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Joint Space-time Trellis Code Detection and MIMO Equalisation via Particle Filtering

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***Abstract*— In this contribution we address the problem of MIMO equalisation and transmit diversity. More specifically, we develop a method that jointly decodes Space Time Trellis Coded Modulation (STTCM) encoded signals and equalises the MIMO channels. The optimal solution for such a setup would require construction of a super-trellis that describes a finite state machine resulting from the combination of MIMO channel and STTCM trellises. However, the number of states in this super-trellis is typically gigantic and as such prohibits practical application. Our solution is based on Sequential Monte Carlo technique (aka Particle filter), which has gained much interest recently in the communications community. The structure of the STTCM code is incorporated into the proposal distribution of the particle filter. As a results a significant reduction in complexity has been achieved as compared not only to the brute force super-trellis approach, but also when compared to the conventional "turbo" solution.**

I. Introduction

Multiple antenna (aka MIMO) techniques continue to receive tremendous interest from the industry and the academia alike. In fact, the "from invention to implementation" delay has been remarkably small and difficult to compare with any other invention created by the communications scientist. The MIMO techniques can broadly be classified into transmit diversity and spatial multiplexing (although this division is more historical, and some researchers argue there is no fundamental difference). In this technical paper we take on the detection problem of a classical transmit diversity scheme i.e. Space-time Trellis Coded Modulation (STTCM) [1]. The STTCM attempts to jointly optimise: modulation encoding, transmit and receive diversity. Additionally STTCM can provide some coding gain. STTCM uses Finite State Machine (FSM) to impose first order Markov property onto the encoded data. As the results, the detection process can be performed by a dynamic programming algorithm - the Viterbi algorithm. The detection process gets complicated, when the system transmits the information over a wideband channel. Wide-channel necessitates some form of equalisation at the receiver. Since the wideband characteristics introduce memory in the channel, it can also be modelled as another FSM machine. A concatenation of two FSM machines, is just another FSM machine and the Viterbi algorithm can solve the problem, at least in principle. This approach rapidly becomes infeasible, as the number of states grows exponentially with the number of the channel taps and transmit antennas.

Two sub-optimal solutions are currently available in the literature. In [2] it is shown that the joint trellis approach may still be feasible, however this method is restricted in practice to only very few simple space-time codes. The second approach [3] utilises the turbo principle. The MIMO equalisation and STTCM detection are handled by a separate soft-in - soft out decoders. Those decoders perform an iterative detection exchanging so-called extrinsic information. Since the number of states in the MIMO equaliser scales exponentially with the modulation alphabet and the number of transmit antennas, the complexity of the turbo solution may still be colossal. In an attempt to reduce the complexity of the turbo system [4] detects I and Q branches separately and [5] proposes M-BCJR algorithm in place of BCJR for MIMO equalisation step. In this contribution, we develop a new approach to this problem. We apply Sequential Monte Carlo technique (aka Particle filter) [6], that has recently gained much interest in the communications community [7]. In our solution the structure of the STTCM code is incorporated into the proposal distribution of the particle filter. As the results a remarkable reduction in complexity has been achieved. In fact the complexity of the proposed solution is independent of the channel length and complexity/performance can be tuned by a single parameter: the number of particles.

II. Signal Model and Estimation tasks

Consider a multi antenna wireless system (aka MIMO) that employs space-time coding for improved performance. The schematic of the considered system is depicted in figure 1. The

source collects an information sequence to be transmitted $d = \{d(1), \ldots, d(T)\}$. The information sequence is then encoded by a STTCM to produce a codeword $\mathbf{C} = (\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_T)$, $\mathbf{c}_t = (c_{1,t} \ldots, c_{N_T,t})^T$ is of size $N_T \times 1$ where N_T is the number of transmit antennas and c_t belongs to a signalling alphabet e.g. M-PSK. In this contribution we assume that interleavers are not used, hence the transmitted signal retains Markovian characteristics imposed by the STTCM encoder. This assumption is in fact crucial for the success of the presented method.

