Pal, A., Beach, M. A., & Nix, A. R. (2006). A novel quantification of 3D directional spread from small-scale fading analysis. In International Conference on Communications (ICC), Istanbul, Turkey. (pp. 1699 - 1704). Institute of Electrical and Electronics Engineers (IEEE). 10.1109/ICC.2006.254964

Link to published version (if available):
10.1109/ICC.2006.254964

Link to publication record in Explore Bristol Research
PDF-document

A Novel Quantification of 3D Directional Spread from Small-Scale Fading Analysis

Arindam Pal, Mark Beach and Andy Nix, University of Bristol. email: a.pal@bristol.ac.uk

Aim: Novel metrics are proposed to classify the 3D directional spread properties of the wireless channel as an aid to design of multi-element antenna systems.

Part 1/2: The proposed metrics are derived analytically in this section.

Abstract

A quantification of directional spread that is applicable to an arbitrary distribution of multipath energy in a 3-D directional domain is developed. The proposed metrics are derived from the second order moments of the partial derivatives of the spatial fading function, using an eigenvalue analysis. The analytical relation between the proposed metric and spatial selectivity justifies the use of the metric for analysis and optimisation of space-diversity based MIMO antenna systems.

Motivation

Directional dispersion of multipath energy is an important measure in design of wireless systems, with increasing levels indicating lower correlation between spatial diversity elements. The RMS angular spread metric, however, has several key limitations. It considers angles in only the 2-D plane and must be stated separately for azimuth and elevation angles. Also, it is unsuitable for multiple clusters and large directional spreads. The analysis presented here provides an extension to that provided in [1] and [2].

Theory: Spatial Partial Derivatives

The channel response for a discrete multipath distribution:

$$h = \sum_{s=1}^{N_s} A_s \exp(i\psi_s) = \sum_{s=1}^{N_s} A_s \exp\left(i\left(-\hat{\mathbf{d}}_s \cdot \mathbf{s} + \chi_s\right)\right)$$

A_s: gain of the sth multipath component.
$[d_x\ d_y\ d_z]$: directional-of-arrival as unit vector.
N_s: number of multipath rays.

Partial derivative of the spatial transfer function ($f = |h|^2$):

$$f_x = \frac{\partial f}{\partial x} = \frac{\partial |h|^2}{\partial x} = \sum_{s=1}^{N_s}\sum_{k=1}^{N_s} A_s A_k d_{x_s} \sin(\psi_s - \psi_k)$$

3-D directional derivative in direction $\mathbf{u} = [u_x\ u_y\ u_z]$

$$\mathrm{m}_{\mathbf{u}} = \nabla_{\mathbf{u}} f = f_x u_x + f_y u_y + f_z u_z$$

Mean of $\mathrm{m}_{\mathbf{u}}$ over multiple phase realisations is zero. The variance of $\mathrm{m}_{\mathbf{u}}$ is given by:

$$\sigma_{\mathrm{m}}^2(\hat{\mathbf{u}}) = E\left\{\mathrm{m}_{\mathbf{u}}^2\right\} = \mathrm{E}\left\{f_x^2\right\}u_x^2 + \mathrm{E}\left\{f_y^2\right\}u_y^2 + \mathrm{E}\left\{f_z^2\right\}u_z^2 + \ldots$$
$$\ldots \mathrm{E}\left\{f_x f_y\right\}u_x u_y + \mathrm{E}\left\{f_y f_z\right\}u_y u_z + \mathrm{E}\left\{f_z f_x\right\}u_z u_x$$

The following expressions for the second order moments of partial derivatives can be derived:

$$\mathrm{E}\left\{f_x f_y\right\} = \frac{1}{2}\sum_{s}^{N_s} A_s^2\left(d_{x_s} - \bar{d}_x\right)^2$$

$$\mathrm{E}\left\{f_x^2\right\} = \frac{1}{2}\sum_{s}^{N_s} A_s^2\left(d_{x_s} - \bar{d}_x\right)\left(d_{y_s} - \bar{d}_y\right)$$

Eigenvalue Analysis

$\mathbf{R}$ is the covariance matrix of the partial derivative vector $\nabla f = [f_x f_y, f_z]$.

