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Semi-Hard Interference Cancellation for Uncoded DS-CDMA Systems

Christos Kasparis, Robert J. Piechocki, Andrew R. Nix Department of Electrical and Electronic Engineering University of Bristol MVB 2.19,Woodland Road Bristol,BS8 1UB,UK. Email: c.kasparis@bris.ac.uk Paul N. Fletcher
QinetiQ Ltd.
MVB 2.19,Woodland Road Bristol,BS8 1UB,UK
Email:Paul.Fletcher@bris.ac.uk

Abstract—A new iterative technique for suppressing multi-user interference in uncoded DS-CDMA systems is proposed. In the new approach semi-hard decisions are taken at the output of the linear MMSE detector. This involves taking hard decisions only for symbols which satisfy a reliability criterion, while the rest are left unaltered in their soft form. The semi-hard estimate is subsequently used for parallel interference cancellation, which is then followed by additional linear MMSE filtering. This procedure is repeated for a small number of times. Simulation results show that the proposed detector is capable of approaching closely the single-user performance in a fully loaded multi-user scenario and this is achieved within 5 to 10 iterations. In the same system scenario the new technique outperforms the Probabilistic Data Association (PDA) algorithm, which is considered as a stateof-the art solution. The computational complexity of the proposed detector is of the same order as of a standard linear detector.

I. INTRODUCTION

Multi-User Detection (MUD) techniques offer significant performance improvements and increased bandwidth efficiency in DS-CDMA, relative to the conventional correlation detector. The primary idea in MUD is to take advantage of the fact that Multi-Access Interference (MAI) in the correlator's estimate, has a well defined structure. This allows MAI to be rejected rather than treated as an increase in the noise power, compared to the single user channel. The optimal Maximum-Likelihood (ML) detector [8], was quickly realized to be infeasible to implement due to the exponential dependency of the complexity on the number of users in the system. However, the vast performance gains and bandwidth efficiency offered by the ML detector has since motivated the proposal of suboptimal solutions, which aim to approach the ML performance in a computationally efficient manner.

Early proposed classes of suboptimal detectors had been the linear detectors: the Decorrelator and MMSE detector, the Interference Cancellation (IC) based detectors: the Serial and Parallel Interference Cancellers and a combination of these two classes which has given the popular Decision Feedback (DF) detector. A review on these early proposed classes of detectors can be found in [9]. These early classes of detectors, though they offer an attractive performance-complexity tradeoff, in most realistic scenarios they fail to approach closely the optimal performance offered by the ML detector. A promising Probabilistic Data Association (PDA) detection algorithm has been proposed recently in [3], where it is shown to outperform the powerful Semi Definite (SD) relaxation detector [7] in

less computational complexity $O(n^3)$. The main idea in PDA is to make signal probability updates by approximating the interfering signals plus noise (which have a multi-modal Gaussian distribution) by a single Gaussian distribution with matched mean and covariance.

In this paper, an iterative Interference Cancellation (IC)/MMSE technique in uncoded DS-CDMA is proposed. Soft Parallel Interference Cancellation (PIC) [6] combined with linear MMSE processing has been shown to provide near single-user performance within Turbo Multi-User Detection (MUD) in coded DS-CDMA [1]. In the Turbo detection framework, symbol estimates are constructed from the posterior code-bit probabilities and then used for soft cancelling interference. In the new detector, multi-user interference at the output of the u^{th} user's matched filter is reconstructed and cancelled using Semi-Hard (SH) decisions, which are taken at the outputs of linear MMSE filters. Taking SH decisions has been proposed recently in [2] and it involves identifying which of the soft MMSE estimates satisfy some reliability criterion. Symbols which satisfy the specified criterion are hard decided to the nearest signalling point while those which fail it are left unchanged in their soft form. The SH-IC stage, is again followed by MMSE filtering in order to suppress further residual multi-user interference. This approach results into a complexity order of $O(n^3)$.

