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Adaptive Frequency-Domain Equalization for Single-Carrier MIMO Systems

J. Coon, S. Armour, M. Beach, and J. McGeehan

Centre for Communications Research, University of Bristol

Merchant Venturers Building, Woodland Road, Bristol BS8 1UB, UK

e-mail: justin.coon@bristol.ac.uk

Abstract— **Channel estimation and tracking pose real problems in wideband single-carrier wireless communication systems employing multiple transmit and receive antennas. An alternative to estimating the channel is to adaptively equalize the received symbols. In this paper, we present an adaptive equalization algorithm for implementation in multiple-input multiple-output (MIMO) single-carrier (SC) systems with frequency-domain equalization (FDE). Furthermore, we outline a novel method of reducing the overhead required to train the adaptive equalizer. Other computationally efficient adaptive MIMO SC-FDE algorithms can only be applied to space-time block-coded (STBC) architectures. The algorithm detailed in this paper can be implemented in STBC systems as well as in broadband spatial multiplexing systems, making it suitable for use in high data rate MIMO applications.**

I. INTRODUCTION

Wideband multiple-input multiple-output (MIMO) architectures are very attractive solutions for high date rate wireless communication systems due to their enormous potential for capacity gains relative to single-antenna systems [1]. However, two key signal processing problems have presented themselves as research into wideband MIMO systems has progressed. The first of these problems is equalization of the received symbols. Equalization in a MIMO system is potentially very complex due to the superposition of all of the transmitted streams at each receive antenna. Single-carrier (SC) transmission with frequency-domain equalization (FDE) at the receiver is one technique that offers a low-complexity solution to this problem [2], [3]. The second key problem is that of gaining knowledge of the channel so that equalization can be performed. In this paper, we attempt to solve these two problems by presenting an adaptive equalization algorithm for implementation in MIMO SC-FDE systems.

To date, several adaptive solutions for SC-FDE systems have been studied. Adaptive algorithms based on the least mean squares (LMS) algorithm and the recursive least squares (RLS) algorithm were explored for SC-FDE systems employing receive diversity in [4]. The potential for significant reductions in complexity relative to LMS and RLS time-domain algorithms was shown. A modification of the RLS algorithm was applied to a space-time coded SC-FDE system in [5]. The structure of the space-time block code (STBC) was exploited to reduce the complexity of the algorithm beyond the reduction achieved through FDE alone.

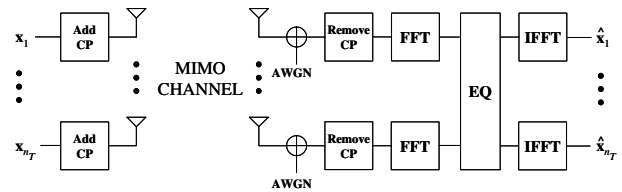One drawback of the algorithms presented in [4] and [5]

Fig. 1. Block diagram of baseband spatial multiplexing SCFDE system.

is that they cannot be applied to MIMO systems with spatial multiplexing (SM) architectures. Consequently, they are not suitable for very high data rate applications where SM systems would be most useful. The algorithm proposed in this paper can be implemented in both SM and STBC architectures. In order to highlight the advantages of this algorithm, we focus on its implementation in an SM system in this paper.

The paper is organized as follows. In section II, we introduce a mathematical model for an SM SC-FDE system. We detail the proposed adaptive algorithm in section III and discuss convergence properties of the algorithm in section IV. In section V, we outline a novel method of reducing the overhead required to train the adaptive equalizer. We address the complexity of the proposed algorithm in section VI and illustrate results obtained from computer simulations in section VII. Finally, we present our conclusions in section VIII.