$$\mathbf{R} = E\left\{(\nabla f)(\nabla f)^T\right\} = \begin{bmatrix} \mathrm{E}\{f_x^2\} & \mathrm{E}\{f_x f_y\} & \mathrm{E}\{f_z f_x\} \\ \mathrm{E}\{f_x f_y\} & \mathrm{E}\{f_y^2\} & \mathrm{E}\{f_y f_z\} \\ \mathrm{E}\{f_z f_x\} & \mathrm{E}\{f_y f_z\} & \mathrm{E}\{f_z^2\} \end{bmatrix}$$

The eigenvalue decomposition of $\mathbf{R}$ is given by

$$\mathbf{R} = \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} [\mathbf{e}_1\ \ \mathbf{e}_2\ \ \mathbf{e}_3]$$

The spread of ∇f over the 3 euclidean co-ordinates, when calculated for many phase realisations of multipath components, takes the form of an ellipsoid. The constituent basis vectors $\mathbf{e}_i$ give the principle directions of the ellipsoid, and the eigenvalues λ_i give the widths in these directions. The eigenvalues increase with the spread of multipath energy along the eigenvectors.

The tr[R] and det[R]

The $\sigma_m^2(\mathbf{u})$ increases with the spread of energy in direction $\mathbf{u}$. It can be shown analytically that the average of $\sigma_m^2(\mathbf{u})$ over all $\mathbf{u}$ is equivalent to the summation of diagonal terms of $\mathbf{R}$, also known as tr[$\mathbf{R}$].

$$E\left\{\sigma_{\mathrm{m}}^2(\hat{\mathbf{u}})\right\} = \frac{1}{3}\left(E\left\{f_{xx}^2\right\} + E\left\{f_{yy}^2\right\} + E\left\{f_{zz}^2\right\}\right)$$

The tr[$\mathbf{R}$] can be expressed as the average directional separation of multipath energy:

$$\mathrm{tr}[\mathbf{R}] = \frac{1}{2}\sum_{s}^{N_s} A_s^2\left[\left(d_x - \bar{d}_x\right)^2 + \left(d_y - \bar{d}_y\right)^2 + \left(d_z - \bar{d}_z\right)^2\right]$$

This tr[$\mathbf{R}$] is also given by the sum of eigenvalues:

$$\mathrm{tr}[\mathbf{R}] = \lambda_1 + \lambda_2 + \lambda_3$$

The tr[$\mathbf{R}$] does not provide an indication of the ratio between the eigenvalues. The det[$\mathbf{R}$] is a useful parameter as it increases with the eigenvalues as well as the uniformity of eigenvalues.

$$\det[\mathbf{R}] = \lambda_1\lambda_2\lambda_3$$

Thus, tr[$\mathbf{R}$] and det[$\mathbf{R}$] concisely characterize the second order moments of the directional derivatives. Unlike $\sigma_m^2(\mathbf{u})$, tr[$\mathbf{R}$] and det[$\mathbf{R}$] are not function of direction.

The UK ORS Scheme is gratefully acknowledged for Arindam Pal's postgraduate scholarship, as well as the facilities and support provided by the Centre for Communications Research at the University of Bristol.

centre for communications research

A Novel Quantification of 3D Directional Spread from Small-Scale Fading Analysis

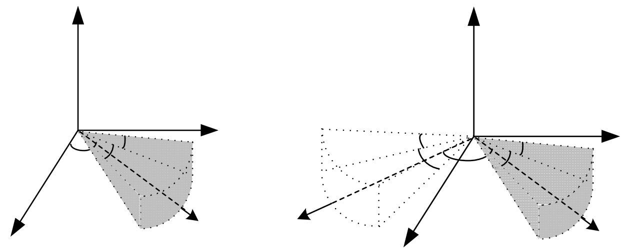
Arindam Pal, Mark Beach and Andy Nix, University of Bristol. a.pal@bristol.ac.uk

University of BRISTOL

Part 2/2: This section provides an illustration of the theory presented in Part 1 using 3D propagation data generated by simulation.

Simulations

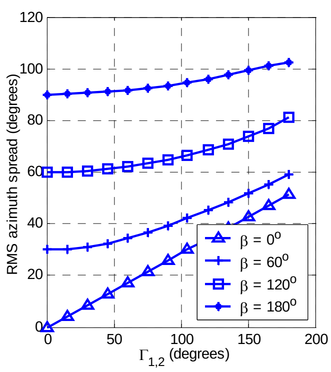
Multipath energy profiles comprising of either one or two uniformly distributed clusters in the range of [-Γ/2, Γ/2] in azimuth and [-Δ/2, Δ/2] in elevation are used.



k is the mean cluster angle and is contained in the azimuth plane. β is the angular separation of clusters.

