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Performance Evaluation of Channel Estimation Techniques in a Multiple Antenna OFDM System

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Abstract—**In this paper a number of pilot-based channel estimation techniques are investigated for a multiple antenna system using the zero-forcing algorithm. The techniques are tested in typical indoor and outdoor (vehicular) channels. Performance results are presented in terms of bit error rate, mean squared error and data throughput. It is found that different pilot strategies offer the best performance in each environment and hence an adaptive system should be considered.**

Keywords—Channel estimation, COFDM, MIMO, Zero Forcing

I. INTRODUCTION

There is currently great interest in multiple antenna systems in the research community due to their ability to exploit the multipath channel. These multiple-input, multiple-output (MIMO) systems have more than one antenna at the transmitter and receiver, and in doing so create multiple channels for data to pass through. This can be used to increase data throughput (as in Layered Space-Time schemes [1,2]) or improved BER performance through increased diversity and coding [3-5]. In this paper MIMO-OFDM is investigated with a zero-forcing (ZF) algorithm as a method of increasing data throughput. The performance of this scheme is evaluated for indoor and outdoor mobile channel scenarios.

The ability of COFDM to efficiently exploit the wideband properties of the radio channel lies at the heart of its popularity. This feature has helped to establish COFDM as the physical layer of choice for broadband wireless communications systems [6-8]. Coherent detection dictates that a per sub-band estimate of the frequency response of the channel is generated for each COFDM symbol. In a single antenna system this is achieved by inserting pilot symbols amongst the data symbols in the COFDM modulation grid. These pilots are distorted by the wideband properties of the radio channel. However, knowledge of their original values enables the distortion at each pilot to be quantified. With suitable interpolation, the channel estimate at all intermediate symbols can be generated [9]. Clearly this becomes more complex when multiple antennas are used at the transmitter and receiver, since multiple channels exist, and the received signal will be a superposition of the signal from each transmit antenna distorted by a number of channels.

This contribution investigates a number of pilot-based channel estimation methods in a wide-area COFDM system. A number of pilot strategies are introduced, using both training sequences and scattered pilots in orthogonal or non-orthogonal implementations. Various interpolation techniques are also identified and tested with these pilot strategies. The performance of our chosen channel estimation techniques are analysed in terms of bit error rate (BER), mean squared error (MSE), and data throughput.

This paper is organised as follows: In Section II, the considered channel environments are presented. The OFDM system parameters for the test system are presented in Section III. In Section IV the pilot strategies and interpolation techniques are described. Simulation results are presented in Section V. Finally Section VI discusses the results and concludes the paper.

II. CHANNEL ENVIRONMENTS

In [10] six different channel models are described, covering a range of indoor and outdoor (vehicular) environments. For this study the environment at either extreme are considered: Environment 1 is an indoor channel. It has an RMS delay spread of 70ns and a maximum mobile velocity of 3km/h, resulting in a maximum Doppler frequency of 5.55Hz when operating in the 2GHz band. Environment 6 is a typical outdoor vehicular channel. The RMS delay spread is much higher, at 4000ns, and the maximum mobile velocity is 120km/h, giving a maximum Doppler frequency of 222Hz. In both cases a classical Doppler spectrum was assumed.

III. THE OFDM SYSTEM

Table II shows the values that were chosen for the OFDM parameters. The OFDM modulation is implemented by means of an inverse FFT with FFT size K=512.

TABLE I. OFDM PARAMETERS FOR 4G.

Parameter	Value
Operating Frequency	2GHz
FFT Size (K)	512
Bandwidth	4096 kHz
Sample period	244 ns
Useful Symbol Duration (T_s)	125 µs
Guard Interval Duration (T_g)	$T_g \approx T/6 \approx 20.8$ µs
Total Symbol Duration (T_{symbol})	T_{symbol} =144.2 µs
Channel Coding	Punctured 1/2 rate convolution code, K= 7, $\{133, 171\}_{\text{octal}}$
Sub-carrier spacing (Δ_f)	8 kHz
No. of Transmit Antennas (M)	2
No. Receive Antennas (N)	2

For the simulations in this paper, QPSK with ½ rate convolution coding will be used. The packet length is variable since an integer number of OFDM symbols must be used, but the number of available sub-carriers will depend on the pilot density. The packet length is therefore defined as the minimum number of OFDM symbols required to transmit 1000 bits of data. A bitwise random interleaver is used across all data bits within a packet in order to mitigate the effects of error bursts, which degrade the performance of the soft-input Viterbi decoder. The channel model assumes Rayleigh fading with a classical Doppler spectrum. The channel is correlated within a packet, and the packets are uncorrelated between one another. The system has 2 transmit and 2 receive antennas and uses the zero-forcing (ZF) algorithm to detect the spatially multiplexed data streams. The power from each transmit antenna is half that of the equivalent single antenna system to ensure the total radiated power is the same.

