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Error Performance of circular 16-DAPSK in Frequency-Selective Rayleigh Fading Channels with Diversity Reception

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Abstract: *An expression for the error probability of raised cosine filtered circular 16-ary differential amplitude and phase shift keying (circular 16-DAPSK) in a frequency-selective Rayleigh fading channel with postdetection diversity reception is obtained. The effect of delay spread, modulation ring ratio and filter roll-off are also considered in the formulation. For a constant bit rate, the average irreducible bit error probability for circular 16-DAPSK is shown to be superior to that of 16-DPSK but slightly worse than Q-DPSK.*

Introduction

Future mobile networks are looking to integrate digital voice with high capacity data based services. Digital modems are prone to the introduction of an irreducible error rate that arises as a result of user motion and/or time dispersion in the channel [1]. The resulting error floors are independent of signal strength and, unless corrected, can limit either the speed of the mobile (Doppler) or the maximum transmission rate (time dispersion). Hence, for a high data rate wireless modem, the effect of delay spread must be carefully considered.

The effect of delay spread on bandwidth efficient modulations schemes has already been investigated by various authors. The error performance of quadrature coherent PSK with bandlimiting raised cosine (RC) pulse shaping was examined in [2],[3]. Both papers concluded that Q-PSK outperform B-PSK under the same channel conditions. An indepth treatment of the error performance of RC filtered Q-DPSK with postdetection diversity combining is covered in [4]. The error performance of M-ary DPSK in frequency selective channels has already been analysed including the effects of Doppler shift [5], where diversity combining was considered with time-limited pulse shapes. Recently, work in [6] has formulated the error performance of RC filtered M-DPSK in a frequency-selective Rayleigh fading channel with diversity reception. The irrducible bit error probability of B-DPSK is worse than higher level schemes such as 4, 8 and 16-DPSK. No improvement is

observed when the modulation level is greater than four. This trend arises since the impact of a more complex, higher level, constellation pattern can override the advantages of a longer symbol period. The error performance of PSK and APSK (amplitude and phase shift keying) in a frequency-selective Rayleigh fading channel has also been reported in earlier works. In [7], 16-QAM was compared with M-PSK assuming coherent detection. In [8], circular 16-DAPSK was compared with M-DPSK. Both works used RC pulse shaping, however diversity reception was not considered in the comparison. Recently, in [9], the authors have calculated the error probability of circular 16-DAPSK in a frequency-selective Rayleigh fading channel with postdetection diversity reception and time-limited pulse shapes. The results have shown that circular 16-DAPSK performs better when compared with 16-DPSK and the improvement increases when diversity is applied. However, in a pratical situation, a bandlimited signal is normally used to improve bandwidth efficiency in a mobile system. Therefore, to justify the advantage of circular 16-DAPSK over 16-DPSK, it is essential to analysis the error performance of circular 16-DAPSK with bandlimited filtering.

The aim of this paper is to extend the work in [9], so that the error probability can be calculated when bandlimited pulse shapes are applied to circular 16-DAPSK in a frequency-selective Rayleigh channel. The expression takes into account factors such as root mean square (rms) delay spread, ring ratio, raised cosine roll-off factor and post-detection diversity combining. Using such equations, the irreducible error floor introduced by delay spread can be evaluated and the sensitivity to the filter roll-off value determined both with and without the use of diversity combining.

System Modelling

The overall transmission system with postdetection diversity reception is modelled as shown in Fig.1. For mathematical convenience, an equivalent lowpass signal repre-

sentation is used. To achieve both a narrowband spectrum and intersymbol interference (ISI) free transmission in the absence of delay spread, the overall raised cosine filter response is applied. [2]-[4]:

$$h_{RC}(t) = \frac{1}{T_s} \frac{\sin(t/T_s)}{\pi t/T_s} \frac{\cos(\alpha \pi t/T_s)}{1 - (2\alpha t/T_s)^2} \qquad (1)$$

where α $(0 \leq \alpha \leq 1)$ is the roll-off factor and T_s is the symbol duration. In this paper, a square root RC filter is assumed for both transmitter and receiver, and hence their frequency responses can be written as:

$$H_T(f) = T_s\sqrt{H_{RC}(f)}, \qquad H_R(f) = \sqrt{H_{RC}(f)} \qquad (2)$$

where $H_{RC}(f)$ is the frequency response of $h_{RC}(t)$ and the constant T_s is used to normalise the pulse to obtain unit power.

