions of the asynchronous users.
2. The power of the received signals are similar.

The second condition relates directly to the near-far problem which is acknowledged as the principle short-

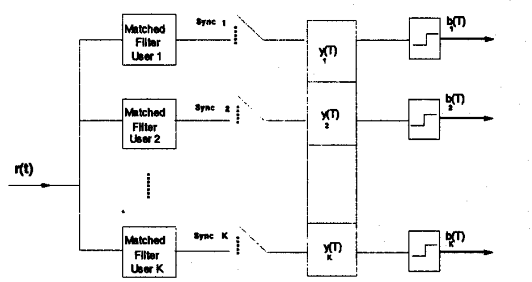

Figure 1: Conventional Multi-User Receiver

coming of DS-CDMA radio networks. It has been experienced that even with quasi-orthogonal codes, large differences in received energies result in unacceptable performance. This is due to the matched filter outputs possessing a spurious component linear in the amplitude of each of the interfering users. It must be noted that in asynchronous systems quasi-orthogonal cross-correlation conditions can seldom be guaranteed.

Many researchers [1, 2, 4, 5] consider the near-far problem to be an inherent shortcoming of the conventional receiver and not of the DS-CDMA concept itself. Two solution concepts have emerged in the literature. One is based one the optimum multi-user detector [1] and approximations to it [4], and use a probabilistic solution approach. The optimum multiuser detector has computational complexity exponential in the number of users. It also demands knowledge of the user received energies. These factors preclude its application in practical systems.

The second approach is based on the class of linear detectors defined by Lupas and Verdu [2]. They use a linear systems approach and show that the decorrelating detector is the optimum linear detector which elim-

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inates multi-user interference. For reasons discussed below, the resulting detectors though optimal do not lend themselves to practical implementation.

We therefore aim to develop a near-far resistant decorrelating algorithm which

1. Can be implemented in a practical radio network
2. Requires knowledge of the PN codes only
3. Does not require knowledge of the received energies

2 Multi-user DS-CDMA Model

We recall the multi-user communication model described in references [1, 2, 6], which is the basis for the class of linear detectors.

2.1 Terminology

$$
\begin{aligned}
b_k(i) &= i^{th} \text{ bit of the } k^{th} \text{ user} \\
w_k(i) &= \text{Transmit energy of } b_k(i) \\
s_k(t) &= \text{Signature waveform of } k^{th} \text{ user} \\
\tau_k &= \text{Delay of } k^{th} \text{ user (relative to some datum)} \\
y_k(i) &= \text{Matched filter output corresponding to } b_k(i) \\
K &= \text{Number of users in system} \\
n(t) &= \text{AWGN with noise power } \sigma^2
\end{aligned}
$$

We number the users according to their delays so that

$$0 \leq \tau_1 \leq \tau_2 ..... \leq \tau_K < T \quad (1)$$

Also define

$$\tilde{s_k}(t) = \begin{cases} s_k(t) & \text{if } 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

2.2 Discrete Time Model

The DS modulated signal for a transmission of length N, received at the base station (after down conversion and de-modulation) can be written as

$$r(t) = \sum_{i=1}^{N} \sum_{k=1}^{K} b_k(i)\sqrt{w_k(i)}\tilde{s}_k(t - iT - \tau_k) + n(t) \quad (3)$$

The matched filter output $y_k(i)$ is then :

$$y_k(i) = \int_{iT+\tau_k}^{iT+T+\tau_k} r(t)\tilde{s}_k(t - iT - \tau_k)dt \quad (4)$$

