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An Error Bound Analysis for M-DPSK in Frequency-Selective Rayleigh Fading Channel with Diversity Reception

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Abstract: *A theoretical upper bound is presented to calculate the error probability for raised cosine filtered M-ary differential phase-shift keying (RC M-DPSK) in frequency-selective Rayleigh fading channels. This analysis also allows the used of postdetection maximal-ratio diversity combining. The effect of delay spread and filter roll-off are also considered in the formulation. The irreducible error probabilities for 2, 4, 8 and 16-DPSK are compared with first and second order diversity reception. The numerical results show similar irreducible error performances for 4 and 8-DPSK with both performing better compared to B-DPSK and 16-DPSK. When diversity is applied, the performance of 4-DPSK is superior to that of other levels. The irreducible error performance of 4, 8 and 16-DPSK is shown to be better than B-DPSK in frequency selective fading channels.*

Introduction

Future mobile networks are looking to integrate digital voice with high capacity data based services. However, unlike analogue transmissions, digital coverage can no longer be predicted by signal strength alone. Digital modems are prone to the introduction of an irreducible error rate that arises as a result of user motion and/or time dispersion in the channel [1]. The resulting error floors are independent of signal strength and, unless corrected, can limit either the speed of the mobile (Doppler) or the maximum transmission rate (time dispersion). Hence, for a high data rate wireless modem, the effect of delay spread must be carefully considered.

The effect of delay spread on M-ary PSK (Phase Shift Keying) has already been investigated by various authors. The error performance of Binary DPSK (B-DPSK) has been analysed in [2],[3]. In [2] the improvement with postdetection maximal ratio combining was studied while in [3], the effect of profile shapes were investigated. Both of the above papers made use of time-limited pulse shaping in their investigations. B-DPSK has also been studied in [4] where a special case of Raised Cosine (RC) filtering was also assumed. The error performance of M-ary coherent PSK with bandlimiting raised cosine pulse shapes was examined in [5],[6],[7]. The frequency selective error performance of raised cosine Q-DPSK with postdetection combining is given in [8], this analysis considered selection,

equal gain and maximal ratio combining. More recently, the error performance of M-ary DPSK in frequency selective channels has been analysed including the effects of Doppler shift [9],[10]. In these papers, diversity combining was considered although time-limited pulse shapes were used.

The aim of this paper is to unify the above work and to produce results for M-ary DPSK in a frequency-selective channel. In this paper, an upper bound error probability is developed which allows factors such as root mean square (rms) delay spread, modulation level, raised cosine roll-off factor and post-detection diversity combining to be integrated into one generic equation. Using such equations, the irreducible error floors introduced by delay spread have been analysed and the sensitivity to filter roll-off and modulation level determined both with and without diversity combining.

System Modelling

The overall transmission system with postdetection diversity reception is modelled as shown in Fig.1. For mathematical convenience, an equivalent lowpass signal representation is used. To achieve both a narrowband spectrum and intersymbol interference (ISI) free transmission in the absence of delay spread, the overall raised cosine filter response is applied. [5]-[8]:

$$h_{RC}(t) = \frac{1}{T_s} \frac{\sin(t/T_s)}{\pi t/T_s} \frac{\cos(\alpha \pi t/T_s)}{1-(2\alpha t/T_s)^2} \qquad (1)$$

where α ($0 \leq \alpha \leq 1$) is the roll-off factor and T_s is the symbol duration. In this paper, a square root RC filter is assumed for both transmitter and receiver, and hence their frequency responses can be written as:

$$H_T(f) = T_s\sqrt{H_{RC}(f)}, \qquad H_R(f) = \sqrt{H_{RC}(f)} \qquad (2)$$

where $H_{RC}(f)$ is the frequency response of $h_{RC}(t)$ and the constant T_s is used to normalise the pulse to obtain unit power.