For circular 16-DAPSK transmission, the equivalent lowpass transmitted signal $u(t)$ can be written as:

$$u(t) = A \sum_{n=-\infty}^{\infty} d_n e^{j\theta_n} h_T(t - nT_s) = \sum_{n=-\infty}^{\infty} u_n \qquad (3)$$

where $d_n = 1$ or β, and β represents the ring ratio. Ad_n represents the amplitude of the transmitted signal and $\theta_n = \pi(i-1)/4$, where $i = 1, 2, \ldots, 8$ is the transmitted phase. The time response of the transmit filter is defined as $h_T(t)$. The differential encoding technique for circular 16-DAPSK is discussed in [10]. The signal generated is sent over L independent frequency-selective Rayleigh fading channels, the statistical characteristics of which are assumed identical.

The Fading Channel: At the nth time interval, the equivalent lowpass received signal in the kth diversity channel, $r_k(t)$, can be written as:

$$r_k(t) = \int_{-\infty}^{\infty} g_k(t, \tau) u(t - \tau) d\tau + z_k(t) \qquad (4)$$

For the kth diversity branch, $g_k(t, \tau)$ represents the channel impulse response which is a zero mean complex Gaussian fading process. $z_k(t)$ denotes zero mean complex additive white Gaussian noise of power spectral density $2N_o$. For strictly frequency-selective fading channels [11] (i.e. fading variations much slower compared to the symbol rate), the auto-correlation function for $g_k(t, \tau)$ is stated below:

$$E[g_k(t, \tau) g_k^*(t - \mu, \tau - \lambda)] = \xi_g(\tau)\delta(\lambda) \qquad (5)$$

for $k = 1, 2, \ldots, L$. $E[\bullet]$ denotes the ensemble average and $*$ denotes the complex conjugate value. $\xi_g(\tau)$ represents the power delay profile. Earlier works have shown that *root mean square delay spread* is an important parameter in the analysis of the error performance of a digital system in frequency-selective fading channels [2]-[4]. This parameter is defined as $\tau_{rms} = [\int_{-\infty}^{\infty} \tau^2 \xi_g(\tau) d\tau]^{1/2}$, where zero mean delay and $\int_{-\infty}^{\infty} \xi_g(\tau) = 1$ are assumed.

Receiver Processing: The receiver block diagram for the kth order diversity branch is shown in Fig.1. The signal $r_k(t)$ is passed through a matched filter with an impulse response of $h_R(t)$ and sampled at time $t = pT_s$, where p is an integer number. The output of the filter is therefore:

$$U_k = \int_{-\infty}^{\infty} r_k(\alpha) h_R(t - \alpha) d\alpha \qquad (6)$$

and similarly, for $r_k(t - T_s)$, the output of the filter is:

$$K_k = \int_{-\infty}^{\infty} r_k(\alpha - T_s) h_R(t - T_s - \alpha) d\alpha \qquad (7)$$

For amplitude detection, a square-law envelope detector is used to extract the envelope of U_k and K_k. The combined outputs after the postdetection combiner, $|U|^2 = \sum_{k=1}^{L} |U_k|^2$ and $|K|^2 = \sum_{k=1}^{L} |K_k|^2$, are then passed to the amplitude decision device, where the amplitude ratio $|U|^2/|K|^2$ is decoded by comparing it with two decision thresholds, $\xi_H = (1 + \beta)/2$ and $\xi_L = 1/\xi_H$. For the phase detection, a conventional 8-DPSK detector is applied. As indicated in Fig.1, the demodulator at each branch forms the product between the two complex Gaussian random variables of (6) and (7), so for L^{th} order diversity, the combiner sums all the demodulator outputs and forms a combined vector, $Z = \sum_{k=1}^{L} Z_k = \sum_{k=1}^{L} U_k K_k^*$

Mathematical Analysis

The error performance of circular 16-DAPSK in frequency-flat fading channels has already been analysed in [12],[13],[14]. The error probability can be calculated using two consecutive signalling periods. However, in a frequency-selective fading channel, due to delay spread, the error is dominated by the time varying ISI, hence the error performance depends on the transmitted sequence, Φ:

$$\Phi = (\ldots, u_{n-2}, u_{n-1}, u_n, u_{n+1}, u_{n+2}, \ldots) \qquad (8)$$

The probability of detecting the amplitude and phase of a data symbol incorrectly can be calculated by using the expressions below:

$$P_a = \frac{1}{N_\Phi} \sum_\Phi P_a(\Phi, d_{n-1}, d_n) \qquad (9)$$

$$P_p = \frac{1}{N_\Phi} \sum_\Phi P_p(\Phi, \theta_{n-1}, \theta_n) \qquad (10)$$