Notation: We use a bold uppercase (lowercase) font to denote matrices (column vectors); frequency-domain variables are denoted by a tilde (e.g. $\tilde{a}$); $\mathbf{F}_m$ is the normalized $m \times m$ DFT matrix where its (k, i)th element is given by $F_{m;k,i} \triangleq (1/\sqrt{m}) \exp(-j2\pi ki/m)$ for $k, i = 0, \ldots, m - 1$; $\mathbf{I}_m$ is the $m \times m$ identity matrix; $\mathbf{0}_{m \times n}$ is an $m \times n$ zero matrix; $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, $(\cdot)_m$, and $|\cdot|$ denote the complex conjugate, transpose, conjugate transpose, modulo-m, and absolute value operations, respectively; $\otimes$ is the Kronecker product operator; $\mathrm{E}\{\cdot\}$ is the expectation operator; $\mathrm{tr}\{\cdot\}$ is the trace operator; $\mathrm{diag}\{x_0, \ldots, x_{m-1}\}$ denotes the $m \times m$ diagonal matrix with the elements $\{x_0, \ldots, x_{m-1}\}$ on the diagonal.

II. SPATIAL MULTIPLEXING SYSTEM MODEL

Consider the broadband MIMO system illustrated in Fig. 1, which has n_T transmit antennas and n_R receive antennas where $n_T \leq n_R$. The baseband sequence at each transmit antenna is modulated onto a single carrier waveform for

transmission across a wireless channel. The received baseband sequences are equalized in the frequency domain. A cyclic prefix is added to each transmitted sequence and removed from each received sequence to facilitate FDE.

Adopting matrix notation, we can mathematically describe this system. Let $\mathbf{x}_q$ be a length-K vector of symbols that is transmitted from the qth transmit antenna. A cyclic prefix of $Q \geq L$ symbols is employed to eliminate interblock interference, where L is the memory order of each of the $n_T n_R$ channel impulse responses (CIRs). The vector $\mathbf{y}_p$ of symbols received at antenna p is therefore given by

$$\mathbf{y}_p = \sum_{q=1}^{n_T} \mathbf{H}_{p,q}\mathbf{x}_q + \mathbf{n}_p. \tag{1}$$

In (1), $\mathbf{n}_p$ is a length-K vector of i.i.d. zero-mean complex Gaussian noise samples with variance $\sigma_n^2/2$ per dimension and $\mathbf{H}_{p,q}$ is a $K \times K$ circulant matrix defined by the CIR between the qth transmit antenna and the pth receive antenna. Specifically, the first row of $\mathbf{H}_{p,q}$ is $(h_{p,q;0}, \mathbf{0}_{1 \times K-(L+1)}, h_{p,q;L}, \ldots, h_{p,q;1})$ where $h_{p,q;i}$ is the ith complex tap coefficient of the CIR between the qth transmit antenna and the pth receive antenna. It is assumed that the channels remain static for at least one block duration.

It is convenient to construct a length-$n_R K$ vector of received symbols $\mathbf{y} \triangleq (\mathbf{y}_1^T, \ldots, \mathbf{y}_{n_R}^T)^T$. A length-$n_T K$ vector $\mathbf{x}$ of transmitted symbols can be constructed in a similar manner where $\mathbf{x} \triangleq (\mathbf{x}_1^T, \ldots, \mathbf{x}_{n_T}^T)^T$. Therefore, we have

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n} \tag{2}$$

where $\mathbf{n} \triangleq (\mathbf{n}_1^T, \ldots, \mathbf{n}_{n_R}^T)^T$ and the (p,q)th submatrix of the $n_R \times n_T$ block matrix $\mathbf{H}$ is $\mathbf{H}_{p,q}$. The equalized symbols can be expressed as

$$\hat{\mathbf{x}} = \mathbf{D}_{n_T}^{-1}\mathbf{\Gamma}^{\mathrm{H}}\mathbf{D}_{n_R}\mathbf{y} \tag{3}$$

where $\mathbf{D}_m = \mathbf{I}_m \otimes \mathbf{F}_K$ and $\mathbf{\Gamma}^{\mathrm{H}}$ is the $n_T K \times n_R K$ equalizer matrix. Because each circulant submatrix of $\mathbf{H}$ is diagonalized by pre- and post-multiplication of a DFT and an IDFT matrix, respectively, it is convenient to express (3) as

