A Constrained Approach to Pre-Compensation for TDD OFDM Systems

M.R.G.Butler, P.N.Fletcher, A.R.Nix, D.R.Bull
Centre for Communications Research
University of Bristol, UK
mike.butler@bristol.ac.uk

Abstract

Pre-compensation techniques can be used to enhance the Bit Error Rate (BER) performance of Time Division Duplex (TDD) Orthogonal Frequency Division Multiplexing (OFDM) communication links. With pre-compensation at the transmitter, the power assigned to each subcarrier is varied according to the Channel State Information (CSI). This paper derives a pre-compensation algorithm for OFDM systems that use coherent Quadrature Phase Shift Keying (QPSK) modulation on each subcarrier. The algorithm attempts to minimize the received BER subject to a constraint on the transmitted signal power. In order to yield a closed-form solution, the minimization is based on an approximation of the BER function, and is sub-optimal. The performance of the algorithm is compared with the fully optimal solution. In addition, the possible exploitation of the technique within a Wireless Local Area Network (WLAN) is discussed.

1. Introduction

Wireless communication systems that operate with a Time Division Duplex (TDD) medium access mechanism use the same frequency spectrum for both up-link and down-link transmissions. Therefore, knowledge of the transmission medium—Channel State Information (CSI)—from the up-link can be used to improve the performance of down-link signaling, and vice-versa. In the case of systems that use Orthogonal Frequency Division Multiplexing (OFDM), a *pre-compensation* algorithm, based on the CSI, can control transmit power on a subcarrier-by-subcarrier basis in order to improve Bit Error Rate (BER) performance.

Various approaches to pre-compensation for wireless OFDM systems have been suggested [1, 4, 5]. A key issue, and one that typically limits the usefulness of the technique, is how best to pre-compensate signals without increasing the transmitted signal power. The algorithm outlined here constrains the output signal power to be identical to an OFDM system with no pre-compensation. Specifically, the method that we derive attempts to minimize the received BER subject to a power constraint. In order to produce a closed-form solution, this minimization is based on an approximation of the complementary error function, and yields a near-optimal method of pre-compensation for an OFDM system using coherent Quadrature Phase Shift Keying (QPSK) subcarrier modulation. As a result of the approximation used, the solution presented here is similar to the one given in [4], which is the optimum solution for Differential Phase Shift Keying (DPSK) signaling.

The paper is organized as follows: section 2 introduces a basic approach to pre-compensation and, using a simple OFDM transmission model, outlines the associated output power problem. Section 3 then derives a near-optimal pre-compensation algorithm based on constraining the output power. The performance of this technique is compared with the basic approach presented in section 2, and with the fully optimal solution. Finally, section 4 discusses some issues relating to the implementation of the pre-compensation algorithm in a Wireless Local Area Network (WLAN).

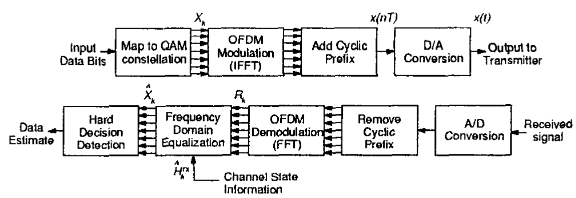

Figure 1. OFDM communication system

2. Pre-Compensation by Channel Inversion

Figure 1 shows a block diagram of an *uncoded* OFDM communication system. The Fast Fourier Transform (FFT) is used both to synthesize the multicarrier signal and to demodulate the received signal. We denote the information-bearing QAM constellation points by X_k, and N is the number of points in the FFT. If, in order to combat channel delay spread, the system utilizes a cyclic prefix of length N_g samples, then the OFDM symbol length is $N_s = N + N_g$ samples, and the OFDM transmit signal, sampled at the Nyquist rate, for one symbol can be expressed as

$$x(nT) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi k(n-N_g)/N};\ n = 0 \ldots N_s - 1 \tag{1}$$

where T is the sampling period, and $1/\sqrt{N}$ scales $x(nT)$ to have unity average power. If we assume that the cyclic prefix is sufficiently long that successive OFDM symbols do not interfere, then the received vector for the kth subcarrier is

$$R_k = H_k X_k + \eta_k \tag{2}$$

where $H_k = A_k e^{j\phi_k}$ is the channel frequency response at the kth subcarrier frequency and η is sampled Additive White Gaussian Noise (AWGN).