where N_Φ is the number of possible sequences. $P_a(\Phi, d_{n-1}, d_n)$ is the conditional probability of making an amplitude decision error when the transmitted amplitude sequence goes from d_{n-1} to d_n. $P_p(\Phi, \theta_{n-1}, \theta_n)$ is the conditional probability of making a phase error when the transmitted phase goes from θ_{n-1} to θ_n. The expressions for the conditional amplitude and phase error probabilities are already given in [9]. Both expressions are a function of the second central moments between the matched filter outputs, U_k and K_k. The second central moments m_{UU}, m_{KK} and m_{UK} (normalised to $T_s/2N_o$) are shown below:

$$m_{UU} = \Gamma_s \int_{-\infty}^{\infty} \xi_g(0, \tau) \left\{ \sum_{n=-\infty}^{\infty} d_n e^{j\theta_n} H_n \right\}$$
$$\times \left\{ \sum_{n=-\infty}^{\infty} d_n e^{-j\theta_n} H_n \right\} d\tau + 1 \qquad (11)$$

$$m_{KK} = \Gamma_s \int_{-\infty}^{\infty} \xi_g(0, \tau) \left\{ \sum_{n=-\infty}^{\infty} d_{n-1} e^{j\theta_{n-1}} H_n \right\}$$
$$\times \left\{ \sum_{n=-\infty}^{\infty} d_{n-1} e^{-j\theta_{n-1}} H_n \right\} d\tau + 1 \qquad (12)$$

$$m_{UK} = \Gamma_s \int_{-\infty}^{\infty} \xi_g(T_s, \tau) \left\{ \sum_{n=-\infty}^{\infty} d_n e^{j\theta_n} H_n \right\}$$
$$\times \left\{ \sum_{n=-\infty}^{\infty} d_{n-1} e^{-j\theta_{n-1}} H_n \right\} d\tau \qquad (13)$$

where $H_n = T_s h_{RC}(pT_s - \tau - nT_s)$ and $\Gamma_s = (1 + \beta) A^2 T_s / 4 N_o$. Γ_s represents the average received symbol energy-to-noise power spectral density ratio per channel.

Numerical Results and Discussion

Analysis in the last section has shown that the calcuated error performance is dependent on the number of interference symbols considered in equations (11)–(13). Although the equations derived in the last section allow us to considered any number of interference symbols, in this paper only adjacent-pulse-limited ISI channels [5],[6],[11], are considered for numerical evaluation. Without loss of generality, the symbol at time $n = 0$ is used for detection, hence the sampling time, $pT_s = 0$. The average amplitude and phase error probabilities for circular 16-DAPSK can be written in the following form:

$$P_a = \frac{1}{2^{4k}} \sum_\Phi P_a(\Phi, d_{-1}, d_0) \qquad (14)$$

$$P_p = \frac{1}{2^{4k}} \sum_\Phi P_p(\Phi, \theta_{-1}, \theta_0) \qquad (15)$$

where $k = 4$ represents the number of bits per symbol

and $\Phi = (u_{-2}, u_{-1}, u_0, u_1)$ represents the truncated transmitted sequence. (Note: For expressions $P_a(\Phi, d_{-1}, d_0)$ and $P_p(\Phi, \theta_{-1}, \theta_0)$ see [9, Eq.2 and 9].) The second central moments are simplified to:

$$m_{UU} = \Gamma_s \int_{-\infty}^{\infty} \xi_g(0, \tau) D_{UU} d\tau + 1 \qquad (16)$$

$$m_{KK} = \Gamma_s \int_{-\infty}^{\infty} \xi_g(0, \tau) D_{KK} d\tau + 1 \qquad (17)$$

$$m_{UK} = \Gamma_s \int_{-\infty}^{\infty} \xi_g(T_s, \tau) D_{UK} d\tau \qquad (18)$$

with

$$D_{UU} = d_{-1}^2 H_{-1}^2 + d_0^2 H_0^2 + d_1^2 H_1^2$$
$$+ 2[d_{-1} d_0 H_{-1} H_0 \cos(\theta_0 - \theta_{-1})$$
$$+ d_{-1} d_1 H_{-1} H_1 \cos(\theta_1 - \theta_{-1}) + d_0 d_1 H_0 H_1 \cos(\theta_1 - \theta_0)]$$