For RC M-DPSK transmission, the equivalent lowpass transmitted signal $u(t)$ can be written as:

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$$u(t) = A \sum_{n=-\infty}^{\infty} e^{j\theta_n} h_T(t - nT_s) = \sum_{n=-\infty}^{\infty} u_n \qquad (3)$$

where $\theta_n = 2\pi(i-1)/M$ with $i = 1, 2, \ldots, M$. M is the number of different transmitter phase states and can be written as below:

$$\theta_n = \theta_{n-1} + \Lambda_{in} \qquad (4)$$

where Λ_{in} is given by $2\pi(i-1)/M$ (for $i = 1, 2, \ldots, M$) and is used to represent the absolute phase of the data symbol s_i at time nT_s. The time response of the transmit filter is defined as $h_T(t)$ and A represents the amplitude of the transmitted signal. The signal generated is sent over L independent frequency-selective Rayleigh fading channels.

The Fading Channel: At the nth time interval, the equivalent lowpass received signal in the kth diversity channel, $r_k(t)$, can be written as:

$$r_k(t) = \int_{-\infty}^{\infty} g_k(t,\tau) u(t) d\tau + z_k(t) \qquad (5)$$

For the kth diversity branch, $g_k(t,\tau)$ represents the channel impulse response which is a zero mean complex Gaussian fading process. $z_k(t)$ denotes the zero mean complex additive white Gaussian noise of power spectral density $2N_o$. The fading signals at the different diversity branches are assumed independent and have identical statistical characteristics. For strictly frequency-selective fading channels [3] (i.e. fading variations much slower compared to the symbol rate), the auto-correlation functions for $g_k(t,\tau)$ is stated below:

$$E[g_k(t,\tau)g_k^*(t-\mu,\tau-\lambda)] = \xi_g(\tau)\delta(\lambda) \qquad (6)$$

for $k = 1, 2, \ldots, L$. $E[\bullet]$ denotes the ensemble average and $*$ denotes the complex conjugate value. $\xi_g(\tau)$ is the power delay profile. Earlier works have shown that *root mean square delay spread* is an important parameter for analysis of the error performance of the digital system in frequency-selective fading channels [5]-[8]. This parameter is defined as $\tau_{rms} = [\int_{-\infty}^{\infty} \tau^2 \xi_g(\tau)d\tau]^{1/2}$, where zero mean delay and $\int_{-\infty}^{\infty} \xi_g(\tau) = 1$ are assumed in the equation.

Receiver Processing: The receiver block diagram for the kth order diversity branch is shown in Fig.1. The signal $r_k(t)$ is passed through a matched filter with an impulse response of $h_R(t)$ and sampled at time $t = pT_s$, where p is an integer number. The output of the filter is therefore:

$$U_k = \int_{-\infty}^{\infty} r_k(\alpha) h_R(t-\alpha) d\alpha \qquad (7)$$

and similarly, for $r_k(t-T_s)$, the output of the filter is:

$$K_k = \int_{-\infty}^{\infty} r_k(\alpha - T_s) h_R(t - T_s - \alpha) d\alpha \qquad (8)$$

As indicated in Fig.1, the demodulator at each branch forms the product between the two complex Gaussian random variables of (7) and (8), so that for L^{th} order diversity, the combiner sums all the demodulator outputs and forms a combined vector, which can be expressed as:

$$Z = \sum_{k=1}^{L} Z_k = \sum_{k=1}^{L} U_k K_k^* \qquad (9)$$

If the transmission system is ideal (i.e. no fading and AWGN), the phase of Z_k or Z is simply the phase of the actual data symbol at time nT_s, Λ_{in}. Therefore, Z can be used as a decision variable for the diversity receiver.