Pre-compensation is achieved at the transmitter by multiplying each symbol constellation by a pre-compensation coefficient. We assume that the coefficients are real—they only alter the amplitude of the signal constellations. If the coefficient for subcarrier k is denoted by C_k, then the received vectors are

$$R_k = C_k H_k X_k + \eta_k \tag{3}$$

A straightforward approach to pre-compensation is to multiply each OFDM subcarrier by the inverse of the channel magnitude response at that frequency. In this case, the coefficients are set according to

$$C_k = \frac{1}{|H_k|} = \frac{1}{A_k} \tag{4}$$

We refer to this method as "pre-compensation by channel inversion". Note that with this approach, for coherent detection the received vectors R_k still need to be 'equalized' at the receiver to account for the phase distortion of the channel.

In order to demonstrate the effects of this technique, we use a simple two-ray channel model given by

$$h(t) = \alpha_1 \delta(t) + \alpha_2 \delta(t - 2T) \tag{5}$$

where $\alpha_2 = 7\alpha_1/8$ and α_1 and α_2 are selected so that the channel has an average power gain of one. We assume that the OFDM signal comprises 64 subcarriers ($N = 64$), with QPSK signaling on each subcarrier. We further assume that both the transmitter and receiver have exact knowledge of the channel response. Figure 2 shows the BER, or probability of error, performance of the OFDM system with and without pre-compensation by channel inversion. The average power of the transmitted subcarriers is S, and the received signal is corrupted by AWGN of single-sided noise spectral density N_0.

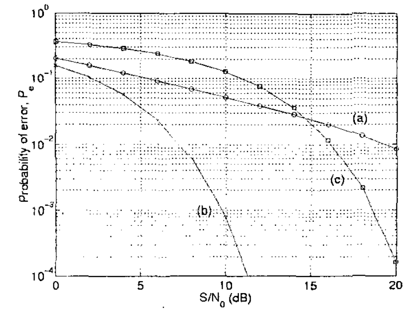

Figure 2. BER performance of OFDM system: (a) No pre-compensation; (b) Pre-compensation by channel inversion; (c) Pre-compensation by channel inversion with normalized power

The BER performance with pre-compensation by channel inversion is vastly improved. Indeed, because we have assumed perfect CSI, the performance is equivalent to QPSK corrupted by AWGN only. However, this improvement is achieved only with a significant (more than 17.0 dB) increase in the average transmit power. In practical OFDM systems, difficulties with amplifier linearity make it unacceptable to vary the output power according to the channel fading, particularly to such a large degree.

In order to limit the output power fluctuations, if the constraint

$$\sum_{k=0}^{N-1} C_k^2 = N \tag{6}$$

is imposed on the pre-compensation coefficients, then the average transmit power is normalized to unity. If we first set the pre-compensation coefficients according to channel inversion, and then adjust them to satisfy (6), we have

$$C_k = \frac{1}{A_k} \sqrt{\frac{N}{\sum_{k=0}^{N-1} \frac{1}{A_k^2}}} \tag{7}$$

The BER performance of this method is also shown in Figure 2. A significant performance degradation is observed. This is because the channel inversion technique forces all received subcarriers to have constant power. As a result, a disproportionate amount of transmit power is assigned to subcarriers that are deeply faded and, when the power constraint is enforced, the average received signal-to-noise ratio (SNR) is substantially reduced. An approach that has been suggested in order to reduce this problem involves limiting the signal power that can be assigned to subcarriers, or transmitting zero power on subcarriers that are faded below some threshold [1, 5]. In the next section, we will outline a more optimal way of selecting the pre-compensation coefficients that fulfils the output power constraint given in (6).