In [2] SH decisions were used as a prior solution in a Tikhonov Regularization (TR) estimator. The TR estimator is a biased linear estimator, similar in structure to the MMSE estimator, but it allows a prior solution to be proposed to the problem, which serves for cancelling the bias. Optimal parameter selection in TR provides a generalized MMSE detector when a prior solution is available. The iterative detector in [2], was shown to provide substantial performance gains relative to the MMSE detector and this was achieved without an increase in the complexity order. Simulation results show that the SH-IC/MMSE detector which is proposed here outperforms the solution proposed in [2] and also succeeds in approaching closely the single user performance in a fully-loaded DS-CDMA system. Moreover, this near-optimal performance is achieved within 5 to 10 iterations while the detector in [2] requires 20-30 iterations to converge. Simulation results also

¹the complexity of the SD relaxation detector is $O(n^{3.5})$

show that the SH-IC/MMSE detector, in the fully-loaded scenario, outperforms the PDA algorithm that was proposed in [3].

II. DS-CDMA UPLINK MODEL

A. Synchronous non-dispersive scenario

A discrete-time baseband model for the up-link of a synchronous DS-CDMA is used. U active users are assumed in the system, where each transmits a bit sequence $\mathbf{b}_u, 1 \leq u \leq U$ of length N. $\mathbf{b}_u, \forall u$ are mapped on the BPSK constellation to produce symbol sequences \mathbf{a}_u . Each of the symbol sequences is then modulated by a high rate spreading code \mathbf{s}_u of length L chips, which is unique for user u. The spreading codes are chosen and normalized so that they satisfy the condition : $\mathbf{s}_i \cdot \mathbf{s}_j^T = \left\{ \begin{smallmatrix} 1, & i=j \\ \varrho_{ij} < 1, & i\neq j \end{smallmatrix} \right\}$. The spread sequences are transmitted through unity impulse response channels: $h_u = 1, \forall u$. This fact makes the observation with time index $n, 1 \leq n \leq N$ dependent only on information with the same time index. This allows to construct the system model by considering only a single symbol duration:

$$\mathbf{r} = \sum_{u=1}^{U} A_u a_u \mathbf{s}_u + \mathbf{n} \tag{1}$$

where A_u is the received amplitude of user u and \mathbf{n} is zero mean AWGN with covariance $\sigma^2\mathbf{I}$. The sufficient statistic for estimating the user information is extracted by matched-filtering \mathbf{r} with the user codes $\mathbf{s}_u, \forall u$. So the despread channel observation is given us:

$$z = RAa + v \tag{2}$$

where **R** is a $[U \times U]$ semi-definite positive matrix in which the ij entry $(1 \le i, j \le U)$ is given as : $\mathbf{s}_i \cdot \mathbf{s}_j^T$, $\mathbf{A} = diag(A_1, A_2, ..., A_u)$ and $\mathbf{v} \sim N(\mathbf{0}, \sigma^2 \mathbf{R})$.

B. Multipath channel scenario

In this case, the spread symbols are transmitted through multipath channels, which in the discrete time domain can be modelled as FIR filters of memory order M:

$$\mathbf{h}_{u} = \sum_{m=1}^{M+1} h_{u}^{m} \cdot \delta[(k-m)T_{c}]$$
 (3)

where k is a discrete unit delay variable, h_u^m are the filter's tap weights, T_c represents the chip period and $\delta[k] = {1, k=0 \brace 0, k\neq 0}$. The relative delays between user transmissions are also incorporated in the channel responses. The receiver observes a superposition of the filtered signals in AWGN:

$$\mathbf{r} = \sum_{u=1}^{U} \sum_{n=1}^{N} A_u a_u[n] \cdot (\mathbf{s}_u * \mathbf{h}_u) + \mathbf{n} = \sum_{u=1}^{U} \sum_{n=1}^{N} A_u a_u[n] \cdot \mathbf{g}_u + \mathbf{n}$$
(4)

