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LOW COMPLEXITY TWO-DIMENSIONAL DIGITAL FILTERS USING UNCONSTRAINED SPT TERM ALLOCATION

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ABSTRACT

Previous work by the authors has demonstrated how circularly symmetric and diamond-shaped low-pass linear phase 2-D FIR filters can be designed using coefficients comprising the sum or difference of two signed power-of-two (SPT) terms. This has employed a minimax error criterion in conjunction with an optimisation process based on the use of genetic algorithms. The resulting filters exhibit superior performance to those designed using other methods reported such as simulated annealing and linear programming. This paper extends this work by demonstrating the further improvements possible if the constraints are relaxed, such that only the overall number of SPT terms is constrained, while the distribution between filter coefficients is unconstrained.

1 INTRODUCTION

Two-dimensional linear-phase FIR filters are widely used in image and video processing systems with applications including scanning-rate conversion, compression coding, motion estimation and recognition systems. For hardware implementations, it is often desirable to minimise the filter complexity. One method of achieving this is by constraining the filter coefficients to be simple combinations of power-of-two terms, thereby enabling more complex multipliers to be replaced by shift and addition/subtraction operations.

The use of a signed power of two (SPT) representation for the coefficients implies a discrete and non-uniformly populated solution space which is incompatible with many optimum filter design methods. In order to satisfy these constraints, coefficients have, in the past, been produced either by rounding those generated by optimal continuous designs or by searching the solution space using simulated annealing [2] or linear programming methods [3]. The rounding approach is known to yield sub-optimal filters, while the search-based methods require a large computational effort to effectively search the solution space.

Genetic algorithms (GAs) [1] have been identified as a useful tool for efficiently optimising large discrete

multimodal search spaces. In previous work by the authors [6,7], GAs were used to design 2-D FIR filters where each filter coefficient was restricted to two SPT terms. In this paper we extend this work by constraining the total number of SPT terms while allowing their distribution among the coefficients to be unconstrained. Section 2 outlines the basis of the design problem and section 3 introduces a representation suitable for GA based optimisation. Several example designs are considered in section 4 and comparisons with simulated annealing and linear programming approaches are made.

2 FILTER DESIGN FORMULATION

The impulse response of a two dimensional digital filter is denoted $h(\mathbf{n})$, with $\mathbf{n}^T = (n_1, n_2)$. The filter transfer function is given by:

$$H(\mathbf{f}) = \sum_{\mathbf{n}} h(\mathbf{n}) e^{-j2\pi \mathbf{f}^T \mathbf{n}} \quad (1)$$

with $\mathbf{f}^T = (f_1, f_2) \in \mathfrak{R}$.

In order to ensure a low-pass linear-phase response the coefficients must obey even symmetry in both the horizontal and vertical directions. For a filter with an odd number of taps in both directions this can be expressed as:

$$h(n_1, n_2) = h(n_1, -n_2) = h(-n_1, n_2) = h(-n_1, -n_2)$$
$$\text{with } n_1 = -N_1, \dots, 0, \dots, N_1, \text{ and } n_2 = -N_2, \dots, 0, \dots, N_2.$$

If the filters are further constrained to be horizontally and vertically equivalent then it can be seen that symmetry must also exist about the diagonals, thus: $h(n_1, n_2) = h(n_2, n_1)$ and $N_1 = N_2 = N$. Therefore coefficient values need only be specified for the range:

$$0 \leq n_1 \leq N$$
$$0 \leq n_2 \leq n_1 \quad (2)$$

```

o o o o x
o o o x x
o o x x x
o o o o o
o o o o o

```

Fig 1: Independent coefficient locations (x), assuming octal symmetry for a 5x5 filter.

This is known as octal symmetry and is shown in figure 1 for a 5x5 filter ($N=2$) where only 6 of the total 25 coefficients need to be specified.

In order to reduce architectural complexity, the coefficients, $h(\mathbf{n})$, are assumed to be sums or differences of finite number of power-of-two terms.

$$h(\mathbf{n}) = \sum_{k=1}^{p_n} c_k \cdot 2^{-g_k} \quad c_k \in \{-1,0,1\},$$

$$g_k \in \{1,2,\dots,B\}$$
(3)

where B denotes the maximum shift value used. The total number of SPT terms is however constrained to be a maximum P as follows:

$$\sum_{n_1=0}^N \sum_{n_2=0}^{n_1} p_n \leq P$$
(4)

For comparison with previous work [2,3,6,7], we have restricted P to give an average of two SPT terms per coefficient.