3. Constrained Pre-Compensation

In this section, we introduce an algorithm that selects pre-compensation coefficients, constrained according to (6), which minimize the BER, or average probability of error, of the received data. Again, we assume perfect CSI estimation at the transmitter and receiver. In addition, we assume that the transmitter has knowledge of the receiver noise spectral density, N_0.

With the pre-compensation coefficients C_k applied at the transmitter, and the channel magnitude response given by A_k, the kth subcarrier has an average received signal power of $C_k^2 A_k^2 S$. For coherent QPSK detection the phase distortion caused by the channel requires phase equalization of the received data vectors. However, this does not affect the subcarrier SNR. The probability of error of the kth subcarrier is

$$P_{e,k} = \frac{1}{2}\operatorname{erfc}\left(C_k A_k \sqrt{S/2N_0}\right) \tag{8}$$

and the average probability of error is given by

$$P_e = \frac{1}{N}\sum_{k=0}^{N-1} \frac{1}{2}\operatorname{erfc}\left(C_k A_k \sqrt{S/2N_0}\right) \tag{9}$$

where $\operatorname{erfc}(u)$ is the complementary error function.

In order to select the coefficients that minimize (9) subject to the constraint (6), we use the Lagrange multiplier method. Firstly, we construct the Lagrangian:

$$L = \frac{1}{N}\sum_{k=0}^{N-1} \frac{1}{2}\operatorname{erfc}\left(C_k A_k \sqrt{S/2N_0}\right) - \lambda\left(N - \sum_{k=0}^{N-1} C_k^2\right) \tag{10}$$

The Lagrange equations for this problem are then, for each k

$$\frac{\partial L}{\partial C_k} = -\frac{A_k\sqrt{S/2N_0}\exp\left(-C_k^2 A_k^2 S/2N_0\right)}{N\sqrt{\pi}} + 2\lambda C_k = 0 \tag{11}$$

No closed-form solution exists for these equations. They can be solved numerically, although this is a complex operation and is impractical for large N. However, consider the following asymptotic expansion for $\operatorname{erfc}(u)$

$$\operatorname{erfc}(u) \simeq \frac{\exp\left(-u^2\right)}{\sqrt{\pi}u}\left[1 - \frac{1}{2u^2} + \ldots \pm \frac{1\cdot 3\cdot 5\ldots(2n-1)}{2^n u^{2n}}\right] \tag{12}$$

Omitting the multiplying factor $1/u$ and truncating after the first term of the series, we approximate (9) by

$$P_e' \approx \frac{1}{N}\sum_{k=0}^{N-1} \frac{1}{2}\frac{\exp\left(-C_k^2 A_k^2 S/2N_0\right)}{\sqrt{\pi}} \tag{13}$$

The Lagrangian becomes

$$L' = \frac{1}{N}\sum_{k=0}^{N-1} \frac{1}{2}\frac{\exp\left(-C_k^2 A_k^2 S/2N_0\right)}{\sqrt{\pi}} - \lambda\left(N - \sum_{k=0}^{N-1} C_k^2\right) \tag{14}$$

and the Lagrange equations, for each k

$$\frac{\partial L'}{\partial C_k} = -\frac{C_k A_k^2 S/2N_0 \exp\left(-C_k^2 A_k^2 S/2N_0\right)}{N\sqrt{\pi}} + 2\lambda C_k = 0 \tag{15}$$

Solving the Lagrange equations for C_k gives

$$C_k^2 = \frac{\ln\left(A_k^2 S/2N_0\right) - \ln\left(2N\pi\lambda\right)}{A_k^2 S/2N_0} \tag{16}$$

The value of λ for any set of A_k and N_0 can be determined by substituting this expression into the constraint (6). It is then straightforward to find the coefficients from (16).