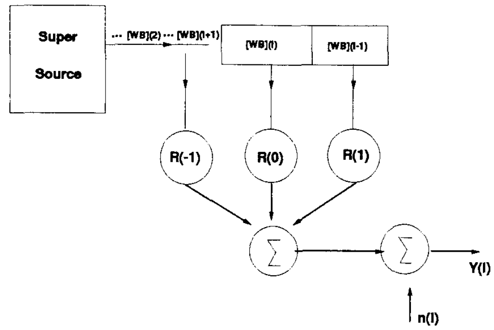

Figure 2: Discrete Time Model

Expanding we have :

$$y_k(l) = S_1 + S_2 + S_3 \quad (5)$$

where

$$
\begin{aligned}
S_1 &= \sum_{i=1}^{N} \int_{-\infty}^{\infty} b_k(i)\sqrt{w_k(i)}\tilde{s}_k(t-iT-\tau_k)\tilde{s}_k(t-lT-\tau_k)dt \\
S_2 &= \sum_{i=1}^{N} \sum_{j=1_{j\neq k}}^{K} \int_{-\infty}^{\infty} b_j(i)\sqrt{w_j(i)}\tilde{s}_j(t-iT-\tau_j)\tilde{s}_k(t-lT-\tau_k)dt \\
S_3 &= \int_{-\infty}^{\infty} n(t)\tilde{s}_k(t-lT-\tau_k)dt
\end{aligned}
$$

Let $R(l)$ be the $K * K$ normalised signal cross-correlation matrices such that

$$R_{k,j}(l) = \int_{-\infty}^{\infty} \tilde{s}_k(t - \tau_k)\tilde{s}_j(t - lT - \tau_j)dt \quad (6)$$

It follows that

$$R(l) = 0 \text{ for } \mid l \mid > 1 \quad (7)$$

$$R(-l) = R^T(l) \quad (8)$$

We use a vector notation for the matched filter outputs, transmit data and noise (eg $\underline{y}(l) = [y_1(l), y_2(l), ..., y_K(l)]$), and also define a transmit energy matrix.

$$W(l) = diag\left(\sqrt{w_1(l)}, \sqrt{w_2(l)}, ..., \sqrt{w_k(l)}\right) \quad (9)$$

S_1 of (5) is the term we seek for the k^{th} user. S_2 is the MA interference and S_3 the thermal noise. Substituting from (6,7) in (5)

$$\underline{y}(l) = R(-1)W(l+1)\underline{b}(l+1) + R(0)W(l)\underline{b}(l) + R(1)W(l-1)\underline{b}(l-1) + \underline{n}(l) \quad (10)$$

where $\underline{n}(l)$ is the matched filter output noise vector at time instant $t = lT$ with auto-correlation matrix given by :

$$E\left[n(i)n^T(i)\right] = \sigma^2 R(i - j) \quad (11)$$

This leads us to the discrete time model of Figure 2. The model represents the entire reverse (multiple mobiles to base) link up to and including the the matched filtering process. It must be noted that a single element of $\underline{y}(l)$ consists of the following components:

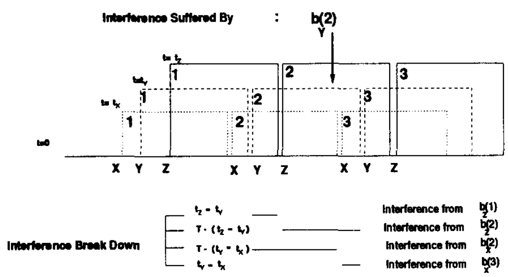

Figure 3: Multi-User Interference

1. The required signal $\sqrt{w_k(l)}b_k(l)$.
2. Interference from $l+1^{th}$,l^{th} and $l-1^{th}$ bits of other users.
3. correlated additive noise.

The formulation of these interfering components is demonstrated by Figure 3. The partial correlation matrices can be computed from information available at the base station as follows.

$$
\begin{aligned}
R(0)_{i,j} &= \frac{1}{N_c}\sum_{h=\tau_i}^{N_c-\tau_j} s_i(h-\tau_i)s_j(h-\tau_j) \\
R(1)_{i,j} &= \frac{1}{N_c}\sum_{h=\tau_i}^{\tau_j} s_i(h-\tau_i)s_j(h-\tau_j) \\
R(-1) &= R(1)^T
\end{aligned} \tag{12}
$$

where N_c is the number of chips per bit.