$$\begin{aligned}
\hat{\mathbf{x}} &= \mathbf{D}_{n_T}^{-1}\mathbf{\Gamma}^{\mathrm{H}}\tilde{\mathbf{y}} \\
&= \mathbf{D}_{n_T}^{-1}\mathbf{\Gamma}^{\mathrm{H}}\left(\widetilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{n}}\right)
\end{aligned} \tag{4}$$

where $\tilde{\mathbf{y}} = \mathbf{D}_{n_R}\mathbf{y}$, $\widetilde{\mathbf{H}} = \mathbf{D}_{n_R}\mathbf{H}\mathbf{D}_{n_T}^{-1}$, $\tilde{\mathbf{x}} = \mathbf{D}_{n_T}\mathbf{x}$, and $\tilde{\mathbf{n}} = \mathbf{D}_{n_R}\mathbf{n}$. The (p,q)th diagonal $K \times K$ submatrix of $\widetilde{\mathbf{H}}$ is $\widetilde{\mathbf{H}}_{p,q} \triangleq \mathrm{diag}\{\tilde{h}_{p,q;0}, \ldots, \tilde{h}_{p,q;K-1}\}$, where the discrete frequency response of the channel is given by $\tilde{h}_{p,q;k} = \sum_{i=0}^{L} h_{p,q;i}\exp(-j2\pi ki/K)$. This mathematical model for an SM SC-FDE system is used throughout this paper.

III. ADAPTIVE ALGORITHM

The adaptive FDE algorithm described in this section is a version of the RLS algorithm. The goal is to adaptively update

$$\mathbf{\Gamma}^{\mathrm{H}} = \begin{pmatrix} \gamma_{0,0}^* & \cdots & \gamma_{n_R K-1,0}^* \\ \vdots & \ddots & \vdots \\ \gamma_{0,n_T K-1}^* & \cdots & \gamma_{n_R K-1,n_T K-1}^* \end{pmatrix}$$

utilizing training blocks (training mode) or detected data blocks (decision-directed mode). However, the update process requires the revision of $n_T n_R K^2$ elements, which can become computationally cumbersome as the block length K increases. We can reduce the number of elements that must be updated by observing that only the n_R elements of $\tilde{\mathbf{y}}$ corresponding to the kth frequency bin are required to recover the kth frequency component of the block transmitted from antenna q. Therefore,

$$\gamma_{u,v}^* = \begin{cases} a_{u,v} + jb_{u,v} & , \quad (|u-v|)_K = 0 \\ 0 & , \quad \text{otherwise} \end{cases}$$

and the number of elements in $\mathbf{\Gamma}^{\mathrm{H}}$ that must be updated is reduced to $n_T n_R K$.

To update the non-zero elements of $\mathbf{\Gamma}^{\mathrm{H}}$, consider the classical cost function defined by

$$J_v(t) = \sum_{\ell=1}^{t} \zeta(t,\ell) |E_v(\ell,t)|^2, \qquad \forall v = 0, \ldots, n_T K - 1 \tag{5}$$

where t and ℓ are time indices denoting a given block interval and $\zeta(t,\ell) = \rho^{t-\ell}$ is the standard weighting factor, which is included for implementation of the algorithm in decision-directed mode [6]. A *block interval* is defined here as the interval of time in which one block from each transmit antenna is sent. In (5), the error term $E_v(\ell,t) = \tilde{x}_v(\ell) - \sum_u \gamma_{u,v}^*(t)\tilde{y}_u(\ell)$ for every u such that $(|u-v|)_K = 0$. The notation $\tilde{x}_v(\ell)$ denotes the vth element of the vector $\tilde{\mathbf{x}}$ at time ℓ.

