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# SURVIVAL ANALYSIS MODELLING WITH HIDDEN CENSORING

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ABSTRACT. There are well-established survival analysis methodologies for data sets which are complete, with accurate information on censoring. But what if they are not complete? In this paper we consider how to analyse cases where “hidden censoring” occurs, where individuals have effectively left the study but the hospital is unaware of this. We develop a new Markov chain-based methodology for generating survival curves and hazard functions, and demonstrate this using a breast cancer dataset from the Kurdistan region of Iraq.

## 1. INTRODUCTION

The modelling of survival has a long history, and there are well-established methodologies for estimating survival probabilities for individuals with a given medical condition (Cox and Oakes, 1984; Crowder, 2012; Crowder et al, 1991; Lawless, 2003). Essentially identical methods are also used for modelling the failure of items, such as components in machines (see e.g. Barlow and Proschan, 1975; Bedford and Cook, 2009). The most fundamental functions used in survival analysis are the *survivor function*, which is the probability that an individual survives beyond time  $t$ , and the *hazard function*, which is the risk of death (per unit time). Thus if our individual has lifetime distribution  $T$ , following standard terminology (see e.g. Chapter 2 of Cox and Oakes, 1984), the survivor function is

$$(1) \quad S(t) = P[T > t]$$

and the hazard function is

$$(2) \quad h(t) = -\frac{\frac{d}{dt}S(t)}{S(t)}.$$

Expressing the relationship in equation 2 the other way round, we obtain

$$(3) \quad S(t) = e^{-\int_0^t h(u)du}.$$

These fundamental properties can be estimated directly from data in a number of ways, but perhaps the simplest and most robust is the Kaplan-Meier estimator, which estimates the hazard function, using the discrete hazard function

$$(4) \quad h_j = \frac{d_j}{n_j},$$

where  $d_j$  is the number of observed deaths within a particular (unit) interval, and  $n_j$  is the number of individuals at risk at the start of that period. The survivor function is then estimated by

$$(5) \quad \hat{S}(t) = \prod_{j=1}^t (1 - h_j).$$

This method automatically takes into account any data censoring, where an individual is known to leave the study at a particular time, by reducing the number at risk  $n_j$  by the number of censored individuals  $c_j$ . Thus we update the number at risk as follows:

$$(6) \quad n_{j+1} = n_j - d_j - c_j.$$

A great advantage of this method is that reliable estimates can be obtained without making assumptions about the underlying distribution  $T$ . The only information that we need to apply this methodology is, for all times where we take measurements, knowledge of the values of the total number of individuals at risk at the start of the time period and the total number of deaths within the time period, i.e. all values of  $n_j$  and  $d_j$ . This then enables robust comparisons between survival curves from different studies, perhaps between different types of treatment, different times or different countries, and helps clinicians to assess the effectiveness of different approaches. Significant censoring can be factored in as described above without problems, provided that records are sufficiently good to know when contact with patients has been lost. The focus of this paper is how to tackle problems when you do not have this knowledge, and significant “hidden” censoring occurs unknown to the researchers, using a real example as a case study. In Section 3 we shall present two models, one without and one with censoring, which address this problem. In fact our models, in particular the second model, do make parametric assumptions, based as they are on an underlying Markov chain model. Nevertheless the models, in particular the simpler first model, will be robust to (at least certain types of) deviation from them.

## 2. A KURDISH BREAST CANCER DATASET

Breast cancer is the most common cancer in the West, affecting a large number of women (and men) at some point in their lives (WHO Global