(*) denotes discrete convolution between two sequences, \mathbf{g}_u can be thought of as the effective user code, whose duration however spreads beyond the duration of an information symbol

 T_s resulting in ISI. The sufficient statistic for estimating the user information is extracted by matched-filtering ${\bf r}$ with the effective user codes ${\bf g}_u, \forall u$. In a system level this is equivalent to passing ${\bf r}$ through a bank of U parallel matched-filters with responses ${\bf g}_u^T$. It is useful if (4) is re-written in a compact matrix form:

$$\mathbf{r} = \sum_{u=1}^{U} \mathbf{G}_{u} \mathbf{A}_{u} \mathbf{a}_{u} + \mathbf{n} = \mathbf{G} \mathbf{A} \mathbf{a} + \mathbf{n}$$
 (5)

where \mathbf{G}_u is a $[(LN+M)\times N]$ matrix, each column of which contains a downward shift of \mathbf{g}_u , $\mathbf{A}_u = diag(A_u, A_u, ..., A_u)$

and its size is $N \times N$. In the second equality of (5), $\mathbf{G} = [\mathbf{G}_1, \mathbf{G}_2, .., \mathbf{G}_U], \mathbf{A} = block - diag(\mathbf{A}_1, \mathbf{A}_2, .., \mathbf{A}_U)$ and $\mathbf{a} = [\mathbf{a}_1^T, \mathbf{a}_2^T, .., \mathbf{a}_U^T]^T$. At the matched filter bank's output:

$$\mathbf{z} = \mathbf{G}^T \mathbf{r} = \mathbf{G}^T \mathbf{G} \mathbf{A} \mathbf{a} + \mathbf{G}^T \mathbf{n} = \mathbf{R} \mathbf{A} \mathbf{a} + \mathbf{v}$$
 (6)

In (8) ${\bf R}$ is a $[UN\times UN]$ block filtering matrix, which is symmetric semi-definite positive, and ${\bf v}\sim N\left({\bf 0},\sigma^2{\bf R}\right)$ i.e. colored Gaussian noise. ${\bf z}$ is the sufficient statistic for estimating ${\bf a}$, usually given also knowledge of ${\bf R}$ and ${\bf A}$.

III. SEMI-HARD INTERFERENCE CANCELLATION COMBINED WITH MMSE FILTERING

In the initial iteration of the proposed technique, MMSE MUD [4] is applied in order to produce an initial estimate:

$$\widehat{\mathbf{a}}^{MMSE} = (\mathbf{ARA} + \boldsymbol{\sigma}^2 \mathbf{I})^{-1} \mathbf{Az}$$
 (7)

In order to generate an updated estimate for user u, first semi-hard processing is applied on all elements of $\hat{\mathbf{a}}^{MMSE}$ except for the u^{th} element which is set to zero:

$$\widetilde{\mathbf{a}}_{u} = \left[\widetilde{a}_{1}, ..., \widetilde{a}_{u-1}, 0, \widetilde{a}_{u+1}, ..., \widetilde{a}_{U}\right]^{T}$$
(8)

The criterion for deciding whether soft estimates should be hard decided or left unchanged, can be based on a threshold criterion or on a probability of error criterion. Calculating the probability of error for each symbol is possible because the residual interference plus noise at the output of an MMSE detector can be well approximated by a Gaussian distribution [5]. So In the initial iteration, the interference plus noise term $\mathbf{w} \sim N(\mathbf{0}, \sigma^2(\mathbf{ARA} + \sigma^2\mathbf{I})^{-1})$ (assuming $\hat{\mathbf{a}}^{MMSE} = \mathbf{a} + \mathbf{w}$). The semi-hard estimate in (8) is used to partially cancel interference from the u^{th} matched filter output:

$$\mathbf{z}_{u} = \mathbf{z} - \mathbf{R} \mathbf{A} \widetilde{\mathbf{a}}_{u} \tag{9}$$