The objective is to minimize $J_v(t)$ for each v. Taking the partial derivative of (5) with respect to $\gamma_{u,v}^*(t)$, setting the result equal to zero, and performing some algebraic manipulations yields

$$\mathbf{R}_v(t)\boldsymbol{\gamma}_v(t) = \mathbf{p}_v(t) \tag{6}$$

where

$$\mathbf{R}_v(t) = \sum_{\ell=1}^{t} \rho^{t-\ell}\boldsymbol{\psi}_v(\ell)\boldsymbol{\psi}_v^{\mathrm{H}}(\ell) \tag{7}$$

and

$$\mathbf{p}_v(t) = \sum_{\ell=1}^{t} \rho^{t-\ell}\tilde{x}_v^*(\ell)\boldsymbol{\psi}_v(\ell). \tag{8}$$

In (6) through (8), $\boldsymbol{\gamma}_v(t) \triangleq (\gamma_{u_0,v}(t), \ldots, \gamma_{u_{n_R-1},v}(t))^T$ and $\boldsymbol{\psi}_v(t) \triangleq (\tilde{y}_{u_0}(t), \ldots, \tilde{y}_{u_{n_R-1}}(t))^T$, where the index $u_m \in \{0, \ldots, n_R K - 1\}$ such that $(|u_m-v|)_K = 0$. We can rewrite (7) as

$$\mathbf{R}_v(t) = \rho\mathbf{R}_v(t-1) + \boldsymbol{\psi}_v(t)\boldsymbol{\psi}_v^{\mathrm{H}}(t). \tag{9}$$

Similarly, we can rewrite (8) as

$$\mathbf{p}_v(t) = \rho\mathbf{p}_v(t-1) + \tilde{x}_v^*(t)\boldsymbol{\psi}_v(t). \tag{10}$$

Utilizing (6) through (10), it can be shown that the update equation for the non-zero elements in the vth column of $\mathbf{\Gamma}$ at time t is given by

$$\boldsymbol{\gamma}_v(t) = \boldsymbol{\gamma}_v(t-1) + \mathbf{R}_v^{-1}(t)\boldsymbol{\psi}_v(t)\varepsilon_v(t) \tag{11}$$

where $\varepsilon_v(t) = \tilde{x}_v^*(t) - \boldsymbol{\psi}_v^{\mathrm{H}}(t)\boldsymbol{\gamma}_v(t-1)$.

TABLE I
ADAPTIVE ALGORITHM FOR SM SC-FDE.

Initialization:

$\boldsymbol{\Gamma}(0) = \mathbf{0}_{n_R K \times n_T K}$

$\mathbf{R}_v(0) = \delta \mathbf{I}_{n_R}, \quad \forall v = 0, \ldots, n_T K - 1$ and for small δ

$\rho \leftarrow$ some number close to, but less than, 1

$t \leftarrow 1$

Recursion:

$\forall v = 0, \ldots, n_T K - 1$

$\mathbf{R}_v^{-1}(t) = \rho^{-1} \mathbf{R}_v^{-1}(t-1) - \frac{\rho^{-2} \mathbf{R}_v^{-1}(t-1) \boldsymbol{\psi}_v(t) \boldsymbol{\psi}_v^{\mathrm{H}}(t) \mathbf{R}_v^{-1}(t-1)}{1 + \rho^{-1} \boldsymbol{\psi}_v^{\mathrm{H}}(t) \mathbf{R}_v^{-1}(t-1) \boldsymbol{\psi}_v(t)}$

$\varepsilon_v(t) = \tilde{x}_v^*(t) - \boldsymbol{\psi}_v^{\mathrm{H}}(t) \boldsymbol{\gamma}_v(t-1)$

$\boldsymbol{\gamma}_v(t) = \boldsymbol{\gamma}_v(t-1) + \mathbf{R}_v^{-1}(t) \boldsymbol{\psi}_v(t) \varepsilon_v(t)$

$t \leftarrow t + 1$

To this point, the derivation of the adaptive algorithm has more or less followed that of the standard RLS algorithm [6]. However, it was important to step through this derivation to show the first of several interesting points that will be made about this algorithm: the time-averaged correlation matrix $\mathbf{R}_v(t)$ is an $n_R \times n_R$ matrix. Consequently, for small numbers of receive antennas, the inverse of $\mathbf{R}_v(t)$ can be computed directly with ease. In contrast, the size of the analogous correlation matrix in the standard time-domain RLS algorithm is dependent upon the length of the time-domain filter, which grows large with increasing channel memory (i.e. as L increases) [6]. Therefore, in most broadband applications of interest, the time-domain RLS algorithm can only be implemented via the matrix inversion lemma, whereas the proposed FDE algorithm can be implemented easily for small n_R in any environment. Indeed, for MIMO systems with large numbers of receive antennas, the matrix inversion lemma can be used to compute $\mathbf{R}_v^{-1}(t)$ as well, which is shown along with a summary of the proposed algorithm in Table I.