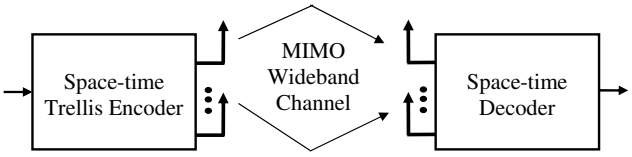

Fig. 1. Space-time coded system signalling over a Wideband MIMO channel.

The system sends information over a wideband channel (multidimensional FIR filter) which is characterised by a channel matrix $\mathbf{H}$: We assume quasistatic scenario i.e. the wideband channel remains constant for the duration of a frame and changes independently from one frame to another.

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_{1,1}^T & \cdots & \mathbf{h}_{1,N_T}^T \\ \vdots & \ddots & \vdots \\ \mathbf{h}_{N_R,1}^T & \cdots & \mathbf{h}_{N_R,N_T}^T \end{bmatrix} \tag{1}$$

where:

$$\mathbf{h}_{m,n} = \left(h_{m,n}^{(\tau=0)}, h_{m,n}^{(\tau=1)}, \ldots, h_{m,n}^{(\tau=L-1)}\right)^T \tag{2}$$

Define:

$$\mathbf{x}_{t,n} = \left(c_{t,n}, c_{t-1,n}, \ldots, c_{t-L+1,n}\right)^T \tag{3}$$

and

$$\mathbf{x}_t = \left(\mathbf{x}_{t,1}^T, \mathbf{x}_{t,2}^T, \ldots, \mathbf{x}_{t,N}^T\right)^T \tag{4}$$

With the above definitions the received signal at time t, for $t = \{1, 2, \ldots T\}$ is

$$\mathbf{y}_t = \mathbf{H}\mathbf{x}_t + \mathbf{n}_t \tag{5}$$

Where $\mathbf{n}_t \sim \mathcal{CN}\left(0, \sigma^2\mathbf{I}\right)$. The receiver's task is to find a legitimate transmitted sequence (i.e. the codeword) that best explains the sequence of observations. This amounts to finding the maximum of the joint likelihood:

$$\hat{\mathbf{C}} = \underset{\tilde{\mathbf{C}}=\mathbf{c}_1,\ldots,\mathbf{c}_T}{\arg\max} \left\{ f\left(\mathbf{y}_1, \mathbf{y}_2 \ldots, \mathbf{y}_T \,\middle|\, \tilde{\mathbf{C}} = \mathbf{c}_1, \ldots, \mathbf{c}_T\right)\right\} \tag{6}$$

In principle, this task could be solved optimally by the Viterbi algorithm, however the computational complexity of this approach is prohibitive. The conventional sub-optimal solution consists of splitting the decision making process into two parts. The resulting solution is called turbo detection and involves iterative detection and exchange of "soft information" between MIMO equaliser and soft STTCM decoder [3]. Our solution is based on an approximate recursive detection procedure known as a particle filter [6]. The Markovian structure plays a central role in the proposed scheme. A similar idea appeared previously in the literature in a somewhat simpler setup of BPSK signalling over flat-fading channels [8].

III. Monte Carlo methods

Monte Carlo methods have proved to be very efficient at tackling such complex problems. They have been successfully applied in physics for 50 years [9], image processing for nearly 20 years and statistics for over a decade where they have revolutionised Bayesian statistics. The basic principle of Monte Carlo methods consists of replacing the algebraic representation of π by a population based representation. More precisely assume that we know how to produce N samples, the population, distributed according to π, then the probability of any region A of $\mathcal{X}$, *i.e.* $\int_A \pi(x)\, dx$, can be approximated by the number of samples that belong to A. Now if we wish to approximate an integral of the form

$$I(f) = \int_{\mathcal{X}} f(x)\, \pi(x)\, dx$$

(where here $\int$ either means discrete or continuous sum), then a Monte Carlo estimator of $I(f)$ is given by

$$\widehat{I}(f) = \frac{1}{N}\sum_{i=1}^{N} f(x_i).$$

Intuitively this estimator ought to be efficient, as the samples $\{x_i\}$ tend to concentrate on regions of high probability (*i.e.* where information is) and avoid regions of low probability, therefore making the most of the available computational power. This statement can be made mathematically rigourous, and it can be proved that under fairly general conditions, the rate of convergence of this estimator to the true value of the integral is of the order $O\left(\frac{1}{\sqrt{N}}\right)$, that is the rate of convergence is *independent of the dimension* of $\mathcal{X}$.