In order to suppress further multi-user interference additional MMSE filtering is also applied on \mathbf{z}_u :

$$\widehat{\mathbf{a}}_{u}^{MMSE} = \mathbf{q}_{u}^{T} \mathbf{z}_{u} \quad , \mathbf{q}_{u} = \underset{\mathbf{q}_{u} \in \Re^{U}}{\operatorname{arg \, min}} E\{ \left| a_{u} - \mathbf{q}_{u}^{T} \mathbf{z}_{u} \right|^{2} \} \quad (10)$$

In [1] it is has been derived that:

$$\mathbf{q}_u = A_u \mathbf{R}^{-1} (\mathbf{V}_u + \boldsymbol{\sigma}^2 \mathbf{R}^{-1})^{-1} \mathbf{e}_u$$
 (11)

TABLE I SHIC/MMSE ALGORITHM

where e_u is the all zero vector except for the u^{th} element, which is 1, and

$$\mathbf{V}_{u} = \mathbf{A}cov\{\mathbf{a} - \widetilde{\mathbf{a}}_{u}\}\mathbf{A} = \sum_{j \neq u} A_{j}^{2}(1 - \widetilde{a}_{j}^{2})\mathbf{e}_{j}\mathbf{e}_{j}^{T} + A_{u}^{2}\mathbf{e}_{u}\mathbf{e}_{u}^{T}$$
(12)

where the symbol power $|a_u|^2$ is assumed to be normalized to unity. The SH-IC/MMSE estimate can be thus written as (see [1]):

$$\widehat{a}_{u}^{MMSE} = A_{u} \mathbf{e}_{u}^{T} (\mathbf{V}_{u} + \boldsymbol{\sigma}^{2} \mathbf{R}^{-1})^{-1} (\mathbf{R}^{-1} \mathbf{z} - \mathbf{A} \widetilde{\mathbf{a}}_{u}) \quad (13)$$

where the error term in the soft estimate $w_u \sim N(\ \mu_u, \mu_u - \mu_u^2)$ with $\mu_u = A_u^2[(\mathbf{V}_u + \boldsymbol{\sigma^2}\mathbf{R}^{-1})^{-1}]_{uu}$, allowing the probability of error for the symbol from user u to be recalculated as a possible criterion for taking semi-hard decisions in the next iteration. The SH-IC/MMSE algorithm is summarized in Table 1. In case the code-cross correlation matrix is nearly singular a small positive weight can be applied on its main diagonal $\mathbf{R} \approx \mathbf{R} + \delta \mathbf{I}$ in order to allow a stable inversion in (13).

A. Reduced complexity sliding window algorithm for multipath channels

For the multi-path channels scenario, the direct application of the SH-IC/MMSE algorithm is not practical as the size of the matrix which needs to be inverted depends on N (the packet length) as well as U. In this section a reduced complexity sliding window algorithm, which can be also found in [1] for the turbo multi-user detector, is given. The idea in the sliding window detection algorithm is to exploit the fact that the symbol $a_u[n]$ has a limited temporal spread within the observation vector ${\bf r}$, as this is given by (5). In particular, symbol $a_u[n]$ has an influence on ${\bf r}$ only within the normalized (by the chip period) time window:

$$\tau_{u,n} = [1 + (n-1)L, ..., L + M + (n-1)L]$$
 (14)

Thus the system model within the window $\tau_{u,n}$ can be expressed as:

$$\mathbf{r}(\boldsymbol{\tau}_{\mathbf{u},\mathbf{n}}) = \mathbf{G}(\boldsymbol{\tau}_{\mathbf{u},\mathbf{n}},:)\mathbf{A}\mathbf{a} + \mathbf{n}$$
 (15)

where $(\tau_{\mathbf{u},\mathbf{n}},:)$ signifies all the rows within the window's