Figure 3 illustrates this minimization for a pair of subcarriers with channel magnitude responses $A_1 = 1.875$ and $A_2 = 0.125$, and $S/N_0 = 15$dB. These subcarriers correspond to the maximum and minimum values of A_k, respectively, given the channel model described by (5). Based on the actual BER function, P_e, the minimum occurs at $C_1 = 0.2920$. This gives $P_e = 0.1664$, compared to $P_e = 0.2411$ when no pre-compensation is applied (i.e. $C_1 = C_2 = 1$). Our approximation predicts an optimal value of $C_1 = 0.3254$, giving a minimum actual BER, $P_e = 0.1670$. Similar results were observed for other values of A_1, A_2 and S/N_0. Therefore, although our approximate approach is clearly sub-optimal, we have some confidence that it produces a near-optimal result and, at the very least, yields a lower BER than applying no pre-compensation technique.

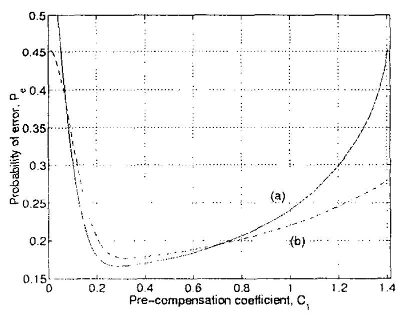

Figure 3. Variation of BER with pre-compensation coefficient C_1 for $H_1 = 1.875$, $H_2 = 0.125$ and $S/N_0 = 15$dB: (a) Based on P_e; (b) Based on P_e'

Figure 4 shows the BER performance of the sub-optimal constrained pre-compensation for channel model (5). Results are shown with the optimization performed across all subcarriers, and with the subcarriers sub-divided into 32 pairs (the pairs were grouped as successive highest and lowest values of A_k). For any pair of subcarriers, with channel magnitude responses A_1 and A_2, the pre-compensation coefficients can be expressed as

$$C_1^2 = \frac{\ln\left(A_1^2\right) - \ln\left(A_2^2\right) + 2A_2^2 S/2N_0}{\left(A_1^2 + A_2^2\right) S/2N_0} \tag{17}$$

and

$$C_2^2 = 2 - C_1^2 \tag{18}$$

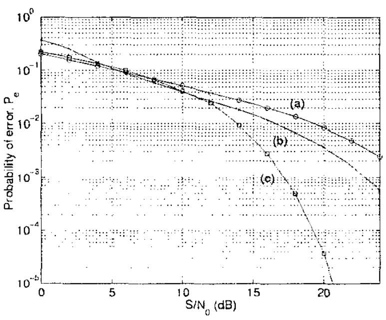

Figure 4. BER performance of OFDM system: (a) No pre-compensation; (b) Sub-optimal pre-compensation across subcarrier pairs; (c) across all subcarriers;

As in [4] we found that, for low values of S/N_0 and A_k, a positive solution for C_k^2 is not guaranteed. In these cases, we discarded the subcarriers that produced negative C_k^2 (i.e. set $C_k = 0$) and performed the optimization across the remaining subcarriers.

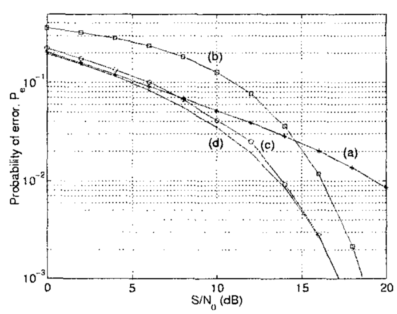

Figure 5. BER performance of OFDM system: (a) No pre-compensation; (b) Pre-compensation by channel inversion with normalized power; (c) Sub-optimal pre-compensation; (d) Optimal pre-compensation;

Figure 5 compares the BER performance of the sub-optimal pre-compensation method (across all subcarriers) with the numerically-calculated optimum solution. The first observation that we can make from this figure is that the optimization based on the approximate BER produces coefficients that perform well compared to the optimal pre-compensation coefficients. Furthermore, the sub-optimal C_k move closer to the optimal coefficients with increasing S/N_0. This is not surprising because, as figure 6 shows, our approximation of erfc (u) becomes increasingly tight as $u > 1$.