Burden of Disease, 2008). There are various risk factors, such as obesity, age and hormone replacement therapy during menopause (Lan et al., 2013; Robb et al., 2007; Rudat et al., 2013). Breast cancer has been well studied and treatments are becoming more sophisticated and successful (De Santis et al., 2014). The American Cancer Society reports that around 250,000 breast cancer cases are diagnosed in the U.S. per year, and of these, almost 10 percent affect women under age 45. While this percentage may sound relatively insignificant in comparison to the total number of women diagnosed annually, it is a noteworthy ratio particularly when compared to other cancers. In women under 40 breast cancer is the leading cause of cancer deaths (Ries et al., 2007). While the UK currently has the 11th highest breast cancer rate with 89.1 of every 100,000 women every year expected to develop breast cancer (NHS Choices, 2011). Breast cancer is also becoming a more common disease in the developing world (Ozmen, 2006). In particular, we are interested in the incidence of breast cancer in the Kurdistan Region of Iraq. This has not received a lot of detailed attention; examples include Majid et al. (2009), Othman et al. (2011), Majid et al. (2012) and Shabila et al. (2012) (see also for example Alwan et al. (2000), Hughson (2012), Hussaion (2009) for other work on breast cancer in the Kurdistan region, and Dey et al. (2010), Rennert (2006) and Sughayer et al. (2006) for work on the incidence of breast cancer elsewhere in the wider region). These papers addressed various important questions, especially related to the incidence of breast cancer, but so far no detailed survival analysis has been carried out for the Kurdish region of Iraq.

We consider a data set of breast cancer patients from Nanakaly Hospital. Nanakaly Hospital is a public sector hospital in Erbil, the capital of the Kurdistan region of Iraq, which was established in 2004 and is funded by the Kurdistan Government. It is a centre concerned with all types of cancer. The hospital registry department collects data regarding the type of cancer and the age of the patients, the time of diagnosis and (if appropriate) the time of death, as well as personal details, and these are registered on the statistical database. We have access to the most recent data on breast cancer, minus the personal information.

Detailed times of death were provided, with censoring only at the end of the study period on 1st June 2014, see Figure 1 for an illustration. Analysing the above data using SPSS, provided the Kaplan-Meier survival curve in Figure 2. The function flattens out to effectively a horizontal line, indicating a hazard rate tending to zero. This is clearly not a realistic survival curve. For comparison, a survival curve for a set of breast cancer data from Schumacher et al. (1994) is shown in Figure 3. The problem with the survival curve from Figure 2 is that we calculated it on the assumption that all individuals other than those who died (or were censored by reaching the

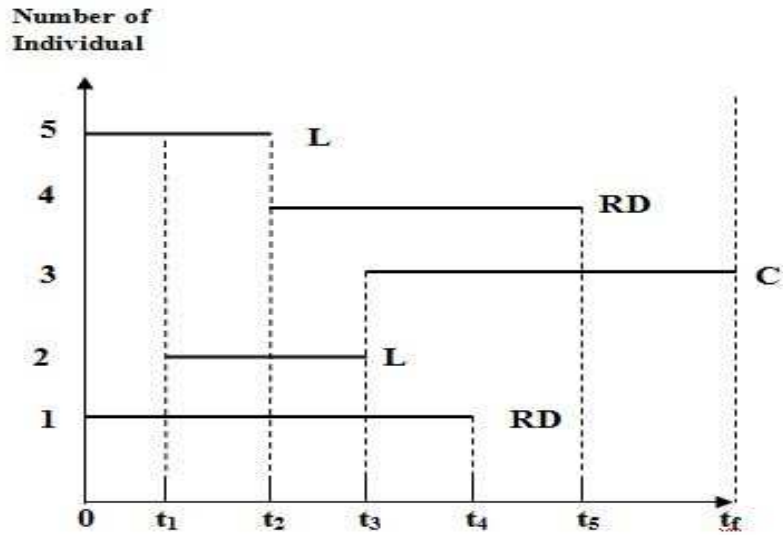


FIGURE 1. Illustrative plot of survival times including end of period censoring (C), recorded death (D) and hidden censoring, individuals unknowingly lost to the study (L), for the Kurdish data from Nanakaly Hospital.