A. Particle Filter

Particle filter (PF) is a recursive Bayesian estimation technique. It is typically used in the context of non-linear and/or non-Gaussian dynamical systems. It can also be used where the underling problem can be modelled as a Hidden Markov Model (HMM), as indeed it is the case in this contribution. To set the background for the application of PF we start with a brief review of HMM and its properties relevant for a sequential detection via PF.

The joint distribution of observations $\{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_T\} \equiv \mathbf{y}_{1:T}$ and hidden states (unobserved signal of interest) $\{\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_T\} \equiv \mathbf{x}_{0:T}$ factors as:

$$f(\mathbf{y}_{1:T}, \mathbf{x}_{0:T}) = f(\mathbf{x}_0) \prod_{t=1}^{T} f(\mathbf{y}_t \,|\mathbf{x}_t)\, f(\mathbf{x}_t \,|\mathbf{x}_{t-1}) \tag{7}$$

The joint posterior distribution is proportional to (7) i.e.

$$f(\mathbf{x}_{0:T} \,|\mathbf{y}_{1:T}) \propto f(\mathbf{y}_{1:T}, \mathbf{x}_{0:T})$$

The task is to construct a procedure for obtaining $f(\mathbf{x}_{0:t+1}|\mathbf{y}_{1:t+1}) = \Phi\{f(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})\}$

By Bayes theorem:

$$f(\mathbf{x}_{0:t}|\mathbf{y}_{1:t}) = \frac{f(\mathbf{y}_{1:t}|\mathbf{x}_{0:t}) f(\mathbf{x}_{0:t})}{\int f(\mathbf{y}_{1:t}|\mathbf{x}_{0:t}) f(\mathbf{x}_{0:t}) d\mathbf{x}_{0:t}} \tag{8}$$

To find out the required transform Φ, Bayes' theorem is used again:

$$f(\mathbf{x}_{0:t+1}|\mathbf{y}_{1:t+1}) = f(\mathbf{x}_{0:t+1}|\mathbf{y}_{t+1}, \mathbf{y}_{1:t}) = \frac{f(\mathbf{y}_{t+1}|\mathbf{x}_{0:t+1}, \mathbf{y}_{1:t}) f(\mathbf{x}_{0:t+1}|\mathbf{y}_{1:t})}{f(\mathbf{y}_{t+1}|\mathbf{y}_{1:t})} \tag{9}$$

However, the observations are conditionally independent: $f(\mathbf{y}_{t+1}|\mathbf{x}_{0:t+1}, \mathbf{y}_{1:t}) = f(\mathbf{y}_{t+1}|\mathbf{x}_{t+1})$. This leads to the required recursive formula:

$$f(\mathbf{x}_{0:t+1}|\mathbf{y}_{1:t+1}) = \frac{f(\mathbf{y}_{t+1}|\mathbf{x}_{t+1}) f(\mathbf{x}_{t+1}|\mathbf{x}_t)}{f(\mathbf{y}_{t+1}|\mathbf{y}_{1:t})} f(\mathbf{x}_{0:t}|\mathbf{y}_{1:t}) \tag{10}$$

Deceptively, the above recursion is straightforward to perform. However, exact analytical formulae are possible to find only in a very few cases. One is the case when the states evolve according to some linear function and both the state and the observation noise are Gaussian. The other exception is when $\mathbf{x}$ is a discrete random variable, which is exactly our case. However, even though the distribution of interest is defined over a finite grid, number of states prohibits exact calculations.

