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Effects of Computation and Feedback Delay on the Capacity of Multiuser MIMO Systems in a Small Outdoor Cell

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Introduction

- Explore effect of out-of-date CSI on the performance of three multiuser algorithms using measured data from highly-scattering environment
  - Generalized waterfilling (Nash equilibrium)
  - Zero-forcing at TX
  - Successive zero-forcing at TX
- Fit curves to delay losses
- Examine tradeoff between convergence gains and delay losses

Measurement setup

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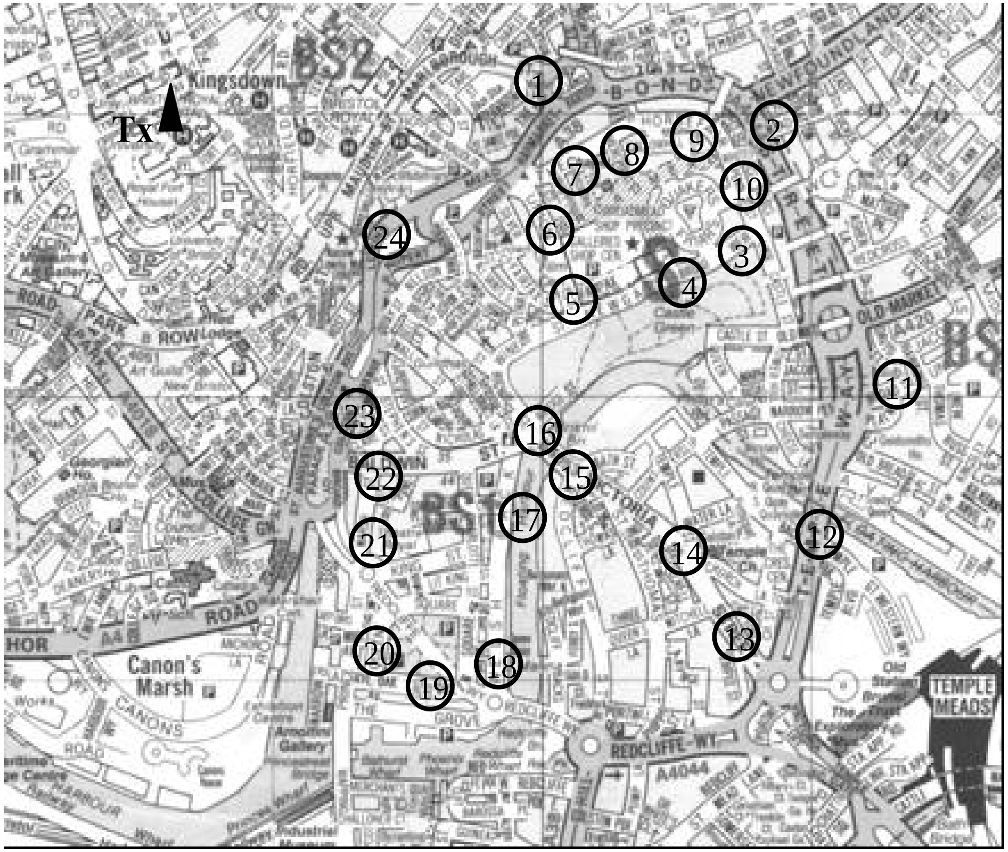
- 4 TX antennas
  - Two dual polarized 65º BW UMTS panel antennas
  - 20λ separation
  - Atop 30m-high building overlooking city centre
- 8 RX antennas
  - UCA,8 monopoles
  - λ/2 radial spacing
- 24 positions, 128 freqs in 20MHZ centred on 1.92GHz



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Algorithms

1. Generalized waterfilling: Nash equilibrium (NE)
   - Non-cooperative ‘game’
   - Users iteratively waterfill until there is only a small change in total system capacity
2. TX-side block-diagonalization (BD)
   - Each user TX’s such that it causes no interference to any other user (an iterative process)
   - $$\mathbf{R}_j^\dagger \mathbf{H}_j \begin{bmatrix} \mathbf{T}_1 & \mathbf{T}_2 & \cdots & \mathbf{T}_L \end{bmatrix} = \begin{bmatrix} \mathbf{0}_1 & \cdots & \mathbf{0}_{j-1} & \mathbf{\Lambda}_j & \mathbf{0}_{j+1} & \cdots & \mathbf{0}_L \end{bmatrix}$$
   - Is RX’d so as to maximise end-to-end channel gains
3. TX-side successive diagonalization (SD)
   - User j transmits such that it causes no interference to users $1, \ldots, j-1$ and compensates for interference from them (non-iterative)