Mathematical Analysis

Equation (4) shows that the information is encoded into the phase differences between θ_n and θ_{n-1}. At the receiver, a correct decision is made only if the difference between the two successive received phases $\hat{\theta}_n$ and $\hat{\theta}_{n-1}$ is such that

$$\Lambda_{in} - \frac{\pi}{M} < \hat{\theta}_n - \hat{\theta}_{n-1} \leq \Lambda_{in} + \frac{\pi}{M} \qquad (10)$$

where $\hat{\theta}_n$ and $\hat{\theta}_{n-1}$ are the noisy distorted versions of the corresponding transmitted phases. The decision rule (10) can be represented in graphical form as shown in Fig.2. An incorrect decision is made if the phase of the decision vector Z (or $\hat{\theta}_n - \hat{\theta}_{n-1}$) falls inside the area R_e. In [11] it has been shown that when ISI free transmission is assumed, a simple upper bound of $P_s(M)$ can be written as:

$$P_s(M) < \Pr\{Z \in R_1\} + \Pr\{Z \in R_2\} \qquad (11)$$

where R_1 and R_2 are the corresponding half plane regions for s_i. (Note: for $M = 2$, only the first term of (11) is needed for evaluation and the bound reduces to an exact solution.) However, in a frequency-selective fading channel, due to delay spread, the error is dominated by the time varying ISI, hence the error performance depends on the transmitted sequence, Φ:

$$\Phi = (\ldots, u_{n-2}, u_{n-1}, u_n, u_{n+1}, u_{n+2}, \ldots) \qquad (12)$$

The probability of detecting the data symbol $s_{i,n}$ incorrectly can be calculated by usinng (11) for particular transmitted sequence and the average symbol error probability can be written as:

$$P_s(M) < \frac{1}{N_\Phi} \sum_\Phi [\Pr\{Z_i \in R_{i1}\} + \Pr\{Z_i \in R_{i2}\}] \qquad (13)$$

for $i = 1, 2, \ldots, M$. N_Φ is the number of possible sequences. $\Pr\{Z_i \in R_{im}\}$, with $m = 1, 2$, is the probability of the decision vector at time nT_s falling into the two corresponding half-plane regions with $\theta_n - \theta_{n-1} = \Lambda_{in}$. For

high SNR, the average bit error probability can be approximated by dividing the symbol error probability by $\log_2 M$.

Formulation of the Upper Bound: To evaluate the upper bound shown in (13), the objective is to calculate the probability of Z_i falling into regions R_{i1} and R_{i2}. Following the approach developed by Edbauer[12] and described in [11], $\Pr\{Z_i \in R_{im}\}$ can be written as:

$$\Pr\{Z_i \in R_{im}\} = \Pr\left\{\sum_{k=1}^{L}(X_k Y_k^* + X_k^* Y_k) < 0\right\} \quad (14)$$

with two new complex Gaussian decision variables $X_k = U_k$ and $Y_k = K_k e^{-j\Psi_{im}}$. Ψ_{im} is the angle of rotation of the constellation mapping that transforms the half plane area into the *Left-Hand-Plane* (LHP) for data symbol s_i. The steps to evaluate (14) are shown in [11], and the final result is shown below:

$$\Pr\{Z \in R_{im}\} = \frac{1}{(1+B)^{2L-1}}\sum_{k=0}^{L-1}\binom{2L-1}{k}B^k \quad (15)$$

where

$$B = 1 + \frac{2\mathrm{Re}\{m_{XY}\}}{\sqrt{m_{XX}m_{YY} - \mathrm{Im}^2\{m_{XY}\}} - \mathrm{Re}\{m_{XY}\}} \quad (16)$$