3 Near-Far Resistance and MA limited Conditions

We target our decorrelating algorithm at MA limited channels where the dominant source of interference is the presence of multiple users rather than thermal noise.

In order to investigate the receiver performance from the near-far problem point of view, we use *near-far resistance* as an added performance criterion as defined in [2]. We start with the definition of *efficiency*, η.

$$\eta = \frac{\text{Effective SNR}}{\text{Actual SNR}} \tag{13}$$

where the Effective SNR is the SNR Required to Support the Same Bit Error Probability in the Absence of Other Users.

The *efficiency* characterises the performance loss due to other users in the system. This leads to yet another performance measure termed the *asymptotic efficiency*, η_0.

$$\eta_0 = \lim_{noise\to 0} efficiency \tag{14}$$

It can thus be seen that η_0 characterises the performance loss in MA limited conditions, and $\eta_0 = 0$ suggests that there is an irreducible probability of error and the receiver is not *Near-Far Resistant.*

4 Linear Detectors

We consider two classes of linear detectors.

4.1 Finite Length Data Sequence

The finite sequence length formulation implies that no data is transmitted for one bit period by any user in the system, prior to or following the data block. In the case of a mobile radio network this would demand a high degree of coorporation between users, which in turn would place unacceptable demands on the networking protocols.

We rewrite (10) in full as :

$$
\begin{bmatrix} \underline{y}(1) \\ \underline{y}(2) \\ \underline{y}(3) \\ \vdots \\ \underline{y}(N) \end{bmatrix}
=
\begin{bmatrix} R(0) & R(-1) & 0 & \cdots & 0 \\ R(1) & R(0) & R(-1) & \cdots & 0 \\ 0 & R(1) & R(0) & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & R(1) & R(0) \end{bmatrix}
\mathcal{W}
\begin{bmatrix} \underline{b}(1) \\ \underline{b}(2) \\ \underline{b}(3) \\ \vdots \\ \underline{b}(N) \end{bmatrix}
+
\begin{bmatrix} \underline{n}(1) \\ \underline{n}(2) \\ \underline{n}(3) \\ \vdots \\ \underline{n}(N) \end{bmatrix}
\tag{15}
$$

where

$$\mathcal{W} = diag\left[W(1), W(2), W(3), \cdots, W(N)\right]$$

$$\underline{y} = \Re\mathcal{W}\underline{b} + \underline{n} \tag{16}$$

It is clear that $\Re$ is $NK * NK$, the inversion of which is not feasible for practical sequence length values.

However $\Re$ is block tri-diagonal, and this property can be exploited.

4.2 Infinite Length Data Sequence

Lupas and Verdu [2] propose a LTI filter implementation of the decorrelation process. The filter has transfer function

$$G(\mathcal{Z}) = \frac{1}{det\left(S(\mathcal{Z})\right)} adj\left(S(\mathcal{Z})\right) \tag{17}$$

Where

$$S(\mathcal{Z}) = R^T(1)\mathcal{Z} + R(0) + R(1)\mathcal{Z}^{-1} \quad (18)$$

Since each element of $S(\mathcal{Z})$ will be a polynomial of the form $az^{-1}+b+cz$, the solution involves inverting a $K * K$ matrix, the elements of which are second order polynomials in z. In addition to the computational complexity involved, this suggests that the system is inflexible to changes in the timing configuration and to the addition/removal of users.

In the next section we propose a novel sliding window decorrelator. Our algorithm is based on adapting finite sequence length theory to the infinite length data sequence. We therefore combine the virtues of low computational complexity, the structure of $\Re$, and minimal corporation between users of an asynchronous system.

5 Sliding Window Decorrelator

From now on we assume an infinite length data sequence. This places no demands on the coperation between users in the system. Our strategy is to partition the infinite sequence into blocks of size $\acute{N}$. Here $\acute{N}$ is such that the solution of the linear system is feasible within a time period of the order of one data bit period. We first define a valid linear system which can be solved within a finite data window (rather than the entire data sequence).