Figure 5 also shows the BER performance with no pre-compensation, and with power-normalized pre-compensation by channel inversion. We notice that neither the optimal, nor the near-optimal, C_k start yielding significantly lower BER than with no pre-compensation until $S/N_0 > 10$dB. This is because, with low values of S/N_0, a large number of subcarriers have high BER and the pre-compensation can have negligible effect. In addition, in the sub-optimal case, at low S/N_0, blocked subcarriers contribute $P_{e,k} = 0.5$. As S/N_0 increases, however, P_e becomes dominated by a small number of subcarriers that are deeply faded, and power can be effectively redistributed from subcarriers with low BER to those with high BER. In the limit, the power distribution tends towards that given by channel inversion—this is reflected in figure 5. Indeed, it is straightforward to show that, as $S/N_0 \rightarrow \infty$, the approximately optimal pre-compensation coefficients tend asymptotically towards those given in (7).

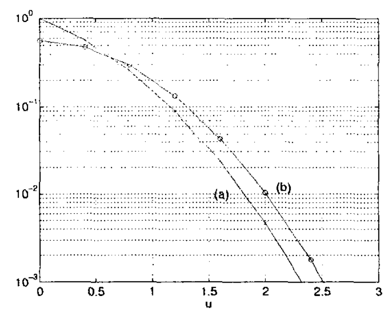

Figure 6. The complementary error function: (a) Actual value $\mathrm{erfc}\,(u)$; **(b) Approximation** $exp\left(-u^2\right)/\sqrt{\pi}$

4. Implementation within a WLAN

It is possible that the pre-compensation technique outlined above could be applied to the European HiperLAN/2 5 GHz Wireless Local Area Network (WLAN) standard [2]. This uses a centrally-controlled TDD approach to Medium Access Control (MAC) and, typically, will be configured as a cellular radio access network, with mobile terminals connected to wired infrastructure via fixed access points.

We assume that pre-compensation would be implemented within access points rather than mobile terminals (from a processing and complexity point of view, this arrangement seems logical).

The HiperLAN/2 physical layer allows the use of seven transmission modes, which operate with various forward error correction (FEC) coding rates and modulation formats (BPSK, QPSK, 16-QAM and 64-QAM). For coherent detection, accurate CSI estimation is required, and this is facilitated by the regular transmission of a known preamble sequence. Although the derivation above is for QPSK modulation, the extension of the pre-compensation routine for the other modulation modes is straightforward. In the case of 16-QAM, for example, (9) becomes

$$P_e \simeq \frac{1}{N} \sum_{k=0}^{N-1} \frac{3}{8} \mathrm{erfc} \left(C_k A_k \sqrt{\frac{S}{10N_0}} \right) \qquad (19)$$

The performance of the pre-compensation routine is ultimately limited by imperfections in the CSI estimation. Within a HiperLAN/2 access point, not only is this corrupted by AWGN from the reception of up-link preambles, but also by the fact that the channel varies with time. Thus in the delay between the reception of an up-link preamble from a mobile terminal and the subsequent (pre-compensated) down-link transmission to that terminal, the channel state can change. Measurements at 5.2 GHz [3] have shown that, assuming regular up- and down-link communication within the HiperLAN/2 MAC, the coherence time for indoor channels is typically long enough to ensure the validity of the CSI. Nevertheless, the affect of CSI degradation on the pre-compensation routine is clearly an issue that requires more attention.

Clearly, there are a number of other factors, not investigated here, that would influence the usefulness of the pre-compensation routine within HiperLAN/2. These include: how to measure the noise spectral density at the mobile terminal receiver (the pre-compensation algorithm requires knowledge of this), and how to communicate it to the access point; the performance of the routine under more realistic channel fading conditions; and what affect the pre-compensation has on the performance of the FEC coding, particularly if the CSI is used when computing branch metrics within the decoding process.

5. Summary

This paper has outlined a method of pre-compensation for OFDM systems in which knowledge of the CSI is available at the transmitter. Although the proposed algorithm gives pre-compensation coefficients that are sub-optimal, it has been shown that they perform well compared with the optimal coefficients. Some issues relating to the implementation of the routine within a WLAN have been highlighted, and areas requiring further research have been suggested.

6. Acknowledgement

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