$$D_{KK} = d_{-2}^2 H_{-1}^2 + d_{-1}^2 H_0^2 + d_0^2 H_1^2$$
$$+ 2[d_{-2} d_{-1} H_{-1} H_0 \cos(\theta_{-1} - \theta_{-2})$$
$$+ d_{-2} d_0 H_{-1} H_1 \cos(\theta_0 - \theta_{-2}) + d_{-1} d_0 H_0 H_1 \cos(\theta_0 - \theta_{-1})]$$

$$D_{UK} = d_{-2} d_{-1} H_{-1}^2 e^{j(\theta_{-1} - \theta_{-2})} + d_{-1} d_0 H_0^2 e^{j(\theta_0 - \theta_{-1})}$$
$$+ d_0 d_1 H_1^2 e^{j(\theta_1 - \theta_0)} + d_{-2} d_1 H_{-1} H_1 e^{j(\theta_1 - \theta_{-2})}$$
$$+ d_{-1} d_0 H_{-1} H_1 e^{j(\theta_0 - \theta_{-1})} + d_{-1}^2 H_{-1} H_0 + d_0^2 H_0 H_1$$
$$+ d_{-2} d_0 H_{-1} H_0 e^{j(\theta_0 - \theta_{-2})} + d_{-1} d_1 H_0 H_1 e^{j(\theta_1 - \theta_{-1})}$$

Assuming $\xi_g(\tau)$ is defined with a rectangular power delay profile as in [11], the average bit error probability for 4-, 16-DPSK and circular 16-DAPSK with 1st and 2nd order diveristy is shown in Fig.2 as a function of normalised rms delay spread, d_b (a roll-off factor of 0.5 is used). The rms delay spread is normalised in terms of the bit period (i.e. $d_b = \tau_{rms}/T_b$) so that the error performances can be compared for the same information throughput for different levels of modulation scheme. The result shows that when the system operates without diversity, the perform of circular 16-DAPSK is close to that of QDPSK. The effect of filter roll-off factor, α, on the error probability is shown in Fig.3 for $d_b = 0.2$. As expected, the error probability is reduced as α is increased. For circular 16-DAPSK, the modulation ring ratio, β, has been optimised for the above results. A value of 2.2 was found to result in the best performance. The numerical results shown in Fig.2 and 3 have indicated that circular 16-DAPSK outperforms 16-DPSK when bandlimited pulse shaping is applied to the systems. This trend confirms the result given in [9] where simpler time-limited pulse shapes were used. The advantage of using circular 16-DAPSK rather than 16-DPSK is further enhanced when postdetection diversity reception is applied. For an rms delay spread of 50ns (indoor environments) and a target bit error probability of 1e-5, asumming second order diversity reception,

a QDPSK modem can transmit up to 1.2Mbps whereas for 16-DPSK only 800kbps can be achieved. Using circular 16-DAPSK, the maximum allowable transmission rate is 1.04Mbps. For outdoor applications, a 64kbps data rate and a target bit error probability of 1e-3 were assumed. Once again, assumming second order diversity reception, QDPSK and 16-DPSK can tolerate rms delay spreads of $3.125\mu s$ and $1.875\mu s$ respectively. Applying circular 16-DAPSK, an rms delay spread of $2.66\mu s$ can be tolerated.

Conclusions

The theoretical error performance of raised cosine filtered circular 16-DAPSK with postdetection diversity combining has been analysed for a frequency-selective Rayleigh fading channel. The numerical results have shown that although the error probability of circular 16-DAPSK is slightly worse when compared with Q-DPSK under the same bit rate transmission (2nd order diversity), it is superior when compared to 16-DPSK. The advantage of circular 16-DAPSK over 16-DPSK is enhanced when postdetection diversity reception is applied. Although circular 16-DAPSK and 16-DPSK both operate with symbol periods twice that of Q-DPSK, it is interesting to note that the error performance of these schemes are not significantly improved. However, for the same bit rate, both 16-level schames use half the bandwidth occupied by Q-DPSK.