IV. MIMO CHANNEL ESTIMATION

The system block diagram is shown in fig. 1. Since the data is divided into two streams and transmitted simultaneously, the maximum data throughput will be twice that of the equivalent single antenna system.

A. *Equalizer Structure*

The equalized symbol vector $\tilde{\mathbf{X}}$ is given by

$$\tilde{\mathbf{X}} = (\mathbf{XH} + \mathbf{V})\tilde{\mathbf{H}}^{-1} \tag{1}$$

where

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_{1,1} & \cdots & \mathbf{H}_{M,1} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{1,N} & \cdots & \mathbf{H}_{M,N} \end{pmatrix} \tag{2}$$

is the overall channel matrix of a system with M transmit and N receive antennas. The sub-matrix $\mathbf{H}_{i,j}$ is a diagonal matrix defining the frequency response of the K sub-carriers between the ith transmit and jth receive antenna. $\tilde{\mathbf{H}}$ is the estimate of the channel matrix $\mathbf{H}$, and $\mathbf{X}$ and $\mathbf{V}$ are $2K$ row vectors of transmitted symbols and white Gaussian noise respectively. Capital letters on vectors indicate the frequency domain.

B. *Pilot Strategies*

The pilot strategies investigated in this paper and their associated packet structures are now described.

1) Independent Training Sequences

In this scheme each antenna will take its turn to transmit a known training sequence while all other antennas are switched off. These training sequences will be transmitted in the middle of the packet (midamble), since these symbols will be most correlated to the packet as a whole and hence offer better performance than the case where training occurs at the start (preamble). In the system under test, M=2 and hence 2 OFDM symbols will be dedicated to training. In order to meet the packet length requirement of 1000 bits, a further 2 OFDM symbols will be required for data symbols.

To further improve performance, FFT noise reduction can be employed on the received training sequences. The spectrum of the superimposed noise is typically white, whereas the actual channel response is strictly band-limited due to the maximum excess delay of the channel. If this delay is known, out-of-band components can be eliminated and hence the corrupting effect of noise is reduced [11].

2) Orthogonal Training Sequences

Training sequences can be transmitted from both antennas simultaneously if they are orthogonal in space and time. One family of suitable pilots is the set of Orthogonal Space-Time Pilot Matrices (OSTPM) [12]. Such matrices are the STBC matrices [4] and the Hadamard matrices [12]. In this paper we use a Hadamard matrix to produce sequences that are orthogonal in space and time according to:

$$\mathbf{P} = \begin{pmatrix} P_k & P_k \\ P_k & -P_k \end{pmatrix} \tag{3}$$

where P_k is the known (BPSK) pilot symbol at sub-carrier k. The channel can therefore be resolved at the receiver by simple linear processing of the received symbols:

$$\begin{aligned} Y_{1,k} &= P_k H_{1,1,k} + P_k H_{2,1,k} + V_{1,k} \\ Y_{2,k} &= P_k H_{1,1,k} - P_k H_{2,1,k} + V_{2,k} \end{aligned} \tag{4}$$

$$\begin{aligned} \tilde{S}_{1,k} &= Y_{1,k} + Y_{2,k} = 2P_k H_{1,1,k} + V_{1,k} + V_{2,k} \\ \tilde{S}_{2,k} &= Y_{1,k} - Y_{2,k} = 2P_k H_{2,1,k} + V_{1,k} - V_{2,k} \end{aligned} \tag{5}$$

This method assumes that the channel remains constant over two consecutive COFDM symbols. For high Doppler spreads this will not be the case and so performance will be degraded.