IV. CONVERGENCE PROPERTIES

Several convergence properties of the proposed algorithm were studied. The properties that are of greatest interest are the mean-square error (MSE) convergence and the rate at which the matrix $\boldsymbol{\Gamma}^{\mathrm{H}}$ converges to the mean solution $\overline{\boldsymbol{\Gamma}}^{\mathrm{H}}$. First, consider the MSE convergence, from which the rate of convergence follows.

A. MSE Convergence

The vth weight-error vector is defined by $\mathbf{e}_v(t) = \boldsymbol{\gamma}_v(t) - \overline{\boldsymbol{\gamma}}_v$ where $\overline{\boldsymbol{\gamma}}_v = \mathrm{E}\{\boldsymbol{\gamma}_v(t)\}$ and the vth weight-error correlation matrix is given by

$$\mathbf{K}_v(t) = \mathrm{E}\left\{\mathbf{e}_v(t) \mathbf{e}_v^{\mathrm{H}}(t)\right\}. \tag{12}$$

The MSE of the vth equalizer vector relative to the mean solution $\overline{\boldsymbol{\gamma}}_v$ can be found by taking the trace of $\mathbf{K}_v(t)$. Assuming that

1) the vectors $\boldsymbol{\psi}_v(1), \ldots, \boldsymbol{\psi}_v(t)$ are i.i.d., and
2) $\boldsymbol{\psi}_v(1), \ldots, \boldsymbol{\psi}_v(t)$ where $t \geq n_R$ are drawn from a stochastic process with a zero-mean Gaussian distribution with an ensemble-averaged correlation matrix $\boldsymbol{\Phi}_v = \mathrm{E}\{\boldsymbol{\psi}_v \boldsymbol{\psi}_v^{\mathrm{H}}\}$,

the MSE of the vth equalizer vector can be expressed as

$$\begin{aligned} \mathrm{MSE}(t) &= \frac{\sigma^2}{t - n_R - 1} \mathrm{tr}\left\{\boldsymbol{\Phi}_v^{-1}\right\} \\ &= \frac{\sigma^2}{t - n_R - 1} \sum_{p=1}^{n_R} \frac{1}{\lambda_{v,p}}, \quad t > n_R + 1 \end{aligned} \tag{13}$$

where $\lambda_{v,1}, \ldots, \lambda_{v,n_R}$ are the eigenvalues of $\boldsymbol{\Phi}_v$. The term σ^2 denotes the variance of a zero-mean measurement error process $\epsilon_{0,v}(t)$ that is adopted from a multiple linear regression model, which is given by [6]

$$\tilde{x}_v(t) = \overline{\boldsymbol{\gamma}}_v^{\mathrm{H}} \boldsymbol{\psi}_v(t) + \epsilon_{0,v}(t). \tag{14}$$

Since $\boldsymbol{\Phi}_v$ is positive definite, the eigenvalues of $\boldsymbol{\Phi}_v$ are positive. Therefore, we can write

$$\sum_{p=1}^{n_R} \frac{1}{\lambda_{v,p}} \geq n_R \left(\prod_{p=1}^{n_R} \frac{1}{\lambda_{v,p}}\right)^{1/n_R} \tag{15}$$

which is met with equality if and only if $\lambda_{v,1} = \lambda_{v,2} = \cdots = \lambda_{v,n_R}$. Furthermore, we have

$$\prod_{p=1}^{n_R} \frac{1}{\lambda_{v,p}} = \det\left(\boldsymbol{\Phi}_v^{-1}\right) = [\det(\boldsymbol{\Phi}_v)]^{-1}. \tag{16}$$