Particle filters are designed to solve exactly this problem. Suppose that samples $\left\{\mathbf{x}_{0:t}^{(i)}, i = 1:P\right\}$ are drawn independently from a normalised importance function .

$$\pi(\mathbf{x}_{0:t}|\mathbf{y}_{0:t}) \tag{11}$$

Then the posterior distribution at time t can be approximated by:

$$\hat{p}(\mathbf{x}) = \sum_{i=1}^{P} \tilde{w}_t^{(i)} \delta\left(\mathbf{x} - \mathbf{x}_{0:t}^{(i)}\right) \tag{12}$$

$$\tilde{w}_t^{(i)} = \frac{w_t^{(i)}}{\sum_{j=1}^{P} w_t^{(j)}} \tag{13}$$

where:

$$w_{t+1}^{(i)} = \frac{f\left(\mathbf{y}_{t+1}\middle|\mathbf{x}_{t+1}^{(i)}\right) f\left(\mathbf{x}_{t+1}^{(i)}\middle|\mathbf{x}_t^{(i)}\right)}{\pi\left(\mathbf{x}_{t+1}^{(i)}\middle|\mathbf{x}_{0:t}^{(i)}, \mathbf{y}_{1:t+1}\right)} w_t^{(i)} \tag{14}$$

is the unnormalised incremental importance weight.

IV. Particle filtering applied to joint MIMO equalisation and STTCM detection

As mentioned in section II our aim is to find the maximum of the joint likelihood. With the assumption that all code sequences are *apriori* equally likely, the maximum of the joint likelihood will coincide with the maximum of the joint posterior distribution i.e. the MAP estimate. Hence our strategy is to tract the joint posterior distribution $f(\mathbf{x}_{0:t}|\mathbf{y}_{0:t})$ in a computationally feasible way, i.e. using particle filter. The final decision will be taken using an approximation to the joint posterior, simply by choosing a particle with its whole trajectory for which the associated weight at time $t = T$ is maximal.

The algorithm proceeds as follows:

1) Draw samples from the importance function. In our case it is the prior, which is augmented by the knowledge of the STTCM structure i.e.

$$\mathbf{x}_t^{(j,i)} \sim \pi\left(\mathbf{x}_t|\mathbf{x}_{t-1}^{(i)}\right) \mathcal{I}\left(\mathbf{x}_t|\mathbf{x}_{t-1}^{(i)}\right)$$

Where the indicator function $\mathcal{I}\left(\mathbf{x}_t|\mathbf{x}_{t-1}^{(i)}\right)$ takes value 1 if the transition $\mathbf{x}_{t-1} \Rightarrow \mathbf{x}_t$ is compatible with the STTCM structure and 0 otherwise.

2) Update and re-normalise the particle weights.

$$w_t^{(j,i)} \propto \tilde{w}_{t-1}^{(i)} f\left(\mathbf{y}_t\middle|\mathbf{x}_t^{(j,i)}\right)$$

$$\tilde{w}_t^{(j,i)} = \frac{w_t^{(j,i)}}{\sum_{k=1}^{K}\sum_{l=1}^{P} w_t^{(k,l)}}$$

3) Resample P particles according to their weights from $K \times P$ particles.

4) Append the trajectory of each particle $\left\{\mathbf{x}_{0:t-1}^{(i)}\right\}$ with the resampled particles $\left\{\mathbf{x}_t^{(i)}\right\}$

5) if $t < T$ repeat steps 1 to 4, otherwise choose $\left\{\mathbf{x}_{0:T}^{(i)}\right\}$ as the final decision for which the associated weight has the maximal value.

It is reasonable in our case to replace the sampling step in 1) by an exploration of all possibilities that are compatible with the STTCM code. This also explains the notation K - is the number of transitions leaving each state in the space time code. In step 3) the random resampling procedure can be replaced with a max function i.e. P particles are retained according to the value of their weights. In fact, such procedure will produce a biased estimator, but not necessarily worse in terms of Frame Error Rate - as we will see in the next section such procedure (deterministic procedure) will lead to improved FER.

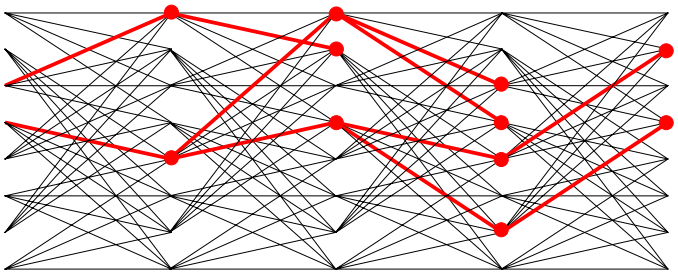

Fig. 2. Graphical interpretation of the particle filtering algorithm.