The second central moments m_{XX}, m_{YY} and m_{XY} (normalised to $T_s/2N_o$) are shown below:

$$m_{XX} = \Gamma_s \int_{-\infty}^{\infty} \xi_g(0,\tau)\left\{\sum_{n=-\infty}^{\infty}e^{j\theta_n}H_n\right\}$$
$$\times \left\{\sum_{n=-\infty}^{\infty}e^{-j\theta_n}H_n\right\}d\tau + 1 \quad (17)$$

$$m_{YY} = \Gamma_s \int_{-\infty}^{\infty} \xi_g(0,\tau)\left\{\sum_{n=-\infty}^{\infty}e^{j\theta_{n-1}}H_n\right\}$$
$$\times \left\{\sum_{n=-\infty}^{\infty}e^{-j\theta_{n-1}}H_n\right\}d\tau + 1 \quad (18)$$

$$m_{XY} = \Gamma_s e^{j\Psi_{im}} \int_{-\infty}^{\infty} \xi_g(T_s,\tau)\left\{\sum_{n=-\infty}^{\infty}e^{j\theta_n}H_n\right\}$$
$$\times \left\{\sum_{n=-\infty}^{\infty}e^{-j\theta_{n-1}}H_n\right\}d\tau \quad (19)$$

where $H_n = T_s h_{RC}(pT_s - \tau - nT_s)$ and $\Gamma_s = A^2 T_s/2N_o$ is the average received symbol energy-to-noise power spectral density ratio per channel.

Numerical Results and Discussion

Analysis in the last section has shown that the calcuated error performance is dependent on the number of interference symbols considered in equations (17)–(19). Although the equations derived in the last section allow us to considered any number of interference symbols, in this paper

only adjacent-pulse-limited ISI channels [3], [9] are considered for numerical evaluation. Without lost of generality, the symbol at time $n = 0$ is used for detection, hence the sampling time, $pT_s = 0$. The upper bound symbol error probability for RC M-DPSK can be written in the following form:

$$P_s(M) < \frac{1}{2^{1k}}\sum_{\Phi}[\Pr\{Z_i \in R_{i1}\} + \Pr\{Z_i \in R_{i2}\}] \quad (20)$$

for $i = 1, 2, \ldots, M$. $k = \log_2 M$ is the number of bits per symbol and $\Phi = (u_{-2}, u_{-1}, u_0, u_1)$ represents the transmitted sequence. The second central moments are simplified to:

$$m_{XX} = \Gamma_s \int_{-\infty}^{\infty} \xi_g(0,\tau)D_{XX}d\tau + 1 \quad (21)$$

$$m_{YY} = \Gamma_s \int_{-\infty}^{\infty} \xi_g(0,\tau)D_{YY}d\tau + 1 \quad (22)$$

$$m_{XY} = \Gamma_s e^{j\Psi_{im}} \int_{-\infty}^{\infty} \xi_g(T_s,\tau)D_{XY}d\tau \quad (23)$$

with

$$D_{XX} = H_{-1}^2 + H_0^2 + H_1^2 + 2[H_{-1}H_0\cos(\theta_0 - \theta_{-1})$$
$$+ H_{-1}H_1\cos(\theta_1 - \theta_{-1}) + H_0 H_1\cos(\theta_1 - \theta_0)]$$

$$D_{YY} = H_{-1}^2 + H_0^2 + H_1^2 + 2[H_{-1}H_0\cos(\theta_{-1} - \theta_{-2})$$
$$+ H_{-1}H_1\cos(\theta_0 - \theta_{-2}) + H_0 H_1\cos(\theta_0 - \theta_{-1})]$$

$$D_{XY} = H_{-1}^2 e^{j(\theta_{-1} - \theta_{-2})} + H_0^2 e^{j(\theta_0 - \theta_{-1})} + H_1^2 e^{j(\theta_1 - \theta_0)}$$
$$+ H_{-1}H_1[e^{j(\theta_1 - \theta_{-2})} + e^{j(\theta_0 - \theta_{-1})}] + H_{-1}H_0$$
$$+ H_0 H_1 + H_{-1}H_0 e^{j(\theta_0 - \theta_{-2})} + H_0 H_1 e^{j(\theta_1 - \theta_{-1})}$$

Assumming $\xi_g(\tau)$ is defined with a rectangular power delay profile as in [3], the average bit error probability for 2, 4, 8 and 16-DPSK is shown in Fig.3 as a function of normalised rms delay spread, d_b. The graph also shows the impact of roll-off factor, a, and 1st and 2nd order diversity. The rms delay spread is normalised in terms of the bit period (i.e. $d_b = \tau_{rms}/T_b$) so that the error performances can be compared for the same information throughput for different levels of modulation schemes [6]. The results obtained are in good agreement with the simulation results given in [13].