In order to minimize (15), and therefore (13), we must maximize $\det(\boldsymbol{\Phi}_v)$. Applying Hadamard's Inequality, we have

$$\det(\boldsymbol{\Phi}_v) \leq \prod_{p=1}^{n_R} \phi_{v;p,p} \tag{17}$$

where $\{\phi_{v;p,p}\}_{p=1}^{n_R}$ are the diagonal elements of $\boldsymbol{\Phi}_v$. Equation (17) is met with equality if and only if $\boldsymbol{\Phi}_v$ is diagonal, in which case $\phi_{v;p,p} = \lambda_{v,p}$ for $p = 1, \ldots, n_R$. Therefore, in order to achieve the minimum MSE, $\boldsymbol{\Phi}_v$ must be a diagonal matrix with equal elements on the diagonal for all v. Consequently, improper design of the training sequences and/or ill-conditioned channels may lead to poor MSE convergence.

B. Rate of Convergence

To determine the rate at which the algorithm converges, the mean-squared *a priori* estimation error, given by

$$J_v'(t) = \mathrm{E}\left\{|\xi_v(t)|^2\right\} \tag{18}$$

is computed for all v where $\xi_v(t) = \epsilon_{0,v}(t) - \mathbf{e}_v^{\mathrm{H}}(t-1) \boldsymbol{\psi}_v(t)$ is the *a priori* estimation error. Expanding (18) and evaluating the resulting expectations yields

$$\begin{aligned} J_v'(t) &= \sigma^2 + \mathrm{tr}\left\{\mathbf{K}_v(t-1) \boldsymbol{\Phi}_v\right\} \\ &= \sigma^2 \left(1 + \frac{n_R}{t - n_R - 2}\right), \quad t > n_R + 2. \end{aligned} \tag{19}$$

This expression for the rate of convergence can be used to reduce the number of symbols required to train the equalizer as will be shown in the next section.

V. REDUCING TRAINING OVERHEAD

Equation (19) suggests that for $n_R \gg 2$ approximately $2n_R$ block intervals must be used for training before the mean-squared *a priori* estimation error reaches within 3 dB of its final value. From this result, a more practical solution would be to estimate the channel by sequentially transmitting one training block from each transmit antenna while the others are silent, which would require n_T block intervals, then construct an equalizer from the estimate. However, we conclude from (19) that the rate of convergence is not dependent on the lengths of the transmitted symbol vectors. Therefore, the transmitted *time-domain* blocks can have any length κ, as long as K frequency components can be obtained from each block. Thus, we have three cases: $\kappa > K$, $\kappa = K$, and $\kappa < K$.

$\kappa > K$: Choosing $\kappa > K$ during equalizer training but using a K-point DFT during equalization is inefficient. Although the resolution of the training sequences in the frequency domain is higher, a larger DFT is needed for training than for transmission. Furthermore, this increased resolution is not exploited during equalization of the received data symbols.

$\kappa = K$: In this case, the full block length is used to train the equalizer. The algorithm converges to the minimum mean-squared *a priori* estimation error floor as described by (19).

$\kappa < K$: Frequency-domain interpolation can be used to obtain K frequency components from each length-κ sequence. This is a standard technique and has been presented in the literature [5]. In this case, the eigenvalues in (13) are generally smaller than if the full block size were used because a length-κ sequence of constant-modulus training symbols has less energy than a length-K sequence. Consequently, the MSE floor is higher for $\kappa < K$ than for other values of κ.

Now, consider a system employing the proposed algorithm where κ is initially less than K. Specifically, let $\kappa = \kappa_0$ in the set $\mathcal{K} = \{\kappa_0, \kappa_1, \ldots, \kappa_{\theta-1}\}$ where $Q \leq \kappa_0 < \kappa_1 < \cdots < \kappa_{\theta-1} \leq K$. After τ training block intervals, κ is incremented to $\kappa = \kappa_1$. Following another τ training block intervals, κ is incremented to $\kappa = \kappa_2$ and so on. By periodically incrementing κ, convergence to a high error floor is prevented, which is a problem with systems implementing frequency-domain interpolation with a non-varying block size $\kappa < K$ as is the case in [5].