We rewrite (15) using a variable M, which represents the offset from the start of the sequence, of the begining of the current window.

$$\begin{bmatrix} \underline{y}(M+1) \\ \underline{y}(M+2) \\ \underline{y}(M+3) \\ \vdots \\ \underline{y}(M+\acute{N}) \end{bmatrix} = \begin{bmatrix} R(0) & R(-1) & 0 & \cdots & 0 \\ R(1) & R(0) & R(-1) & \cdots & 0 \\ 0 & R(1) & R(0) & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & R(1) & R(0) \end{bmatrix} \mathcal{W} \begin{bmatrix} \underline{b}(M+1) \\ \underline{b}(M+2) \\ \underline{b}(M+3) \\ \vdots \\ \underline{b}(M+\acute{N}) \end{bmatrix} + \mathcal{N} \quad (19)$$

where

$$\mathcal{N}^T = [\underline{n}(M+1), \underline{n}(M+2), \underline{n}(M+3), \cdots, \underline{n}(M+\acute{N})]$$

and

$$\mathcal{W} = diag[W(M+1), W(M+2), W(M+3), \cdots, W(M+\acute{N})]$$

Since both the energies and the bit values are unknown at the receiver, we seek both these values. The

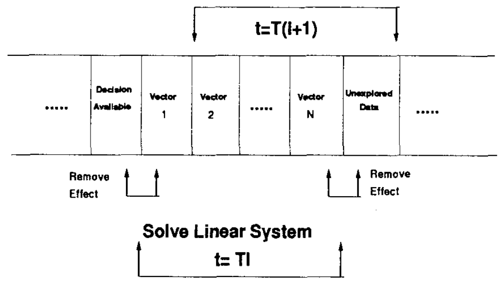

Figure 4: Schematic Representation of Sliding Window Decorrelator

linear system (19) in invalid as it stands, since according to the discrete time model (10), $\underline{y}(M+1)$ and $\underline{b}(M)$ are related. Similarly for $\underline{y}(M+\acute{N})$ and $\underline{b}(M+\acute{N}+1)$.

Define a continuous valued transmit data vector as follows.

$$\underline{WB}(i) = \left[\sqrt{w_1(i)}b_1(i), \sqrt{w_2(i)}b_2(i), ..., \sqrt{w_K(i)}b_K(i)\right]$$

It is clear that the requried correction terms are $R(1)\underline{WB}(M)$ in the case of $\underline{y}(M+1)$, and $R(-1)\underline{WB}(M+\acute{N}+1)$ in the case of $\underline{y}(M+\acute{N})$.

Now define a modified input vector X by

$$\begin{aligned} \underline{x}(M+1) &= \underline{y}(M+1) - R(1)\underline{WB}(M) \\ \underline{x}(M+\acute{N}) &= \underline{y}(M+\acute{N}) - R(-1)\underline{WB}(M+\acute{N}+1) \\ \underline{x}(M+i) &= \underline{y}(M+i) \text{ for } 1 < i < \acute{N} \quad (20) \end{aligned}$$

We have reduced our problem to

1. Obtaining reasonable estimates of the terms $\underline{WB}(M)$ and $\underline{WB}(M+\acute{N}+1)$
2. Solving the linear system $\underline{X} = \Re\mathcal{W}\underline{b} + \underline{n}$, $\Re$ block tri-diagonal.

5.1 Estimation of Explored Data

Provided the receiver is initialised in a suitable manner, it would have already solved the decorrelation problem for $\underline{WB}(M)$. As shown in Figure 4 an estimate is available to the best accuracy of the method.

Clearly our estimate holds with equality when the noise power spectral density (NPSD) is zero. Hence we conform to the near-far resistant condition when making this estimate. Referring to Figure 4 it is clear that by allowing the window to slide by two vectors at each iteration (for reasons discussed below), we obtain

$\acute{N}/2$ estimates for each vector $\underline{WB}(i)$. We propose an averaging process over the estimates obtained. It is likely that the effects of the random thermal noise will be reduced as a result of the averaging process.