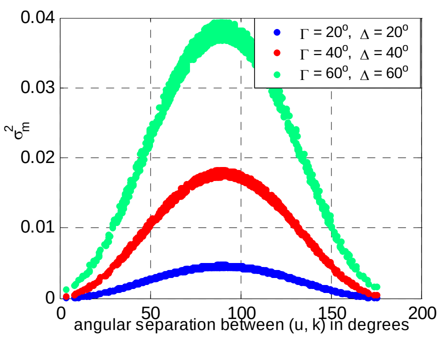
RMS Angular Spread

The RMS azimuth spread of a uniformly distributed cluster increases linearly with the azimuth angular width, as shown by the β=0º case. For a bi-cluster distribution, the RMS spread improves with the separation of clusters (β) as well as the constituent cluster widths ($\Gamma_{1,2}$):

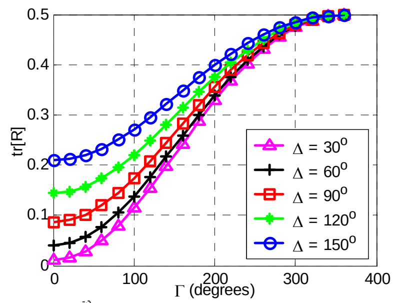


Rotation of cluster or linear array

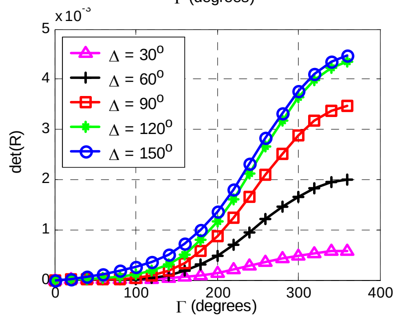
$\sigma_m^2(\mathbf{u})$ is maximum when the cluster mean angle is perpendicular to the line of observation, which is equivalent to the line of a linear array. Note that larger $\sigma_m^2(\mathbf{u})$ leads to lower antenna correlation.



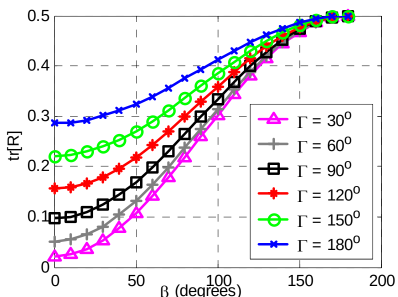
tr[R] and det[R]



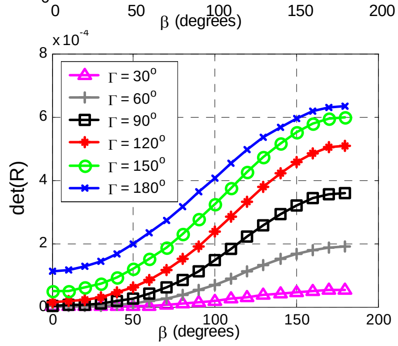
uni-cluster distribution: tr[**R**] increases with cluster azimuth and elevation widths. tr[**R**] achieves its upper bound for cluster azimuth width of 360º, regardless of the elevation width.



det[**R**] increases with cluster azimuth and elevation widths. Unlike tr[**R**], det[**R**] continues to improve with the elevation width when the azimuth width is 360º.



bi-cluster distribution: tr[**R**] and det[**R**] both increase with constituent cluster spreads as well as cluster angular separation. At β = 180º, tr[**R**] is maximum regardless of the constituent cluster widths.



Unlike tr[**R**], det[**R**] continues to improve with the constituent cluster widths even at maximum angular separation between clusters

Summary

The joint use of tr[**R**] and det[**R**] has been proposed as a quantification of the 3-D directional spread, where **R** is the covariance matrix of the partial derivatives of the spatial transfer function.

Key References

[1] B. H. Fleury, "First- and second-order characterization of direction dispersion and space selectivity in the radio channel", *IEEE Trans. on Information Theory*, vol. 46, iss. 6, pp. 2027-2044, September 2000.

[2] G. D. Durgin and T. S. Rappaport, "Theory of Multipath Shape Factors for Small-Scale Fading Wireless Channels," *IEEE Transactions on Antennas and Propagation*, vol. 48, no. 5, pp. 682-693, May 2000.

The UK ORS Scheme is gratefully acknowledged for Arindam Pal's postgraduate scholarship, as well as the facilities and support provided by the Centre for Communications Research at the University of Bristol.