3) Scattered Pilots

In this scheme the minimum number of pilots is used in order to maximize the number of symbols available for data. The

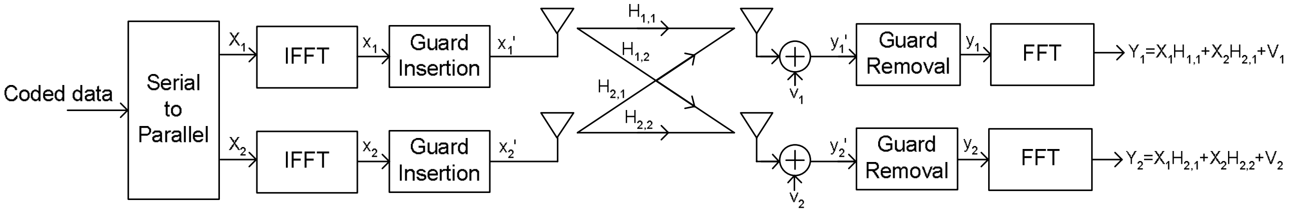


Figure 1: MIMO structure

pilots are spread in time and frequency according to the Nyquist theorem [9]. The transmit antennas must transmit pilots on different sub-carriers, and must transmit zeros on the sub-carriers being used for pilots by other antennas. A typical OFDM symbol structure is as follows:

$$\begin{aligned} X_1 &= \begin{pmatrix} P & 0 & d & d & P & 0 & d & d & \cdots \end{pmatrix} \\ X_2 &= \begin{pmatrix} 0 & P & d & d & 0 & P & d & d & \cdots \end{pmatrix} \end{aligned} \tag{6}$$

where X_1 and X_2 are the OFDM symbols transmitted from antennas 1 and 2, P indicates the positions of known BPSK pilot symbols, and d indicates the position of data symbols. Using this implementation the pilot symbols can be resolved separately and interpolation can be used to gather the intermediate unknown samples (see Section IV.C.2). Note that the number of symbols dedicated to pilots and zeros will increase with the number of transmit antennas. The *maximum* spacing between pilots on one OFDM symbol is dictated by the Nyquist theorem and will therefore determine the maximum number of transmit antennas :

$$M \le s_f \le \frac{K}{L} \tag{7}$$

where s_f is the spacing of pilots in frequency, and L is the channel length.

Low Doppler spread channels only require one OFDM symbol per packet to contain pilots since the packet length is well within the coherence time of these channels. High Doppler (vehicular) channels will require pilots on the first and last OFDM symbols of each packet, and interpolation is performed to gain knowledge of intermediate samples. Since entire OFDM symbols are not dedicated to training, only three OFDM symbols are required to meet the minimum packet length requirements.

4) Orthogonal Space-Time Scattered Pilots

Combining the orthogonality technique described in subsection 2 with the scattered pilot scheme of subsection 3 results in orthogonal scattered pilots. Pilots can be transmitted on the same sub-carrier at the same time if the following OFDM symbol contains an orthogonal sequence. As this method does not require zero carriers, it offers a lower peak-average power ratio (PAPR) than the previous method. However as two consecutive OFDM symbols are required to make one channel estimate, it will not perform so well in high Doppler environments.

C. Interpolation Techniques

Scattered pilot techniques will require interpolation in order to resolve intermediate channel values. A number of separable 1D interpolators and a 2D interpolator are investigated here:

1) 1D Interpolation

In this implementation the channel is interpolated across frequency first, and then across time. Two methods of frequency interpolation are investigated: Spline interpolation fits a Bezier curve between data points. This method has the lowest computational complexity. Alternatively FFT (or time-domain) interpolation works by windowing the time domain channel estimate to suppress noise. Since all the channel information is held in only the delay components up to the maximum excess delay, this method will interpolate the frequency domain samples. As there are no null carriers in the system under test, this can be performed in one calculation and iteration is not required [13]. Due to the radix-4 butterfly algorithm used in the FFT/IFFT process, the pilot spacing must be a power of 2 for this method to work. In the noiseless case, this estimate will be without error [9].

Interpolation in time is only possible if there is more than one estimate available. In this paper we have at most two samples in time and so linear interpolation is the only available option. If only one channel sample is available (e.g. training sequences) then this estimate must be assumed to be correct for the entire packet.