5.2 Prediction of Unexplored Data

The only information available to aid the estimation of $\underline{WB}(M+\acute{N}+1)$ is the corresponding vector of matched filter outputs. Since we assume that MA interference is the dominant source of interference, and underpins our investigation, we cannot approximate $\widehat{\underline{WB}}(M+\acute{N}+1)$ by $\underline{y}(M+\acute{N}+1)$.

We use the notation $\hat{\underline{W}}(.)$ and $\hat{\underline{b}}(.)$, as it is necessary to make independent estimates for these vectors.

5.3 Prediction of User Energies

It is necessary to predict the energy vector $W(M+\acute{N}+1)$ based on previous data. Linear predictive methods have been used [7] for the prediction of fading envelopes. An alternative is to use polynomial extrapolation. However it was found that the added complexity of these extrapolation methods was not justified by the marginal gain in performance. We prefer to use :

$$\underline{W}(M+\acute{N}+1) \approx \hat{\underline{W}}(M+\acute{N}-1) \qquad (21)$$

as it is simple to implement and sufficiently accurate under most conditions (other than in deep and sudden fades).

5.4 Prediction of Future Data Bit Values

Here we focus on the prediction/estimation of $\underline{b}(M+\acute{N}+1)$. We propose a solution which identifies a novel use for the predictive properties of a non-catastrophic convolutional code. We first consider the following simplistic approach.

5.4.1 Conventional Receiver Approximation

Since the conventional receiver is not *near-far resistant*, this approximation brings in an element of near-far dependence. The suitability of including the conventional detector as a decision step in the algorithm (especially in MA limited conditions) is brought into question. We write the approximation specifically as :

$$\hat{\underline{b}}(M+\acute{N}+1) = sgn\left(\underline{y}(M+\acute{N}+1)\right) \qquad (22)$$

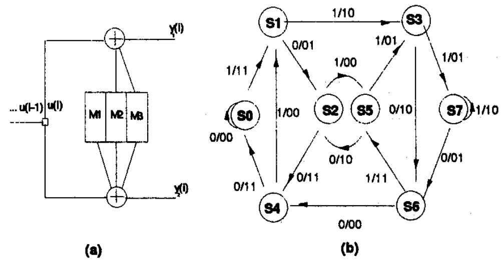

Figure 5: (a) Convolutional Encoder (b) State Diagram for (2,1,3) Code

5.4.2 Prediction Using Added Data Redundancy

If redundancy is added to the data stream using a convolutional code, it is possible to make a prediction based on prior data with a greater degree of certainty. Error correcting codes are a common feature in communication systems and the added redundancy will serve the dual purpose of aiding the prediction as well as providing error correcting capability at the next stage of the receiver. A state diagram is shown in Figure 5(b) for a (2,1,3) code. We have observed from state diagrams of codes of constraint length upto 7 that the following holds :

For a code of constraint length $m+1$ and code rate $R=\frac{1}{2}$, observing m consecutive pairs of outputs (2m bits), uniquely defines a path connecting $m+1$ states, and hence uniquely defines the current $(m+1)^{st}$ state on this path. If the next bit of output is also known, this path uniquely determines the second output bit.

This means however that the prediction can be performed only during every other bit period, or alternatively the sliding window should be advanced by two bit vectors at each iteration.

We return to the sliding window system. The two most recent vectors in the window have not been estimated. Hence if $\underline{b}(M+\acute{N}+1)$ is to be predicted, and we relate it to $y_2(i)$ in Figure 5(a), $y_1(i)$ should be situated in $\underline{y}(M+\acute{N}-2)$ or a previous vector. This calls for some form of interleaving of the data. Since we are considering a continuous stream of data, we propose the use of a convolutional interleaver. Adjacent bits in the original data stream will be separated by two bits in the transmit data.