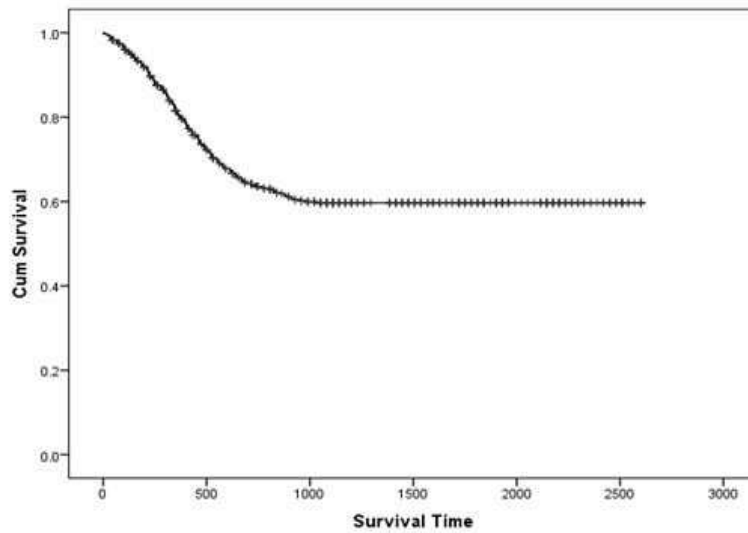


FIGURE 2. The original survival curve for the Kurdish data from Nanakaly Hospital.

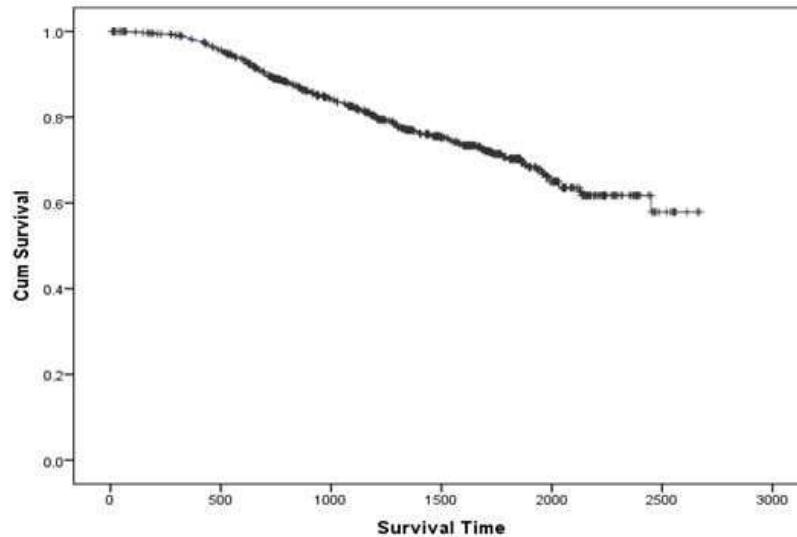


FIGURE 3. Survival curve for the German data from Schmoor et al. (1996) and Schumacher et al. (1994).

end of the study period) were still active in the study, but in fact individuals often did not return to the hospital after initial treatment, and there are no clear records of when the deaths of these individuals occur, or of which individuals these are. Thus there is some secret censoring that we do not have knowledge about. We can think of this to mean that whilst the values of  $d_j$  are accurate, the values of  $n_j$  are not, and we are (after some time, greatly) overestimating them.

This paper will present two related methods for overcoming this problem, and obtaining a realistic survival curve for the Nanakaly data. The data itself will be analysed more fully in a separate paper.

### 3. MARKOV MODELS

**3.1. A Markov model without censoring.** We shall first introduce a continuous time Markov model without overt censoring (Cox and Miller, 1965; Grimmett and Stirzaker, 2001). In our data the only observed censoring was caused by the end of the study period, although as patients were being recruited all the time during the period, the censoring time could be small, and such censoring could occur for any time less than 2602 days, the time from the earliest record considered to the end of the study period. Thus all of the individuals censored and removed from the number at risk in the standard way following equation (6) (see equation (15)) was censoring of this type.

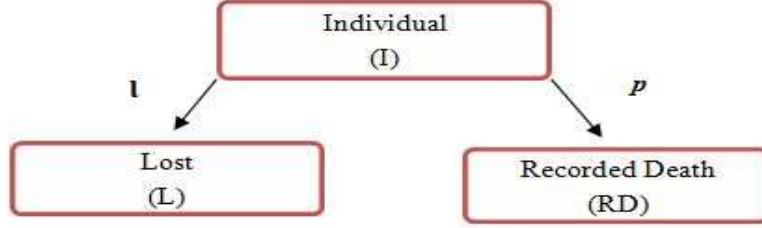


FIGURE 4. The Markov survival model without censoring

Consider a population of individuals in three categories; either at risk (I), died (D) or who have left the study (without our knowledge), which we shall call “lost” (L). Individuals simply move from state  $I$  to the other two states at constant rates  $l$  to  $L$  and  $p$  to  $D$ . We thus have a population as described by Figure 4. We denote the proportion of individuals in states  $I$ ,  $L$  and  $D$  at time  $t$  by  $P_I(t)$ ,  $P_L(t)$  and  $P_D(t)$  respectively. State  $I$  cannot be entered, and is left at constant rate per individual  $l+p$ , thus we obtain the differential equation (see e.g. Chapter 8 of Haigh, 2002)