2) 2D Interpolation

A reduced complexity Wiener filter is investigated as a method of 2 dimensional interpolation. Typically a Wiener filter will make use of all available channel estimates, but at the cost of high computational complexity. The reduced-complexity version proposed uses only the four nearest pilots to interpolate a channel sample. The channel estimate is given by:

$$\hat{\mathbf{h}} = \mathbf{R}_{\mathbf{h}\hat{\mathbf{p}}}(\mathbf{R}_{\mathbf{pp}} + \frac{1}{SNR}\mathbf{I})^{-1}\hat{\mathbf{p}} \tag{8}$$

where $\hat{\mathbf{h}}$ is a column vector of filtered channel estimates, $\hat{\mathbf{p}}$ is a column vector of known channel samples, $\mathbf{R}_{\mathbf{h}\hat{\mathbf{p}}}$ is the cross-covariance matrix between $\mathbf{h}$ and the noisy channel estimates $\hat{\mathbf{p}}$, and $\mathbf{R}_{\mathbf{pp}}$ is the auto-covariance matrix of the known channel samples [14]. The elements of $\mathbf{R}_{\mathbf{h}\hat{\mathbf{p}}}$ and $\mathbf{R}_{\mathbf{pp}}$ are therefore given by:

$$E\left\{p_{k,l}\hat{p}^{*}_{k',l'}\right\} = E\left\{h_{k,l}\hat{p}^{*}_{k',l'}\right\} = \frac{J_0(2\pi f_{\max}(T_s+T_g)(l-l'))}{1+j2\pi\tau_{rms}(k-k')\Delta f} \tag{9}$$

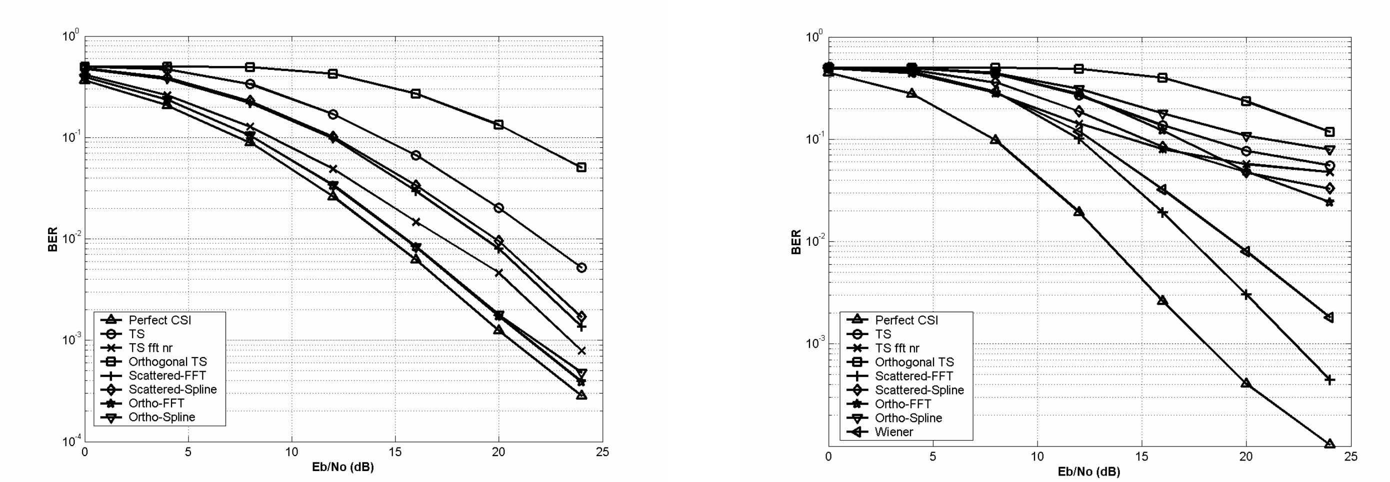
where f_{max} is the maximum Doppler shift, T_s is the OFDM symbol period, T_g is the guard interval, $J_0(x)$ is the zero*th* order Bessel function of the first kind, τ_{rms} is the r.m.s. delay spread of the channel, and Δf is the sub-carrier frequency spacing. l and k are the indices of the pilot/channel sample in time and frequency respectively.