It is necessary to recognise that all $y_1^k(i)$ appear in the same matched filter output vector and all $y_2^k(i)$ appear in the adjacent vector. This will burden the system with little additional networking overhead, as

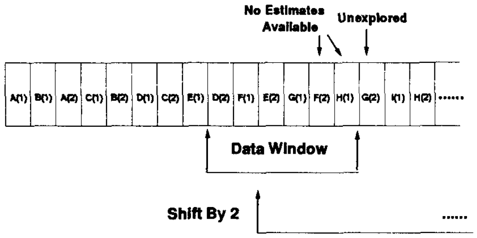

Figure 6: Contents of a Decorrelating Window

obtaining convolutional code synchronisation in this context would only involve applying an unit bit period delay if necessary. Note that this prediction does not rely on a decision made by the conventional detector, and in this respect is independent of the relative energies of the users. Fig 6 shows schematically the contents of a window at a particular instant in time . In order to simplify the notation we denote $y_1(1) : y_2(1)$ as $A(1) : A(2)$, and $y_1(2) : y_2(2)$ as $B(1) : B(2)$, and so on. For this example we use a window length of 6.

The prediction clearly involves two steps :

1. Predict $F(2)$ using $F(1), E(1), E(2), D(1), D(2)$
2. Predict the required $G(2)$ from $G(1)$,$F(1)$,$F(2)$,$E(1)$,$E(2)$

Finally to solve the linear system

$$\underline{X} = \Re W \underline{b} + \underline{n} \tag{23}$$

we exploit the block tridiagonal structure of $\Re$ and use LU decomposition to reduce the solution to the evaluation of the following recurrences.

$$\begin{aligned}
Z_1 &= R(0)^{-1}R(-1), \\
Z_k &= (R(0) - R(1)Z_{k-1})^{-1}R(-1) \\
&\quad k = 2, 3 \ldots \acute{N} \\
W_1 &= R(0)^{-1}\underline{y}(M+1), \\
W_k &= (R(0) - R(1)Z_{k-1})^{-1}\left(\underline{y}(M+k) - R(1)W_{k-1}\right) \\
&\quad k = 2, 3 \ldots \acute{N} \\
\underline{WB}(M+\acute{N}) &= W_{\acute{N}}, \\
\underline{WB}(M+k) &= W_k - Z_k \underline{WB}(M+k+1) \\
&\quad k = \acute{N}-1, \acute{N}-2, \ldots, 1
\end{aligned} \tag{24}$$

It should be noted that the matrix inversions need only be performed once unless there is a change in the timing configuration. Figure 7 shows a schematic representation of the proposed algorithm.

6 Simulation Results

A Multi-User DS-CDMA simulation model has been written. The model uses Gold codes and allows for a capacity comparison between our decorrelating receiver (DR) and the conventional linear correlation receiver (CR) in AWGN, fading, and time-dispersive channels. The mobile radio channel simulation is based on a tapped delay line model [8]. At this stage of our work we emphasize the near-far resistance (as defined earlier) of our receiver algorithm. We also extend our investigation on a preliminary basis to fast fading and mobile radio environments. A thorough investigation on the dependence of the receiver performance on parameters such as vehicle speeds, energy distribution between multi-paths, and spreading factor is the subject of on-going work. The preliminary results however indicate the capacity improvements offered by our algorithm in these channels.

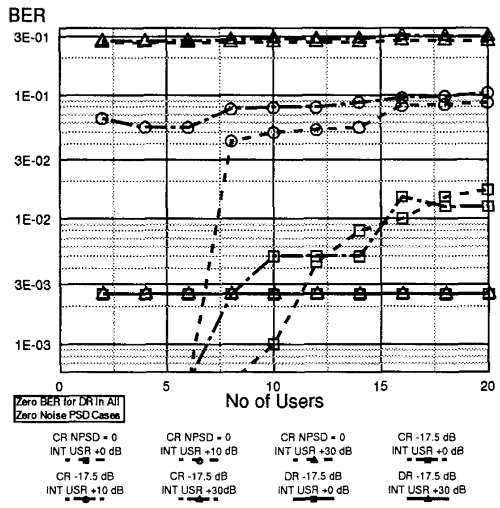

Figure 8: Performance in AWGN with Strong Interfering User)

Figure 8 establishes the near-far resistance of the DR in AWGN. The DR has no irreducible error rate irrespective of the number or relative energies of, the interfering (INT) users. We can hence conclude that the algorithm is near-far resistant by definition.