$$(7) \quad \frac{d}{dt}P_I(t) = -(l+p)P_I(t).$$

Since at time 0 every individual is in the “at risk” category, so that  $P_I(0) = 1$ , we obtain

$$(8) \quad P_I(t) = e^{-(l+p)t}.$$

Since state  $D$  is entered at rate  $pP_I(t)$ , we also have

$$(9) \quad \frac{d}{dt}P_D(t) = pP_I(t),$$

which using equation (8) and the fact that  $P_D(0) = 0$  yields

$$(10) \quad P_D(t) = \frac{p}{l+p} (1 - e^{-(l+p)t}).$$

Since  $P_L = 1 - P_I - P_D$ , we obtain

$$(11) \quad P_L(t) = \frac{l}{l+p} (1 - e^{-(l+p)t}).$$

Suppose that, as in the original survival plot, we consider the data without realising that the category  $L$  exists. We can see from equations (10) and (11) that

$$(12) \quad \frac{P_L(t)}{P_D(t)} = \frac{l}{p}.$$

Let us denote the ratio  $l/p$  by  $\alpha$ , which is the number of “lost” individuals per death. As  $t \rightarrow \infty$  there are no more observed deaths, as all individuals who have not already died have in fact been “lost”. We can obtain an estimate of  $\alpha$  by the ratio of the number of individuals apparently still in the study  $n_\infty$  and the number who have been observed to die  $D_\infty$ , yielding

$$(13) \quad \hat{\alpha} = \frac{n_\infty}{D_\infty}.$$

Thus an easy way to construct a survival curve from the data is to adjust the number at risk, instead of using the formula from equation (6), to instead use

$$(14) \quad \hat{n}_{j+1} = \hat{n}_j - d_j - \hat{\alpha}d_j - c_j,$$

which implies that

$$(15) \quad \hat{n}_{j+1} = \hat{n}_j - \frac{D_\infty + n_\infty}{D_\infty}d_j - c_j.$$

We can see that using this updating method  $\hat{\alpha}$  from equation (13) is the appropriate estimate to use, by observing that after all observed deaths have been accounted for, using equation (15) an extra  $n_\infty$  individuals will have been removed from the at risk category. This is precisely the number of individuals which we noted had been “lost”.

The survivor function is then just calculated using equation (5), with  $h_j$  calculated using

$$(16) \quad h_j = \frac{d_j}{\hat{n}_j}.$$

This method will work well even if the underlying death rate and the rate of loss of individuals vary in time, as long as they vary in proportion with each other. If this is not the case, there would be a bias in the estimates of the hazard function  $h_j$  that we obtain. To tackle this problem we would need to have some more specific information about the way in which the loss of individuals into the  $L$  category differed from the rate of deaths, and this is likely to be problem specific, for example depending upon hospital procedures, and so we do not discuss any specific methodological ideas in this paper.

As mentioned in Section 2, there is overt censoring in this population caused by the end of the study period. This creates a potentially significant problem, because even the “lost” individuals are censored in this way, and so without adjustment the number of individuals at risk can be underestimated due to double counting (effectively the same individual being lost and then censored can be removed twice). This in turn leads to a lower estimate of  $\alpha$  than would otherwise be the case. In the alternative model below