To analyse these graphs, rms delay spread of $100ns$ and $2\mu s$ will be assumed as typical for the indoor and outdoor channel respectively. From Fig.3a, B-DPSK can be seen to result in the highest irreducible error while Q-DPSK and 8-DPSK achieve the lowest values of error. Interesting the performance of 16-DPSK lies between these two curves. Assuming a roll-off factor of 0.3 and an average irreducibe bit error probability of 1e-3, maximum bit rates of $300kbps$ (B-DPSK) and $430kbps$ (Q-DPSK/8-DPSK) can be achieved in the indoor channel. For the outdoor system defined above, bit error probabilities vary

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between 15 and 21$kbps$ depending on modulation level. If the roll-off factor is increased to 1.0, the data rates vary between 700kbps and 1.1Mbps (indoor) and 35 to 55kbps (outdoor).

Fig.3b shows how the error performance can be improved with the use of diversity reception. Diversity is a more powerful technique for combating delay spread, however for $d_b > 0.2$, this technique becomes less effective. The diminished effect of diversity is already explained in [8]. From [1, Fig.7] it has been shown that the correlation of irreducible error with fade depth reduces with increasing normalised delay. Hence for large values of delay spread, errors are no longer confined to deep fades and diversity becomes far less effective. Assumming the previous delay spreads and a roll-off factor of 0.3, bit rates of 1 to 1.7$Mbps$ (indoor) and 50 to 85kbps (outdoor) can be achieved with diversity combining (actual value depending on modulation) These bit rates can be improved to 3 to 4$Mbps$ (indoor) and 150 to 200$kbps$ by increasing the roll-off factor to 1.0. However, unlike the gains offered by diversity, increasing the filter roll-off lowers bandwidth efficiency and is therefore less attractive for mobile radio use.

Conclusions

An error bound for raised cosine filtered M-DPSK with postdetection maximal-ratio diversity combining has been derived for a frequency-selective Rayleigh fading channel. The numerical results have shown that without diversity the irreducible error performance of 4 and 8-DPSK are approximatly equal and superior to that of 2-DPSK and 16-DPSK. Interestingly, when diversity is applied, Q-DPSK outperforms the other levels of DPSK considered.

These results confirm that increasing the level of DPSK does not necessarily improve the resulting irreducible bit error rate (despite lowering the symbol rate for a fixed bit rate). Although for higher levels of DPSK the symbol rate is reduced, increased suseptability to interference (in the form of ISI) can more than offset this advantage. Unless bandwidth efficiency is of paramount importance, of the schemes considered in this study, Q-DPSK offers the best performance in a delay spread limited channel.