Since the filter coefficient space is discrete and sparsely populated, it is useful to incorporate a gain term, G , in the transfer function as follows :

$$H(\mathbf{f}) = \frac{1}{G} \sum_{\mathbf{n}} h(\mathbf{n}) e^{-j2\pi \mathbf{f}^T \mathbf{n}}$$
(5)

For the case of octal symmetry this becomes:

$$H(f_1, f_2) = \frac{1}{G} \{h(0,0)$$

$$+ \sum_{n_1=1}^N 2h(n_1,0) [\cos(2\pi n_1 f_1) + \cos(2\pi n_1 f_2)]$$

$$+ \sum_{n_1=1}^N 4h(n_1, n_1) \cos(2\pi n_1 f_1) \cos(2\pi n_1 f_2) +$$

$$\sum_{n_2=1}^{N-1} \sum_{n_1=n_2+1}^N 4h(n_1, n_2) [\cos(2\pi n_1 f_2) \cos(2\pi n_2 f_1)$$

$$+ \cos(2\pi n_2 f_2) \cos(2\pi n_1 f_1)]\}$$
(6)

If $T(\mathbf{f})$ is the desired frequency response of the filter and $W(\mathbf{f})$ is a weighting function, the cost function of the minmax design process for a 2-D multiplierless FIR filter specified by $\{h(\mathbf{n})\}$ can be formulated as:

$$\delta(\{h(\mathbf{n})\}) = \max_{\mathbf{f}_k} W(\mathbf{f}_k) |H(\mathbf{f}_k) - T(\mathbf{f}_k)|$$
(7)

where \mathbf{f}_k represents a predetermined sampling grid for the evaluation of (7). During the GA optimisation process, a grid containing 85 sample points is used, whereas the final results are evaluated using approximately 32000 points.

3 PROBLEM REPRESENTATION

Each SPT term, $c_k \cdot 2^{-g_k}$ is specified using 1 bit for the sign of c_k and 3 or 4 bits for the shift value, g_k (depending on whether $B>8$). Unlike previous work [6,7] an additional field of 3 or 4 bits is used to specify the associated coefficient index. It is also beneficial to enable some of the SPT terms to assume zero values. This is achieved by using an extra bit for these terms. For example with a 7x7 filter (with $B>8$) there are 10 independent coefficients. If 20 SPT terms are used with 10 allowed to be zero then the total chromosome length is 190 bits. The value of G is determined by taking the average of maximum and minimum passband ripple amplitudes.

4 RESULTS

The representation described above has been incorporated in a GA design framework and used to achieve the results, (GA2), shown in table 1. Also shown are results, (GA1), obtained using a GA [7] with the SPT terms restricted to exactly two per coefficient. For comparison, results are included for other design methods with coefficients restricted to a maximum of two SPT terms. These include simulated annealing (SA) [2], mixed integer linear programming (LP) [3], and coefficient rounding (this takes the optimum continuous minimax coefficients and scales and rounds these to the nearest combination of two SPT terms). Finally, results are shown for for both optimal full-precision filters (opt) [4] and filters designed using the McClellan transform [5] (a commonly used fast sub-optimal method for designing full precision 2-D filters based on an optimal 1-D design). An example frequency response plot for the 7x7 filter (C) designed using the GA approach is shown in figure 2.

As can be observed from table I, GAs are capable of producing results superior to those obtained using competing methods. Even with restricted dynamic range and only two non-zero terms per coefficient, (GA1), their performance approaches that of the optimum (infinite precision) minimax solution, albeit with greatly reduced

implementation complexity. Furthermore, when the distribution of SPT terms is not constrained, (GA2), results improve still further. An example set of coefficient values (for filter C in table I) is presented in table II. This shows how the GA exploits coefficient sensitivities. It should also be noted that, even though an average of 2 terms per coefficient were permitted, the GA solution requires only 1.5 terms, with many coefficients needing only a single term. This has the added advantage for hardware implementations since, in such cases, multipliers may be realised using simple shift operators.

The size of the search space for this problem (measured in terms of the representation used) ranges from 2^{90} for a 5x5 filter with 7 bit dynamic range to 2^{285} for a 9x9 filter with 11 bit dynamic range. To demonstrate the robustness of the GA based design method and to justify its use as a design tool, some run statistics are shown in table III. Twenty different random number seeds were used and the results are presented for different numbers of evaluations for each case. An example of GA convergence for filter H is presented in figure 3 which shows both average and best performance of the population in each generation.

Table I.

Performance comparisons between 2-D Multiplierless FIR filter designs using Full precision minimax optimisation (opt) [4], McClellan Transform Design [5], Linear programming (LP) [3], Simulated Annealing (SA) [2] and Genetic Algorithms (GA1 [6,7] and GA2).