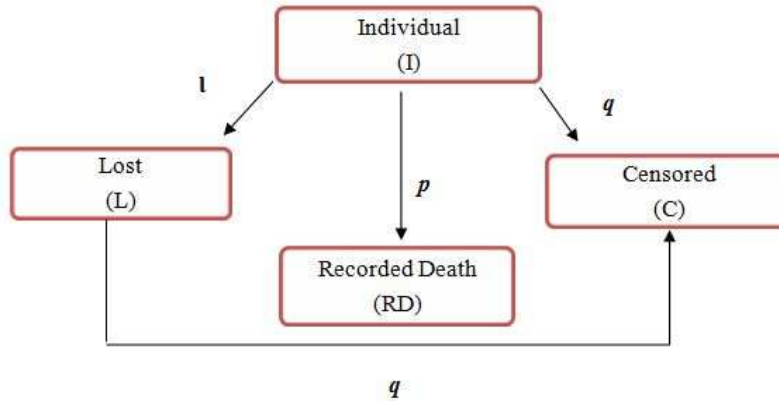


FIGURE 5. The Markov survival model with censoring

we shall see a different, higher, estimate of  $\alpha$ , and there is some discrepancy between the estimated survival curves of the two methods as a result, since the above errors cause an overestimate of the survival curve for the first model. We shall discuss this issue later (see from Figures 6 and 7).

**3.2. A Markov model with censoring.** More generally we would like to allow for observed censoring as well as hidden censoring within our model. Observed censoring occurs either when an individual is still alive at the end of the study period, or where they leave the study before the end, but the hospital is aware of it. Hidden censoring occurs when they leave the study but the hospital is not aware of it. Thus we now add an extra “censored” category  $C$  to our model, where individuals move from  $I$  to  $C$  at rate  $q$ . Importantly, individuals also move from the lost category  $L$  to  $C$  at the same rate  $q$ . This is clearly appropriate for our dataset, since the only overt censoring is due to the end of the study, and thus any individual will reach this at the same time, whether in category  $I$  or  $L$ . We thus now have a population as described by Figure 5. We note that for individuals censored because we know that they have dropped out of the study prior to the end time, it would seem reasonable to assume that these and the “lost” individuals would be entirely separate, and so that the transition rate  $q$  from state  $L$  to state  $C$  would be absent.

Following the transitions in Figure 5, there is a constant rate of departure per individual from state  $I$ , so that we have

$$(17) \quad \frac{d}{dt}P_I(t) = -(l + p + q)P_I(t),$$

and similarly to before, we obtain

$$(18) \quad P_I(t) = e^{-(l+p+q)t}.$$

We also still have

$$(19) \quad \frac{d}{dt}P_D(t) = pP_I(t),$$

which using equation (18) and the fact that  $P_D(0) = 0$  yields

$$(20) \quad P_D(t) = \frac{p}{l+p+q} (1 - e^{-(l+p+q)t}).$$

For the lost category  $L$ , we have entry to the state at rate  $lP_I(t)$  and departure from the state at rate  $qP_L(t)$

$$(21) \quad \frac{d}{dt}P_L(t) = lP_I(t) - qP_L(t).$$

Using standard methods (see e.g. Chapter 8 of Haigh, 2002), together with the fact that  $P_L(t) = 0$ , yields

$$(22) \quad P_L(t) = e^{-qt} \frac{l}{l+p} (1 - e^{-(l+p)t}).$$

Finally, since  $P_C = 1 - P_I - P_D - P_L$ , we obtain

$$(23) \quad P_C(t) = \frac{l+q}{l+p+q} (1 - e^{-(l+p+q)t}) - e^{-qt} \frac{l}{l+p} (1 - e^{-(l+p)t}).$$

It is clear that the death rate for individuals in the at risk category  $I$ , i.e. the correct hazard function, is simply

$$(24) \quad h_c(t) = p.$$

Using equation (3), the true survivor function for our model is thus simply

$$(25) \quad S_c(t) = e^{-pt}.$$

Since we perceive individuals from class  $L$  as being in category  $I$  too, we observe an apparent hazard function of

$$(26) \quad h_a(t) = \frac{P_I}{P_I + P_L} p.$$

Substituting the appropriate terms from equations (18) and (22) and rearranging gives

$$(27) \quad h_a(t) = \frac{p(l+p)}{p + le^{(l+p)t}}.$$

Following equation (3), the apparent survivor function is thus

$$(28) \quad S_a(t) = e^{-\int_0^t h_a(u) du},$$

which rearranges to

$$(29) \quad S_a(t) = \frac{l + pe^{-(l+p)t}}{l + p} = \frac{\alpha + (e^{-pt})^{1+\alpha}}{1 + \alpha}.$$