In general, τ can be varied to optimize convergence, but τ is defined as a constant here. As long as τ is not too large and the system is well-conditioned, the algorithm will continue to converge, in terms of training block intervals, as described by (19). However, the number of symbol intervals used for training will depend on the choice of τ and $\mathcal{K}$. In particular, if $\tau\theta$ *block* intervals are used for training, the number of *symbol* intervals used for training is given by $T_s = \tau\theta Q + \tau \sum_{\vartheta=0}^{\theta-1} \kappa_\vartheta$. By properly varying the length of each training block, the overhead required to train the equalizer can be significantly reduced from the case where $\kappa_\vartheta = K$ for all ϑ.

VI. COMPLEXITY

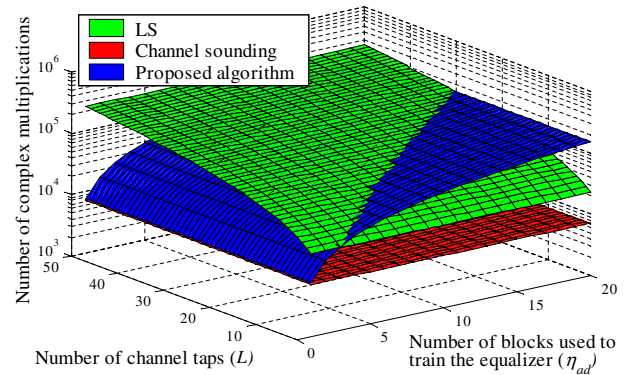The complexity of the proposed algorithm when operating in training mode is compared to that of two techniques that

Fig. 2. Complexity of techniques for a 2 TX × 2 RX system operating in training mode where $\eta_{ls} = \eta_{cs} = 2$ and $K = 256$.

use an estimate of the channel to construct a linear MMSE frequency-domain equalizer [3]. The first of these techniques is a least squares (LS) technique first developed for OFDM systems with transmit diversity in [7], which has been adapted for use in SC systems in this study. The second technique is the aforementioned channel sounding technique (i.e. sequentially transmitting one training block from each antenna).

Complexity is measured in terms of the number of complex multiplications that are performed in each method. Fig. 2 depicts the complexities of the three methods where L is the memory order of the channel. The terms η_{cs}, η_{ad}, and η_{ls} are the numbers of block intervals used for channel estimation and/or equalizer training for the channel sounding technique, the proposed algorithm, and the LS technique, respectively. As seen in Fig. 2, the complexity of the adaptive algorithm is independent of L, making it a low-complexity option for systems operating in channels with large excess delay spread.

VII. SIMULATION RESULTS

Computer simulations were used to observe the rate of convergence of the proposed algorithm and the packet error rate (PER) and bit error rate (BER) of a system employing the algorithm. The ETSI BRAN A [8] channel model was used in the simulations and a Doppler spread of 50 Hz was assumed. Consequently, the channel remained static throughout the transmission of the training blocks and for the duration of one packet in the case of the performance study.

Fig. 3 illustrates the rate of convergence of the proposed algorithm implemented with both a constant training block size and a variable training block size. As a comparison, the convergence curves of the LS and channel sounding techniques are also depicted. Chu sequences were used for training with the latter technique to provide a good estimate with n_T blocks [2], [9]. These sequences are optimal for the channel sounding technique since they have constant-modulus elements in both the time domain and the frequency domain. Random training sequences were used with the other techniques. As a reference, the error produced by a linear MMSE frequency-domain equalizer constructed with perfect channel state information (CSI) is illustrated in Fig. 3.
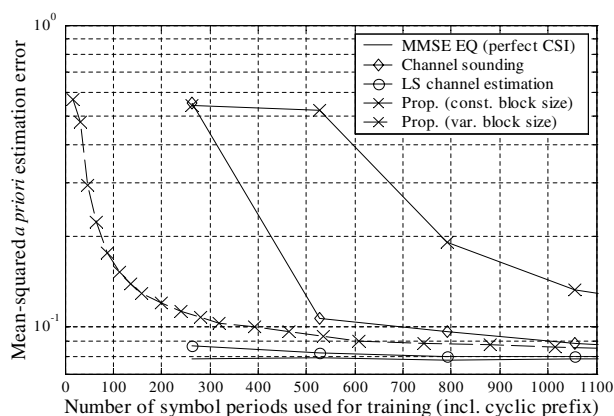
Fig. 3. Convergence of SC-FDE techniques employed in a 2×2 SM system operating in the ETSI BRAN A channel. (SNR per RX antenna = 15 dB, $K = 256$, $\mathcal{K} = \{8, 16, 32, 64, 128\}$, $Q = 8$, $\tau = 4$)