TABLE II

SLIDING WINDOW SHIC/MMSE ALGORITHM FOR MULTI-PATH CHANNEL SCENARIO

indices and all the columns of the matrix. When a SH estimate $\widetilde{\mathbf{a}}_{u,n}$ is available then the observation vector after interference cancellation is given as:

$$\mathbf{r}_{u,n}(\boldsymbol{\tau}_{\mathbf{u},\mathbf{n}}) = \mathbf{r}(\boldsymbol{\tau}_{\mathbf{u},\mathbf{n}}) - \mathbf{G}(\boldsymbol{\tau}_{\mathbf{u},\mathbf{n}},:)\mathbf{A}\widetilde{\mathbf{a}}_{u,n}$$
(16)

Linear MMSE filtering for estimating $a_u[n]$ is now applied on

$$\mathbf{r}_{u,n}(\boldsymbol{\tau}_{\mathbf{u},\mathbf{n}}):$$

$$\widehat{\mathbf{a}}_{u}^{MMSE}[n] = \mathbf{q}_{u,n}^{H} \mathbf{r}_{u,n}(\boldsymbol{\tau}_{\mathbf{u},\mathbf{n}}) \tag{17}$$

where $\mathbf{q}_{u,n} = \underset{\mathbf{q}_{u} \in \mathbb{C}^{(L+M)}}{\min} E\{ \left| a_{u}[n] - \mathbf{q}_{u,n}^{H} \mathbf{r}_{u,n}(\boldsymbol{\tau}_{\mathbf{u},\mathbf{n}}) \right|^{2} \}.$ In [1] it is shown that:

$$\mathbf{q}_{u,n} = [\mathbf{G}(\boldsymbol{\tau}_{\mathbf{u},\mathbf{n}},:)\mathbf{A}\mathbf{V}_{u,n}\mathbf{A}^{H}\mathbf{G}^{H}(\boldsymbol{\tau}_{\mathbf{u},\mathbf{n}},:)]^{-1}\mathbf{g}_{u,n} \quad (18)$$

where $\mathbf{g}_{u,n}$ is the column $c_{u,n} = [(u-1)N + n]$ of the matrix $\mathbf{G}(\boldsymbol{\tau}_{\mathbf{u},\mathbf{n}},:)\mathbf{A}$.

The reader is directed to [1] for an efficient algorithm for calculating the matrix under inversion using the matrix inversion lemma. The sliding window algorithm reduces the computational complexity of the SH-IC/MMSE to $O([L+M]^2)$ per user per symbol per iteration. Table 2 summarizes the sliding window algorithm for performing SH-IC/MMSE detection.

IV. NUMERICAL RESULTS

The proposed SHIC/MMSE multi-user detector has been applied for detection in a synchronous DS-CDMA scenario with U=50 active users. BPSK modulation and real random spreading codes of length L=50 chips have been assumed. The propagation paths are fixed to unity for all users and $A_u=1, \forall u.$ As a criterion for taking semi-hard decisions, a threshold criterion has been used according to which: if $\left|\widehat{a}_{u,k}^{MMSE}\right| > \Delta_{u,k}$ then $\tilde{a}_{u,k} = sign(\hat{a}_{u,k}^{MMSE})$ otherwise $\tilde{a}_{u,k} = \widehat{a}_{u,k}^{MMSE}$, where k is the number of iteration and Δ the

²The index of the symbol $a_u[n]$ in the vector **a** is (u-1)N+n.

threshold value. In the simulations the threshold boundaries $\Delta_{u,k} = k^{-0.3}, \forall u$. Simulation results are provided in figure 1. For comparison the performance of the Bootstrap detector proposed in [2](where SH processing is also applied) is provided as well as the performance of the PDA algorithm proposed in [3]. It is observed that 1 iteration is sufficient for the SH-IC/MMSE detector to provide big performance gain relative to the MMSE detector. After 10 iterations the performance is very closed to the single-user performance. It is also observed that at BER of practical interest (10^{-5}) the SH-IC/MMSE detector outperforms the PDA detector by a significant margin and the Bootstrap detector by 2dB. Complexity-wise the three detectors are of identical order $0(U^3)$.