Figure 2 provides some graphical interpretation to the presented method. The trellis depicts the overall trellis structure i.e. the super-trellis. Even though the algorithm was not presented using trellis, it can easily be cast on that framework.

The red dots represent the support for the filtering distribution, whereas the red thick lines represent trajectories of the particles. The particle filtering is used here as a statistical pruning technique. It should also be clear that particle filtering in this context is similar to so-called T and M algorithms. To be more precise: particle filtering subsumes somewhat heuristically developed T and M algorithms and provides a firm statistical framework.

It is possible to extend the proposed technique to provide soft estimates of the transmitted symbols. One way is to introduce a backward recursion that would perform fixed interval smoothing on the filter grid, as proposed in [10] in the context of somewhat simpler setup.

A. Complexity

The complexity of the optimal solution i.e. Viterbi over super-trellis in a general case is $\mathcal{C}_{opt} = \mathcal{O}\left(SM^{N_T(L-1)}\right)$, where S in the number of STTCM states and M is the cardinality of the signalling alphabet. The complexity of the turbo solution is $\mathcal{C}_{turbo} = I_{\max}\left(\mathcal{C}_{eq} + \mathcal{C}_{STTCM}\right)$, this number strongly depends on the complexity of the MIMO equliser $\mathcal{C}_{eq}$. The complexity order of the soft STTCM decoder is $\mathcal{C}_{eq} = \mathcal{O}(S)$. The complexity order of the particle filter is simply $\mathcal{C}_{part} = \mathcal{O}(N)$, N - number of particles.

V. Numerical Results

In this section we investigate the performance of the proposed techniques. As an example we choose a system with $N_T = N_R = 2$ antennas that uses a 16 state 4-PSK code of [11]. A frame is set to 192 symbols (including 2 terminating symbols). The system transmits the information frame over a wideband channel with $L = 3$ taps, where each tap is *iid* $h_{m,n,\tau} \sim \mathcal{CN}\left(0, \frac{1}{L}\right)$. The channels are kept constant during the duration of a frame. Figure 3 depicts the performance of a turbo system [3], reduced complexity turbo [5] and two benchmarking scenarios. The benchmark "ML narrowband" refers to a case where $L = 1$ (narrowband channel) with ML (Viterbi detection). The benchmark "ML narrow fixed" refers to a case where $L = 1$, and $h_{m,n,1} = 1$. Two re-sampling schemes for the proposed technique are investigated: random re-sampling and deterministic max procedure. Figures 4 and 5 refer to those scenarios respectively. The particle filter with the deterministic max procedure is superior compared to the M-BCJR reduced complexity technique. This is clearly seen in figure 6. At the same time it offers reduced complexity detection.

VI. Conclusions

This paper presents a technique for the detection of space time trellis codes that are transmitted over wideband channels. The detection technique is based on a sequential Monte Carlo technique - particle filtering (aka sequential importance sampling). It offers an improved performance as compared to M-BCJR suboptimal detector. At the same time the detection complexity is reduced significantly. The technique provides a performance/complexity trade-off that is tuned by a single parameter: the number of particles.

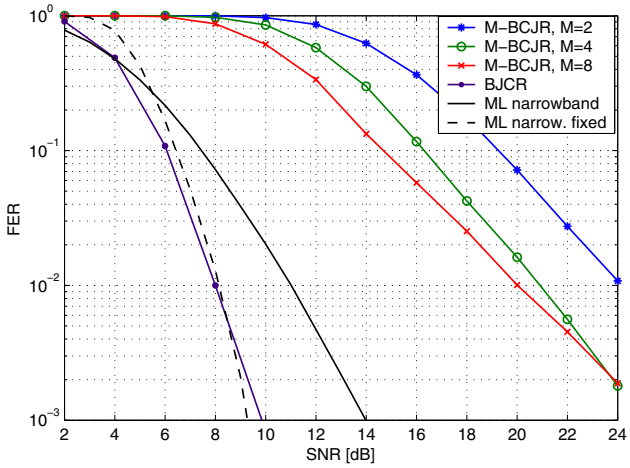

Fig. 3. Performance of the particle filter based joint STTCM detector and MIMO equaliser, $N_T = N_R = 2$, 3 tap Rayleigh faded channel; Turbo BCJR and M-BCJR.