Thus we can express  $S_c(t)$  in terms of  $S_a(t)$  as follows:

$$(30) \quad S_c(t) = ((1 + \alpha)S_a(t) - \alpha)^{1/(1+\alpha)}.$$

The apparent survival curve  $S_a(t)$  from Equation (29) flattens out to a limiting value  $\alpha/(1+\alpha)$ . We can thus estimate  $\alpha$  by equating this theoretical limit with the observed limiting value of the survival curve from the data which we shall denote by  $s_\infty$ , giving:

$$(31) \quad \tilde{\alpha} = \frac{s_\infty}{1 - s_\infty}.$$

This yields the conversion formula from the apparent to the corrected survival curve as

$$(32) \quad S_c(t) = \left( \frac{1}{1 - s_\infty} S_a(t) - \frac{s_\infty}{1 - s_\infty} \right)^{1-s_\infty}.$$

We note that our final solutions for the survivor function and the hazard function do not contain  $q$  at all. In fact, this means that these solutions are unaffected if  $q$  is replaced by a time-dependent function  $q(t)$ . This is important, as the censoring time is in reality directly related to the rate of recruitment into the study, which may be influenced by non-random factors. It also means that we can apply this method to cases without censoring as in the previous method of Section 3.1. We also note the discrepancy between our two estimates of  $\alpha$ . In general when overt censoring occurs,  $\hat{\alpha}$  will be smaller than  $\tilde{\alpha}$ , because the first model neglects the influence of censoring in the estimation procedure.

#### 4. APPLYING OUR MODELS TO THE KURDISH DATA

From the Kurdish data we obtained the following values:  $n_\infty = 232$ ,  $D_\infty = 240$  and  $s_\infty = 0.5969$ , which gives the two alternative estimates of  $\hat{\alpha} = 0.9667$  and  $\tilde{\alpha} = 1.4807$  lost individuals per death. Applying our method from Section 3.1 gives the adjusted survival curve from Figure 6. Using the alternative method from Section 3.2 gives the adjusted survival curve from Figure 7.

We can see that the two alternative survival curves generated by our methods now resemble the survival curve from the German data from Figure 3. The curves in our case are clearly lower than that of the German data, indicating poorer survival rates among the Kurdish patients. There are a number of reasons for this, including later diagnosis, less efficient treatment regimes, and different patient demographics, which we will consider in a

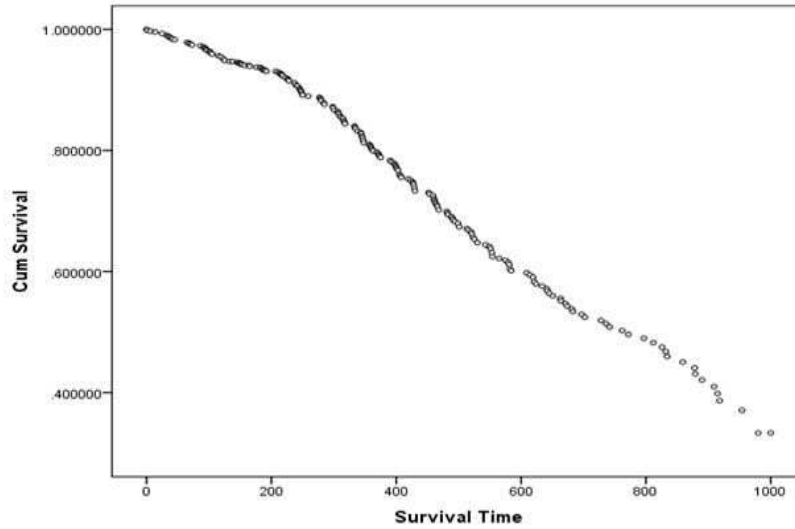


FIGURE 6. An adjusted survival curve for the Nanakaly data using the method without censoring from Section 3.1.