Fig. 4. Performance of 2×2 SM SC-FDE systems operating in the ETSI BRAN A channel.

The benefits of utilizing a variable training block size with the proposed algorithm are evident in Fig. 3. In this example, it is shown that the proposed algorithm and the channel sounding technique converge to the same estimation error after 280 and 528 training symbol intervals, respectively. This difference corresponds to a 47% decrease in training overhead in favor of the proposed algorithm. Unsurprisingly, the LS technique converges very quickly, nearly reaching the reference curve after the first training block interval.

The performances of four systems were studied. Two transmit antennas and two receive antennas were employed in each system. The first two systems devoted two block intervals to channel sounding and LS channel estimation, respectively. The third system adaptively computed the equalizer matrix with 15 training blocks where $\mathcal{K} = \{8, 16, 32, 64\}$ and $\tau = 4$. The fourth system was assumed to have perfect CSI. Each of the three non-adaptive systems employed an MMSE equalizer.

For each system, once the equalizer was constructed, a packet of 1024 data bits was encoded with a half-rate convolutional encoder that uses the generator polynomials $[133]_8$ and $[171]_8$. The encoded bits were randomly interleaved and mapped to QPSK symbols that were then arranged into blocks of $K = 256$ symbols. A cyclic prefix of $Q = 8$ symbols was added to each block prior to transmission. At the receiver, the equalized symbols were mapped to soft bits that were then de-interleaved and passed through a standard Viterbi decoder. The PERs and BERs of the simulated systems are shown in Fig. 4. As observed, the system employing the proposed algorithm performs within 3 dB of the system with perfect CSI.

The crossover of the curves for the proposed algorithm and the channel sounding technique results from the use of frequency-domain interpolation at high SNR. The proposed algorithm in this example is affected by noise and the error floor imposed by frequency-domain interpolation. Therefore, the crossover point can be viewed as the point where noise no longer affects the proposed algorithm as much as frequency-domain interpolation. One might argue that as long as the
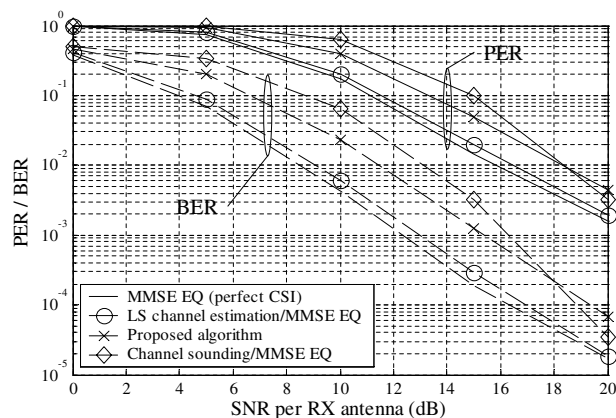
system is operating below the desired PER when this crossover occurs, the fact that the channel sounding technique performs better than the proposed algorithm is irrelevant.

VIII. CONCLUSION

In this paper, we presented a generalized method for adaptively equalizing multi-antenna single-carrier transmissions in the frequency domain. The convergence of this algorithm is comparable to, if not better than, a typical channel sounding technique where the channels between each transmit antenna and all receive antennas are estimated sequentially. The complexity of the proposed algorithm is significantly lower than that of a channel estimation technique based on least squares when the channel has a large excess delay spread.

ACKNOWLEDGMENT

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