In the second simulated scenario, synchronicity among users was again assumed but this time half of the U=24 users were received in 6dB more power than the other half. The code length in this case was L=24 (making the system fully loaded), while the decision boundaries for the SH-IC/MMSE detector were chosen in the same manner as before. The simulation results given in figure 2 show that in this case as well the proposed detector provides the best performance. It is worth noting at this point that the PDA detector does not provide any significant performance improvements after 3-4 iterations, so the performance comparison with PDA is fair despite the difference in the number of iterations.

In the next example a multipath channel scenario is considered. In particular each user channel is modeled by an FIR filter with impulse response \mathbf{h}_u . The channel taps are assumed to be i.i.d. complex Gaussian with zero mean and variance $\frac{1}{2(M+1)}$ per dimension (i.e. Rayleigh faded), where M=4. In this example QPSK signalling is used and all users are received at equal power levels. Furthermore U=10 users are assumed in the system and the random complex spreading codes have length L=10 chips. The results show that in this case the SH-IC/MMSE detector after 15 iterations, provides almost the same performance as the PDA detector after 3 iterations.

In the last example the performance of the SH-IC/MMSE has been compared to the performances of other types of IC/MMSE, where different kinds of functions are imposed on the MMSE estimate prior to IC. These kind of functions have been proposed for iterative PIC without additional MMSE processing; in [10] this function is linear so the soft estimate is used directly for IC while in [11] a hard-limiting function is applied on the soft-estimate. Finally, the unit-clipper function is proposed in [11], [12]. Figure 4 illustrates the performance results in a synchronous AWGN channel scenario where U=L=24 and BPSK signalling is used. It is observed that the SH function provides the best option.

V. Conclusions

Simulation results show that the SH-IC/MMSE multi-user detector can achieve near-single user performance in a fully-loaded uncoded DS-CDMA system. Most importantly this performance is achieved in a low complexity manner and

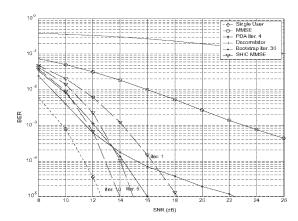


Fig. 1. BER performance of the SHIC/MMSE detector in synchronous DS-CDMA. U=L=50 and all users received at equal power levels.

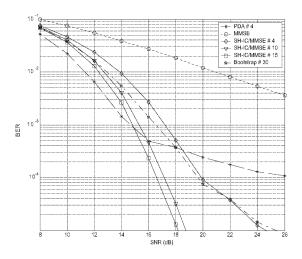


Fig. 2. BER performance of the SHIC/MMSE detector in synchronous DS-CDMA. U=L=24 and half users are 6dB stronger than the rest.

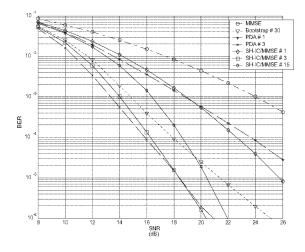


Fig. 3. BER performance of the SHIC/MMSE detector in multipath DS-CDMA. All channels have 5 i.i.d. Rayleigh faded taps. U=L=10, QPSK modulation and equal power for all users.

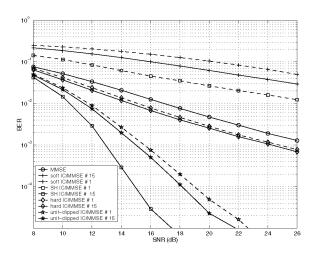


Fig. 4. Performance comparison between the SH-IC/MMSE detector and other types of IC/MMSE detectors which impose different kinds of functions on the soft MMSE estimate prior to IC

within a small number of iterations. In the two synchronous simulated scenarios the technique achieves better performance than the PDA algorithm, while in the multi-path fading scenario similar performance is achieved by both detectors. The results suggest that the SH-IC/MMSE detector is more resilient to low spreading gain conditions where the spreading codes are highly correlated.

VI. ACKNOWLEDGEMENTS

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