Interpolation

- Each location has 1024 snapshots in 8 blocks of 1.3s, with 1.5s recording time between blocks
  - Each snapshot is recorded in approx.10ms
  - Well within the channel's coherence time
- 10ms likely to be longer than calculation + feedback delay for iterative schemes
- Add 10 additional points between each pair of snapshots
  - Each interpolated snapshot approx. 0.9ms
  - Discard snapshot at end of each block

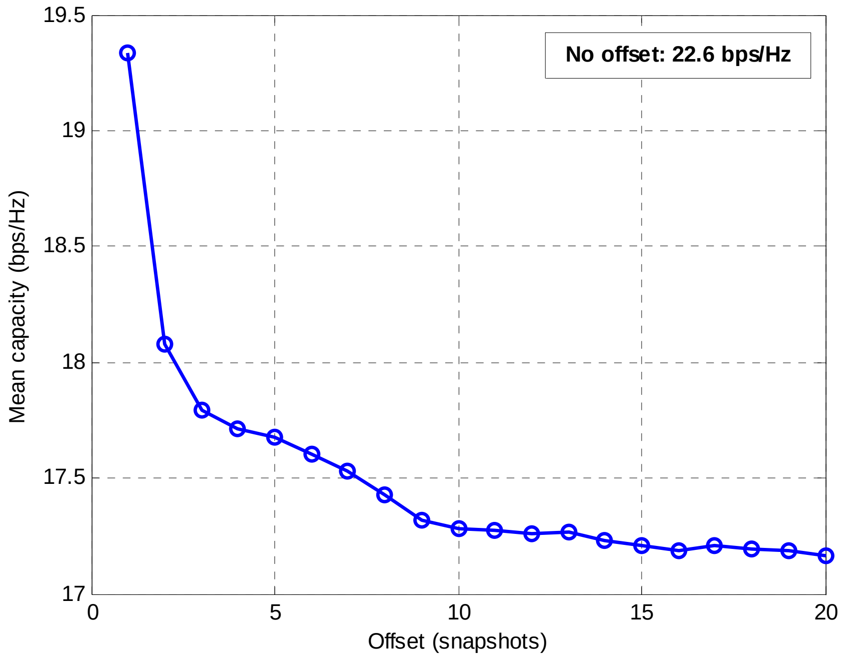
Parameters and assumptions

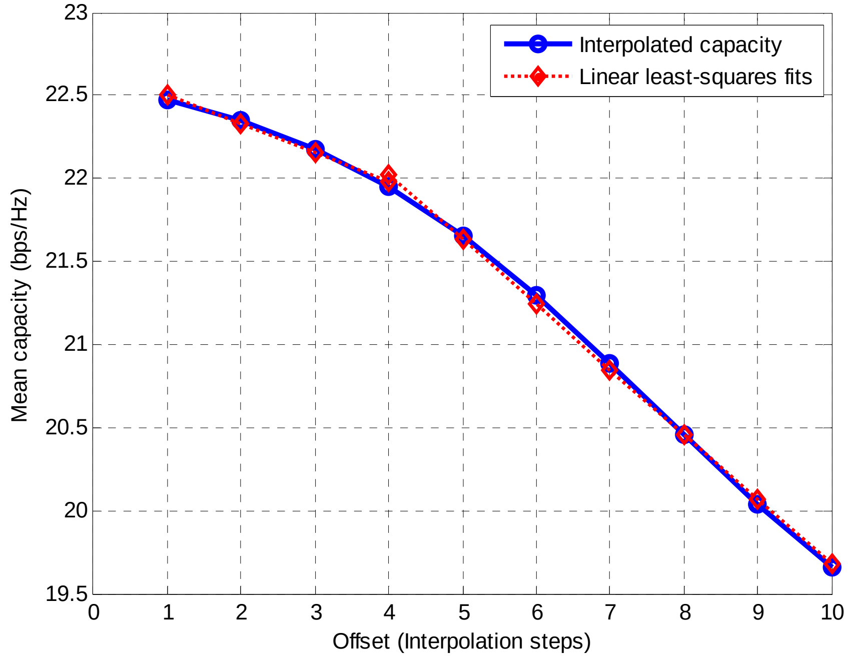
- Normalize channels so each user is RX'd at 20dB SNR
- Users at locations 15, 16, 17 and 18
- 4TX and 4RX antennas
- Measure mean total system capacity
- Restriction on diagonalization schemes:

$$n_R \geq \sum_{j=1}^{L} N_j \quad \text{and} \quad n_{T_j} \geq N_j$$

- N_j substreams from user j, so will restrict $N_j = 1$ and waterfill
- NE convergence to <1% change in total capacity; BD to 10^{-6} off-diagonal elements in aggregate channel matrix (for now)