Acknowledgments

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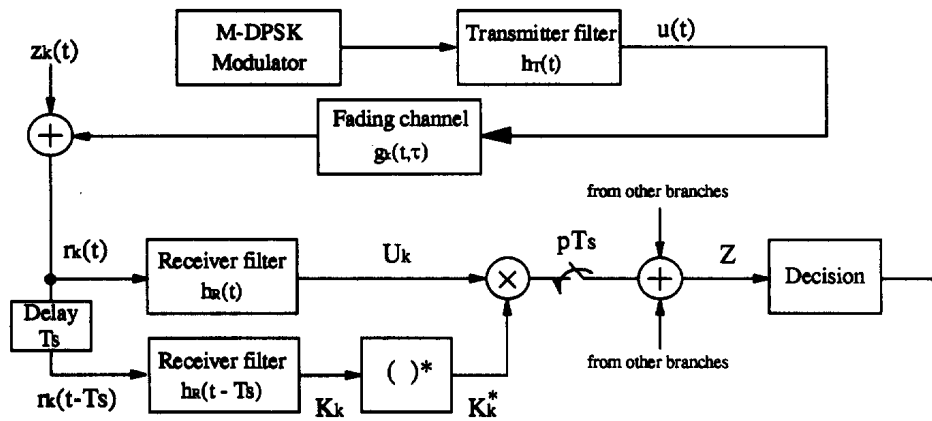
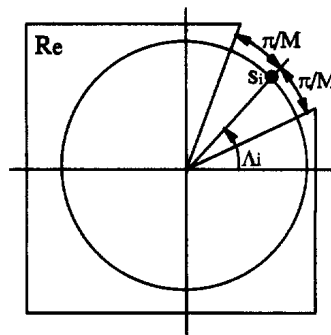Fig.1: Block diagram for M-DPSK transmission system at kth diversity branch in equivalent lowpass representation.
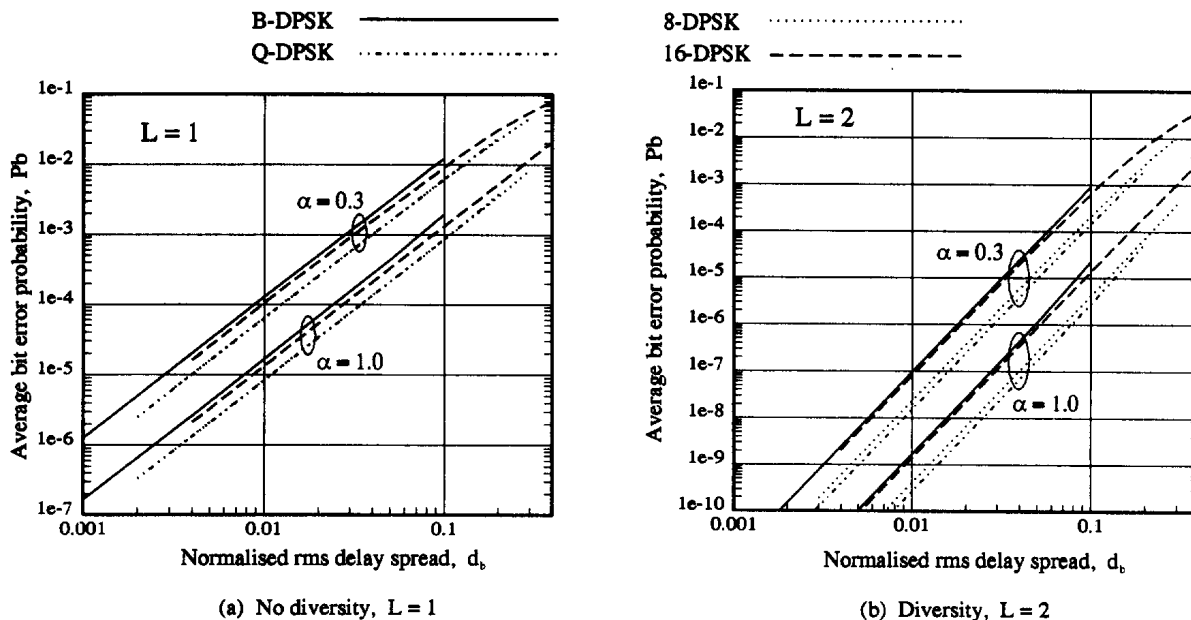

Fig.2: Error region for M-DPSK signal.

B-DPSK ──────── 8-DPSK ·····················

Q-DPSK ·−·−·−·−·−· 16-DPSK − − − − − − −

(a) No diversity, L = 1

(b) Diversity, L = 2

Fig.3: Irreducible error performance of 2, 4, 8 and 16-DPSK versus normalised rms delay spread, d_b.

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