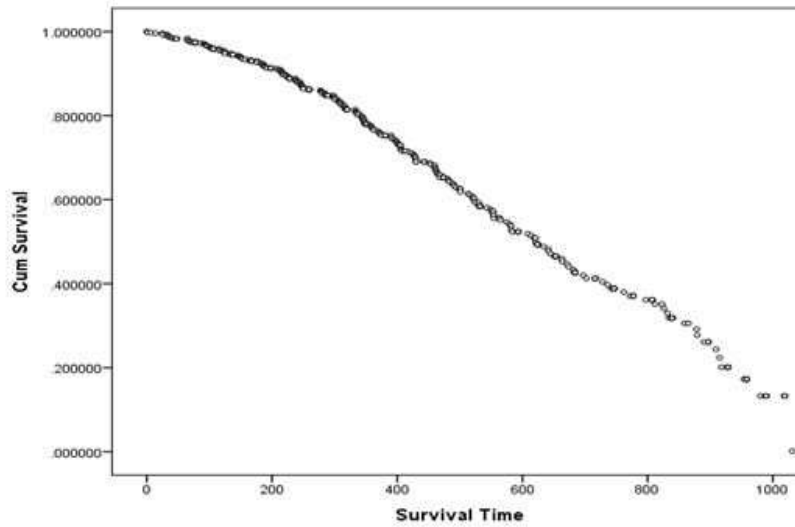


FIGURE 7. An adjusted survival curve for the Nanakaly data using the method with censoring from Section 3.2.

later paper. Comparing the two curves from Figures 6 and 7, we see that initially the two curves are roughly the same, but for later times, the curve in Figure 6 is clearly above that in Figure 7. We should also note that our methods are likely not be very accurate near the end of the curves, i.e. when the last of the recorded deaths occur. Thus in the case of the Nanakaly data, the curves beyond about 700 days are likely to be inaccurate.

We should also note that the above methodology can be applied to any survival curve, so that if we have a survival curve from a subset of the data, or for patients with particular properties, then the method of adjusting the original survival curve is completely unchanged.

## 5. SIMULATIONS

In this section we consider simulations to investigate the validity of our modelling procedure. We consider the example German data from Figure 3, as we have an accurate survival function for this. For each simulation, we chose a distribution and simulated each individual from the German data being “lost” following this distribution. Thus if death happens before the individual is lost, we observe the death, but if the individual is lost first we assume that they are still in the study, and do not observe their death, if it occurs. This thus replicates what happens in the Nanakaly data, and the situation that we are modelling.

The models that we have considered are Markov with constant rate, which would yield an exponentially distributed time of loss. We considered various values of this distribution. One set of simulations considered a mean loss time of 2000 days. Given the length of the German study, this accounted for quite a significant loss of data. One example run of this is shown in Figure 8 where the apparent survival probability after 2000 days has only fallen to approximately 0.8 instead of the true value of just over 0.6 as a result. The survival curves generated for our two models are shown in Figures 9 and 10 respectively. We can see that in both cases, the models significantly correct the survival function from the apparent survival function shown in Figure 8. The first model gives a somewhat conservative correction, which is higher than the true survival function in Figure 3. As explained in the final paragraph of Section 3.1, this is because of the double counting of lost and censored individuals. The estimate of the second model is clearly better, with a closely comparable survival curve. These curves are typical of different simulations with the same mean loss time.

Exponential distributions with higher means yield even better results, as the level of loss is diminished, and so the amount of adjustment that needs to be carried out through our procedure is reduced. For exponential distributions with lower means (in particular below 500 days), and thus very large