V. PERFORMANCE RESULTS

Performance results are presented in terms of BER, MSE and data throughput. The data throughput is given by:

$$D(Eb/No) = \frac{M((KL_p - P)mR - v)}{L_p(T_s+T_g)}(1-\wp(Eb/No)) \tag{10}$$

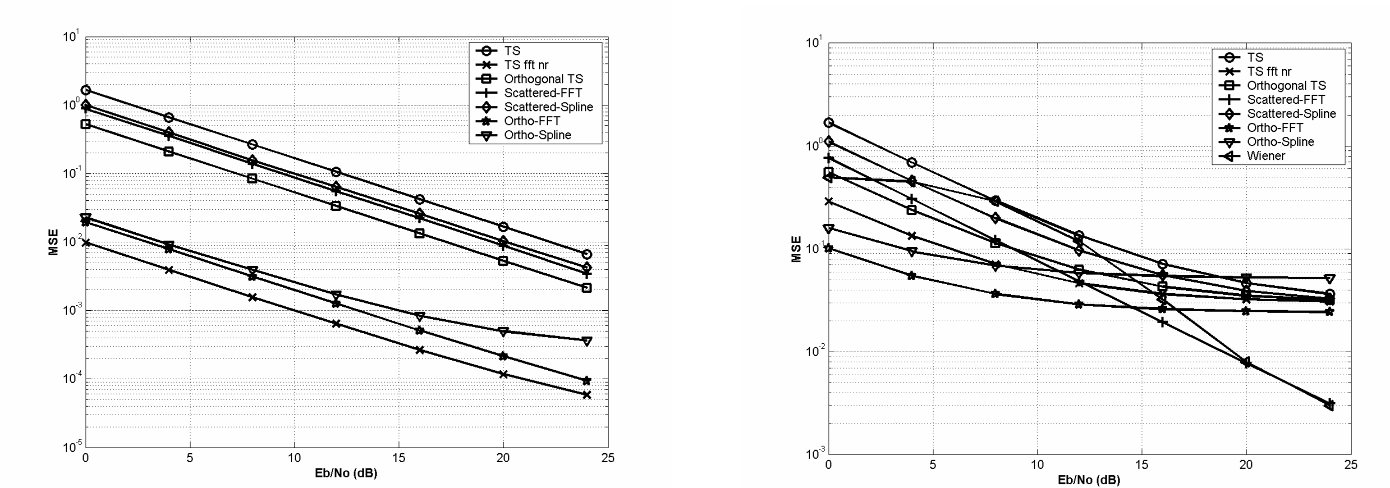
where M is the number of transmit antennas, L_p is the number of OFDM symbols per packet, P is the number of pilot symbols per packet, m is the number of bits per symbol, R is the code rate, v is the number of tail bits added to terminate the code in the zero state, and $\wp(Eb/No)$ is the PER as a function of Eb/No. In the graphs that follow the following abbreviations are used: 'TS' signifies a training sequence, 'fft nr' means FFT noise reduction was implemented. 'Scattered' implies that scattered pilots and zeros were used, and 'Ortho' implies that orthogonal pilots were transmitted on the same sub-carriers.