Nash equilibrium





- Small loss at 1 snapshot increases to about 23% at long delays
- Losses can be approx. linearly smaller by reducing the delay beneath one snapshot

$$C(N_{\text{off}}) \approx \begin{cases} -0.2N_{\text{off}} + 22.7 & : 1 \leq N_{\text{off}} \leq 4 \\ -0.4N_{\text{off}} + 23.6 & : 4 \leq N_{\text{off}} \leq 10 \end{cases}$$

Block diagonalization

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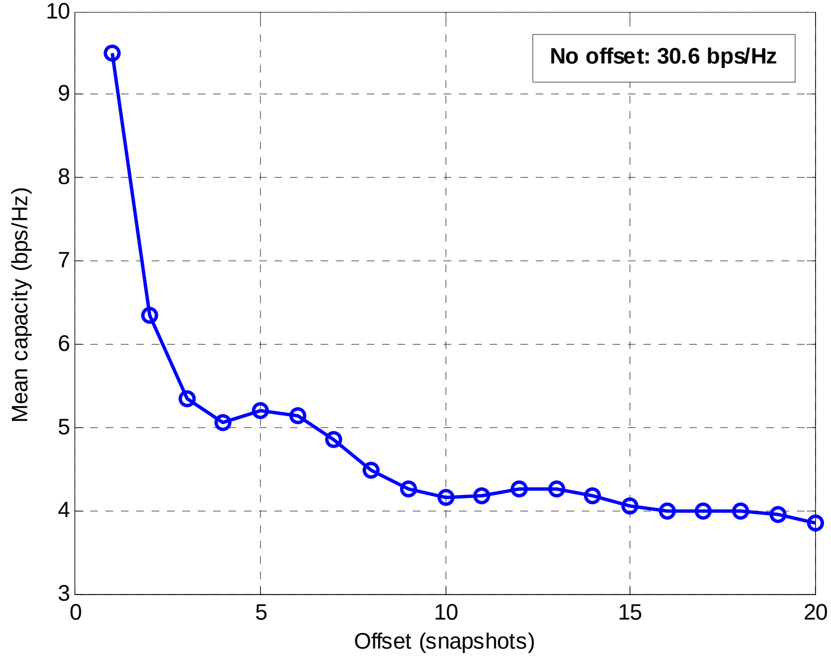

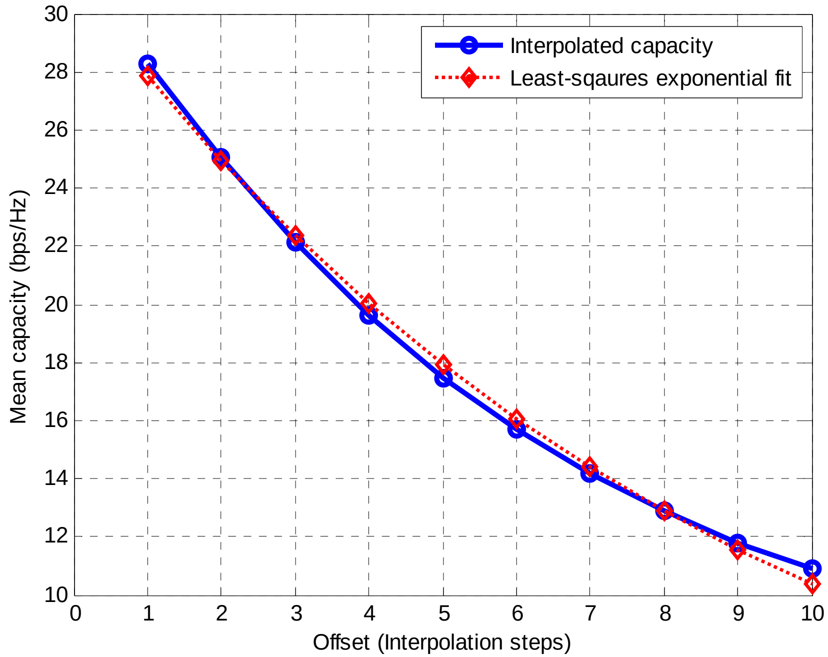

- Losses much greater than NE: down by 70% at one snapshot
  - BD depends on exact orthogonality; NE does not and disturbance from a non-orthogonal situation is less severe
- Reducing delay below 1ms reduces losses quite quickly:

$$C(N_{\text{off}}) \approx 31.1e^{-0.11N_{\text{off}}}$$

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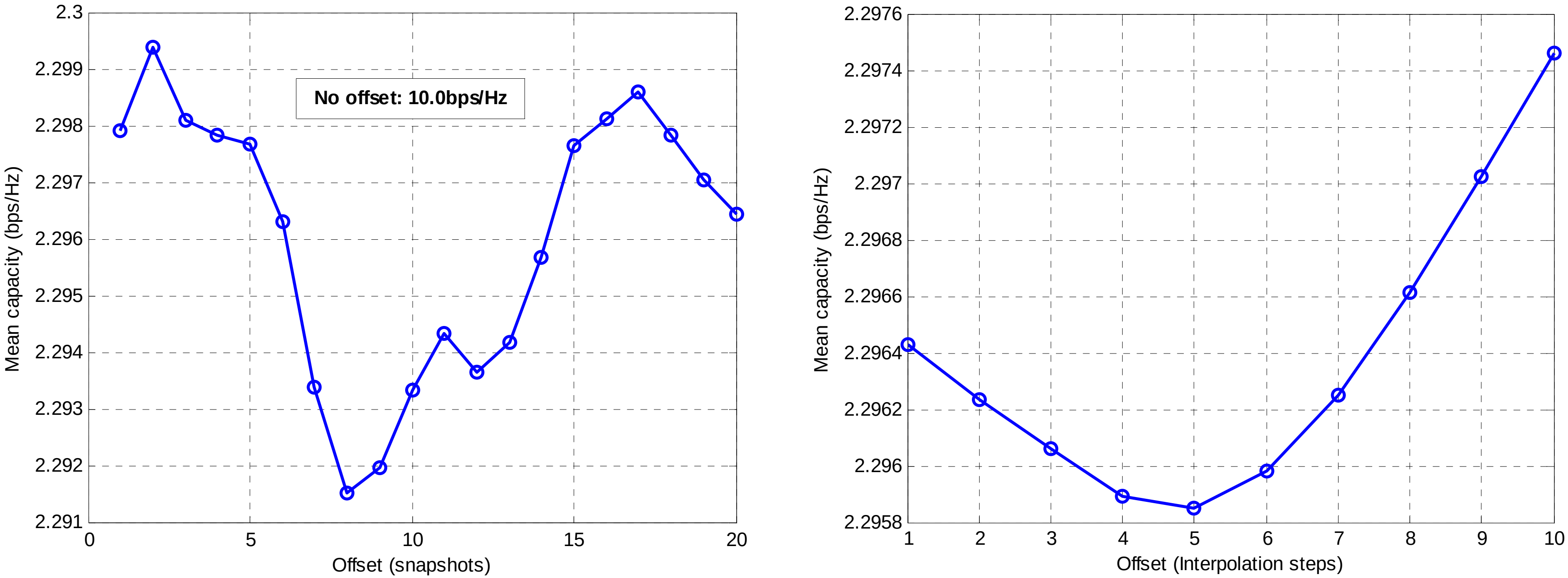
Successive diagonalization



- Losses are substantial at even the smallest interpolation offset
- Fluctuations are essentially random with the channel
- Interpolation 'minimum' is an artefact of the fluctuations
- Delay needs to be <<0.9ms for the per-user capacity to be useful

Convergence –vs– delay

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Convergence (off-diag elements)	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}
Mean iterations	6.4	5.8	5.0	4.2	3.3
Capacity (bps/Hz)	30.6	30.3	30.2	30.1	29.4

Block diagonalization

Convergence (% capacity)	1%	2%	3%	5%	10%
Mean iterations	4.5	4.2	4.0	3.8	3.4
Capacity (bps/Hz)	22.5	22.5	22.4	22.3	22.1

Nash equilibrium

- Little capacity loss for noticeable delay improvement
- Both algorithms should be operated at the weaker convergence levels where the small loss from weaker convergence is made up by the gains from reduced delay

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Conclusions

- Temporal evolution in the channel has distinct effects on the three algorithms:
  - Nash equilibrium is clearly most robust
  - Successive diagonalization is most sensitive
- Characterised the losses due to delay as approximately
  - (Piecewise-) linear for NE
  - Exponential for BD
- Convergence-delay trade-off shows that losses from weaker convergence are very small, but characterization of losses shows that the gains from the shorter delay could be useful
  - Particularly for BD with its exponential delay characteristic

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