For the purposes of this preliminary evaluation in a simulated urban mobile radio channel, we assume a simple open loop power control strategy which compensates for the slow varying mean power variations. No compensation for fast fading is assumed. It has been shown [9] that DS-CDMA is rendered unusable as a multiple access technique due to fast fading if the CR

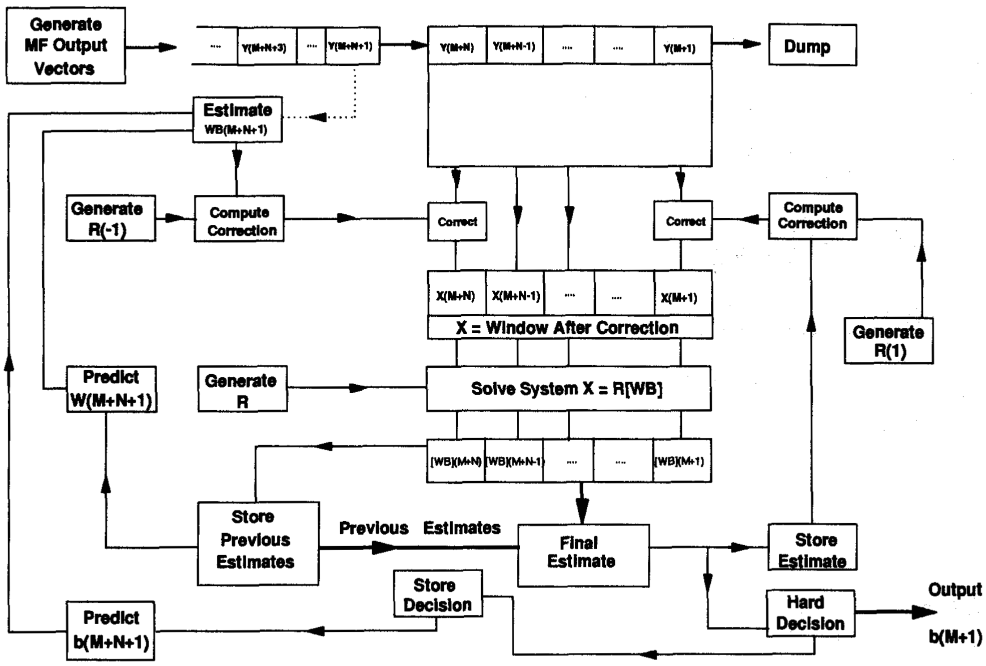

Figure 7: Sliding Window Decorrelating Algorithm

is used. Our simulation uses differential detection to avoid the difficulties associated with multi-user phase tracking.

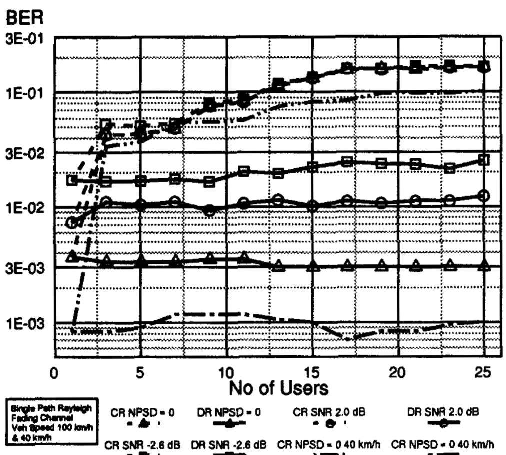

Figure 9: Multiple Users in Fading Channel

We first extend our investigation to a single fading link with no diversity gain available. The resulting channel is similar to that encountered in a sub-urban environment. Each user transmission undergoes inde-