Acknowledgments

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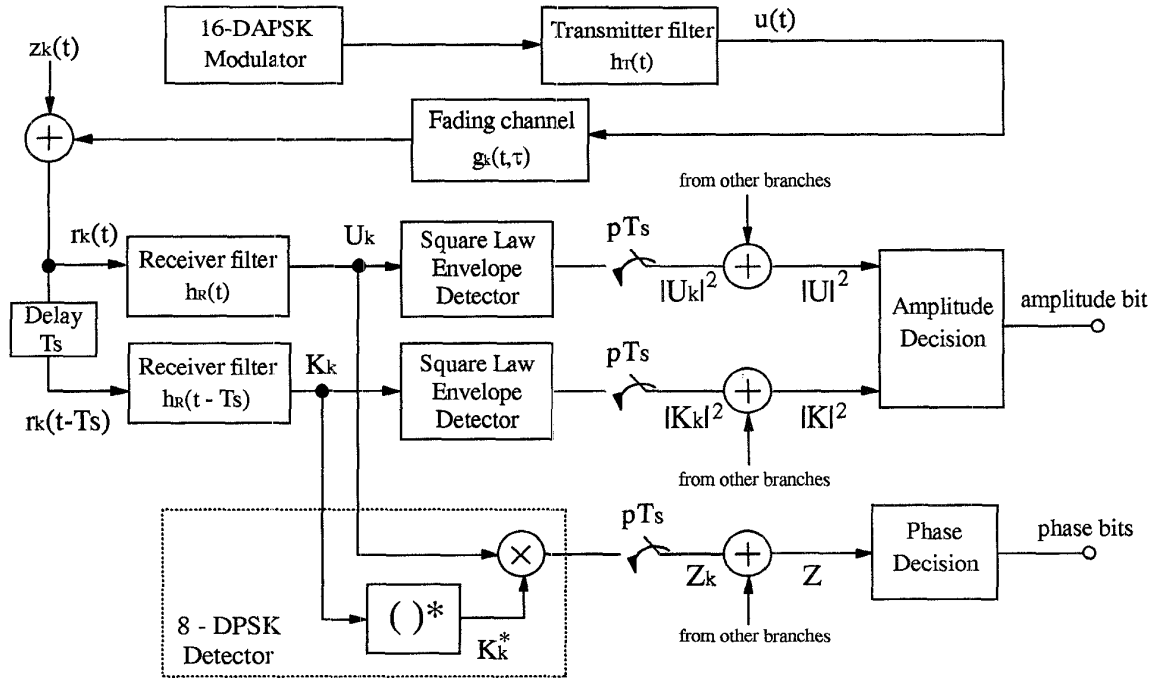
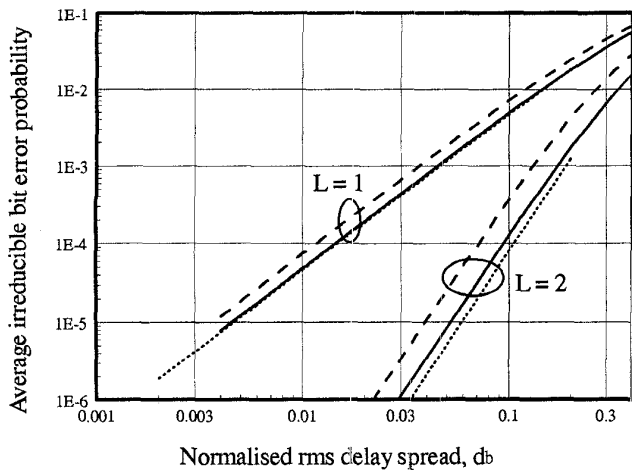Fig.1: Block diagram for circular 16-DAPSK transmission system in equivalent lowpass representation.

Q-DPSK ··················   16-DPSK --------   circular 16-DAPSK ————

Fig.2: Average irreducible bit error probability of 4, 16-DPSK and circular 16-DAPSK versus normalised rms delay spread, d_b.
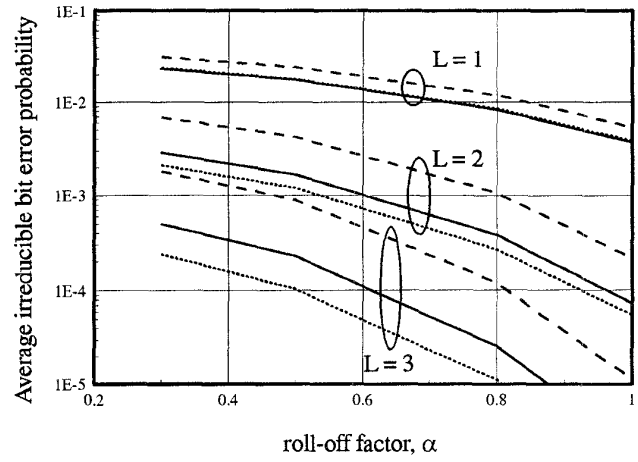Rectangular delay profile and $\alpha = 0.5$.

Fig.3: Average irreducible bit error probability of 4, 16-DPSK and circular 16-DAPSK versus raised cosine filter roll-off.
Rectangular delay profile and $d_b = 0.2$.