(a) Indoor Environment 1

Figure 2: BER Performance

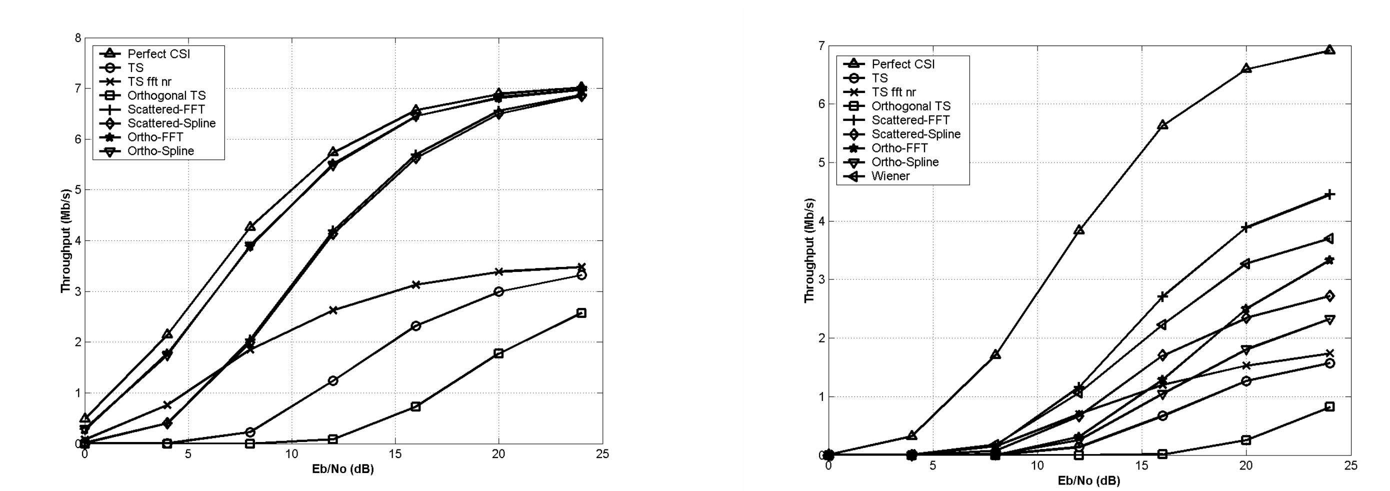
(b) Outdoor Environment 6



(a) Indoor Environment 1

(b) Outdoor Environment 6

Figure 3: MSE Performance



(a) Indoor Environment 1

Figure 4: Throughput Performance

(b) Outdoor Environment 6

While the outdoor vehicular environment (environment 6) achieves better BER performance in the perfect channel knowledge case due to greater channel diversity, the estimators all perform much better in the indoor environment (environment 1). This occurs because the vehicular channel fades more quickly in both frequency and time directions, and is therefore more difficult to track.

It is also worthy of note that the FFT interpolator always outperforms the spline interpolator, and that implementations utilizing the spline interpolator exhibit an error floor (fig. 3a). This is to be expected since this interpolator will introduce high delay 'harmonics' into the channel estimate that are known not to exist. The FFT interpolator strictly removes these components and hence its performance is only limited by the signal to noise ratio.

In the indoor environment, training sequences implemented with FFT noise reduction offer the best MSE performance (fig. 3a), but orthogonal pilots in conjunction with FFT interpolation offer the best BER performance (fig. 1a). However the throughput is the most important metric since it represents the final data rate as seen by the user. It can be seen that the estimation methods employing training sequences suffer a throughput degradation due to the large number of pilots used (fig. 4a), since these pilots will result in fewer symbols being available for user data. Despite offering the best MSE performance, training sequences with FFT noise reduction clearly suffer a decrease in throughput since half of the OFDM symbols are dedicated to training. Orthogonal pilots, both with FFT and spline interpolation offer the greatest throughput, with FFT interpolation being marginally superior, and almost achieving the same data throughput as the perfect channel knowledge case.

Environment 6 shows quite different results. Due to the high Doppler spread, training symbol-based estimators perform very poorly. The high Doppler also affects methods exploiting orthogonality, since this method requires the channel to remain constant across two OFDM symbols which is no longer a valid assumption. The MSE graph for this environment (fig 3b) shows that almost all estimators exhibit an error floor in their performance due to their apparent inability to track the fast time and frequency variations of the channel. The two exceptions are the Wiener filter and scattered pilots with FFT interpolation. The best throughput performance is exhibited by the latter of these two (fig. 4b). It should be noted, however, that this performance is still far less than the case of the ideal channel knowledge, whereas the perfect case was almost matched in the indoor environment. This reflects the difficulty of tracking fast fading channels such as those seen in vehicular environments.

VI. CONCLUSIONS

A number of pilot strategies and interpolation methods have been investigated and their performance analyzed for indoor and vehicular channels. It has been found that in the indoor environment the best estimator in terms of throughput performance is orthogonal pilots combined with FFT interpolation. In the outdoor environment non-orthogonal scattered pilots, again with FFT interpolation, offer the best performance. Since different estimators offer the best performance in different environments, an adaptive pilot system and channel reconstruction algorithm would be well suited to future MIMO terminals expected to operate in such a range of channel environments.

Future work will investigate the implementation of pilots orthogonal in space and frequency (as opposed to space and time) to improve performance in high Doppler spread channels.

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