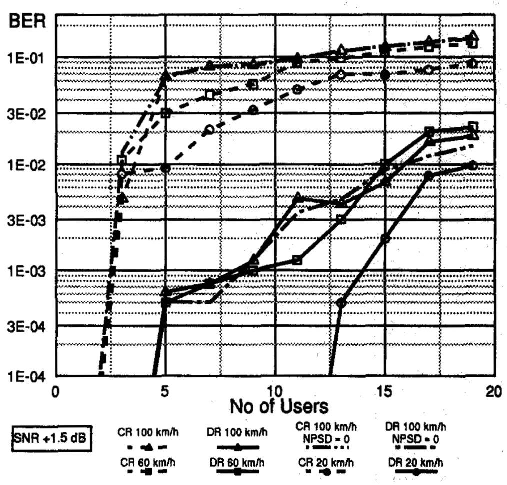

Figure 10: Multiple Users in Mobile Radio Channel

pendent Rayleigh fading at a rate dependent on the vehicle speed. The DR is seen to provide a large capacity improvement (Figure 9), the irreducible error rate in this case being of the same order of magnitude as that of the single user fading channel. In the case of the time dispersive channel a three branch RAKE diversity combining technique is incorporated for both the CR and the DR. No compensation for, or cancellation of, multi-paths outside the depth of the combiner is assumed. The simulation model is of an ur-

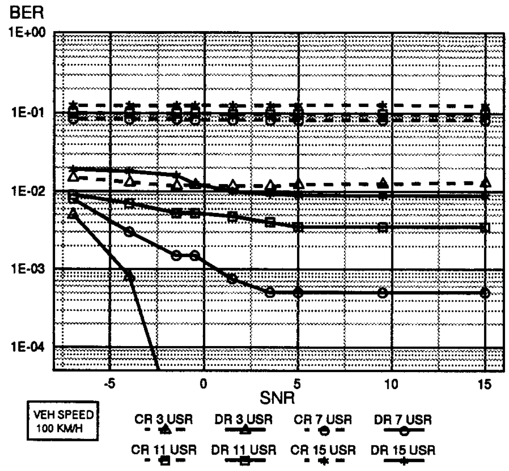

Figure 11: Noise Performance in Mobile Radio Channel

ban 900 MHz channel with a coded data rate of approximately 20kbps (accomodating the half rate code mentioned above) and spreading factor of 63. The results presented are for users travelling at 20,60 and 100 km/h. The mean signal strength parameters used for the tapped delay line model are based on the results presented in [8, 10].

For purposes of clarity we have ommitted the BER of the DR after decoding the convolutional code. It was verified that the BER of the DR was $< 0.4\%$ whereas the BER of the CR exceeded the error correcting capability of the half rate code for more than 5 users. Figures 10,11 show the capacity improvement offered by our algorithm in the absence of closed loop power control, and the effect of thermal noise on the decorrelation, due its misinterpretation (by the algorithm) as MA interference.

6.1 Conclusions

We have demonstrated by simulation that our algorithm is *near-far resistant* in AWGN and single fading link environments. Preliminary evaluation shows that the sliding window algorithm offers considerable capacity enhancements in (time dispersive) mobile radio environments, although residual multi-path activity is seen to degrade performance. Further investigation of anti-multi-path techniques is required so as to negate the detrimental effects of multi-path propagation on the decorrelating process. In agreement with Turin [9] the CR is seen to be unusable in the absence of sufficiently rapid power control, the feasibility of which is questionable at practical vehicle speeds [9, 11]. The DR is hence seen to be a viable alternative to a complex closed loop power control strategy. We can hence conclude that the Sliding Window Algorithm has potential application in future generation point-to-point and mobile radio DS-CDMA systems.

Acknowledgements

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