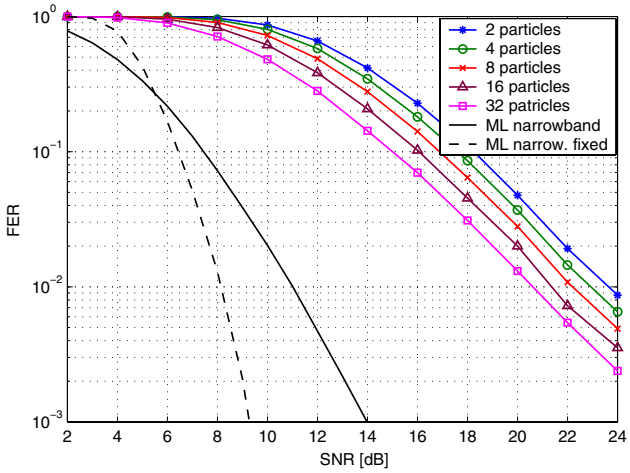

Fig. 4. Performance of the particle filter based joint STTCM detector and MIMO equaliser, $N_T = N_R = 2$, 3 tap Rayleigh faded channel; Stochastic re-sampling procedure.

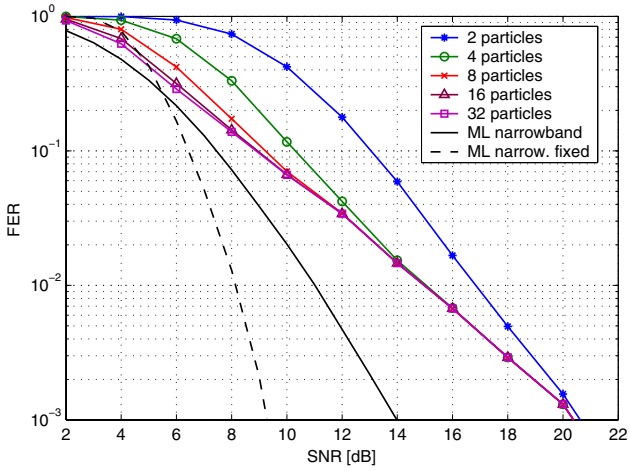

Fig. 5. Performance of the particle filter based joint STTCM detector and MIMO equaliser, $N_T = N_R = 2$, 3 tap Rayleigh faded channel. Deterministic re-sampling (max) procedure.

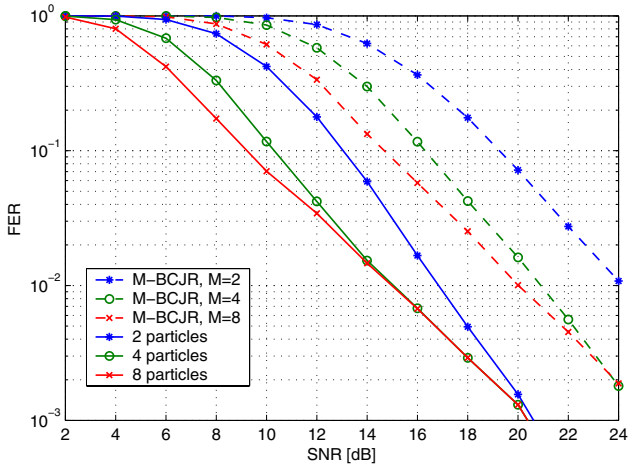

Fig. 6. Performance of the particle filter based joint STTCM detector and MIMO equaliser, $N_T = N_R = 2$, 3 tap Rayleigh faded channel. M-BCJR and particle filtering with the deterministic (max) resampling procedure.

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