Design	A	B	C	D	E	F	G	H
Size	5x5	5x5	7x7	7x7	7x7	7x7	9x9	9x9
Passband radius, fp	0.2	0.15	0.15	0.15	0.175	0.225	0.2	0.15
Stopband radius, fs	0.3	0.35	0.35	0.35	0.325	0.275	0.3	0.35
B	11	7	7	9	11	11	5	7
δ (opt)	0.26705 -11.5 dB	0.13188 -17.6 dB	0.03284 -29.7 dB	0.03284 -29.7 dB	0.06557 -23.7 dB	0.24733 -12.1 dB	0.11417 -18.8 dB	0.02609 -31.7 dB
δ (MCT)	0.31931 -9.9 dB	0.19512 -14.2 dB	0.08210 -21.7 dB	0.08210 -21.7 dB	0.12402 -18.1 dB	0.32910 -9.7 dB	0.19481 -14.2 dB	0.08206 -21.7 dB
δ (rounding)	0.26766 -11.5 dB			0.03831 -28.3 dB			0.24466 -12.2 dB	
δ (LP)		0.14264 -16.9 dB	0.06250 -24.1 dB					0.05444 -25.3 dB
δ (SA)	0.26816 -11.4 dB	0.13929 -17.1 dB	0.06024 -24.4 dB	0.03760 -28.5 dB	0.07969 -22.0 dB	0.27514 -11.2 dB	0.22310 -13.0 dB	0.05106 -25.8 dB
δ (GA1)	0.26737 -11.5 dB	0.13585 -17.3 dB	0.04045 -27.9 dB	0.03754 -28.5 dB	0.07256 -22.8 dB	0.26364 -11.6 dB	0.14044 -17.1 dB	0.04051 -27.8 dB
δ (GA2)	0.26736 -11.5 dB	0.13330 -17.5 dB	0.03678 -28.7 dB	0.03650 -28.8 dB	0.07110 -23.0 dB	0.26050 -11.7 dB	0.12493 -18.1 dB	0.03505 -29.1 dB

Table II.
Filter coefficients for the design 'C'.

Filter Coefficient	Values
$h(0,0)$	$2^{-1}+2^{-2}+2^{-3}-2^{-7}$
$h(1,0)$	$2^{-1}+2^{-4}$
$h(1,1)$	$2^{-2}+2^{-3}$
$h(2,0)$	2^{-3}
$h(2,1)$	2^{-5}
$h(2,2)$	-2^{-4}
$h(3,0)$	-2^{-4}
$h(3,1)$	-2^{-4}
$h(3,2)$	-2^{-5}
$h(3,3)$	-2^{-7}

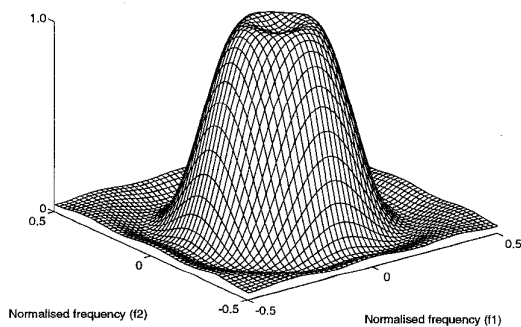


Fig 2: Frequency response for 7x7 filter (C)

Table III.

Run statistics (in terms of pass/stopband ripple normalised to a unity passband gain) for filter H with 20 different random number seeds.

Number of evaluations	Best	Average	Variance
25,000	0.03902	0.06177	0.00026
50,000	0.03108	0.05347	0.00021
75,000	0.03084	0.05016	0.00023
100,000	0.02845	0.04778	0.00021

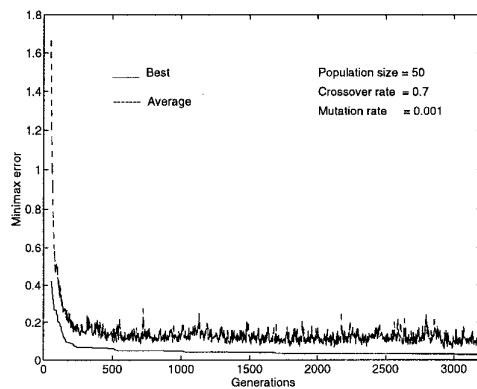


Fig 3: GA convergence characteristics

5 CONCLUSIONS

This paper has demonstrated the power and robustness of genetic algorithms as a means of designing two dimensional digital filters optimised over a finite coefficient space. The resulting filters are superior, in terms of frequency domain performance, to equivalent filters designed using linear programming or simulated annealing and, in many cases approach the optimum infinite precision solution. By constraining the total number of SPT terms available while allowing freedom in their allocation to individual coefficients, further improvements (up to 1.3dB for the filters tried) have been demonstrated. Good convergence characteristics were achieved for all filters, with typical design runs requiring 2000 generations for a population size of 50.

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