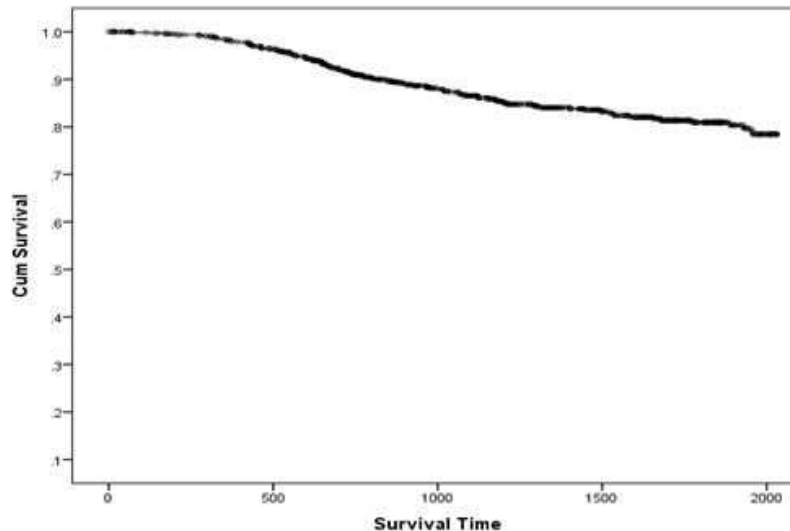


FIGURE 8. The survival curve for a sample simulation of loss from the German data, where loss of individuals occurs following an exponential time with mean 2000 days.

data loss, figures became increasingly less accurate, as a larger proportion of deaths were missed, and this led to overestimates of survival rates.

We also considered non-exponential distributions, for example Gamma  $(2, \theta)$ , for patient loss. When this led to a large number of lost individuals (for sufficiently high means this did not, and thus as above the corrections were not large and were accurate) we would expect our model to perform worse in such circumstances, as this would indicate that the underlying Markov assumption was not correct. This was indeed the case, although the models still corrected the false apparent curves to significant effect, and as for exponential distributions with small means, the effect was generally to produce slightly conservative survival functions, which overestimated the true survival curve.

Thus we see that our models perform well in many circumstances, and even when less accurate, are always an improvement on considering the apparent survival curves from the unadjusted data.

## 6. DISCUSSION

In this paper we have developed a new method for performing a survival analysis on a set of data where there are important unknown factors; namely secret censoring of the data, so that the number of individuals apparently at

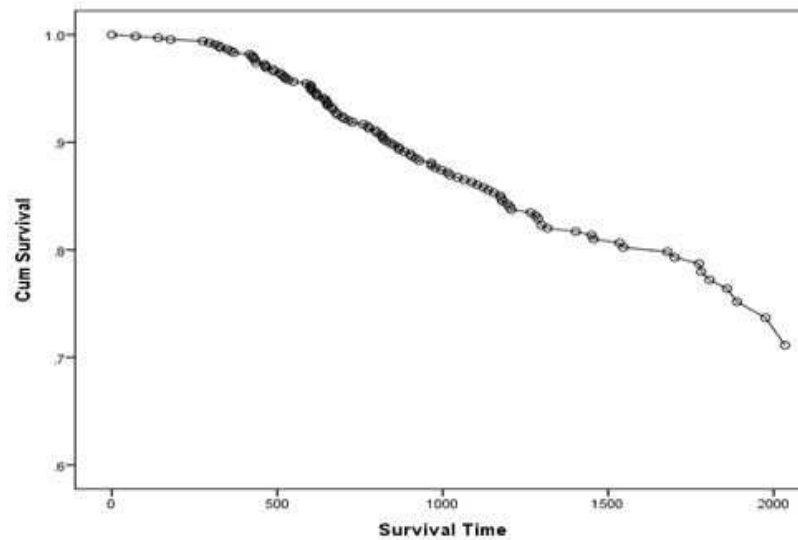


FIGURE 9. An adjusted survival curve for the Gemran data with simulated loss following an exponential distribution with mean 2000 days, using the method without censoring from Section 3.1.

risk is greater than those actually at risk. In particular we have shown how to adjust a Kaplan-Meier analysis to find a survival curve in such circumstances, and also shown how to estimate a true hazard (survivor) function to the biased one obtained directly from the data.

A limitation of our methodology is that it is based upon a Markov chain, and so transition rates are assumed constant over time. In fact some relaxation of these assumptions, such as making the censoring rate  $q$  time dependent, does not affect the model accuracy. Similarly, allowing the parameters  $l$  and  $p$  to be time dependent does not affect the model, provided that they vary with  $\alpha = l/p$  constant. This however is not always reasonable, and it is possible to envisage some situations where this is far from being the case. In such circumstances our predictions would not be reliable. Similarly, if different groups of individuals have different rates with different  $l/p$  ratios, this might also affect the results. We claim, however, that in circumstances where the problems outlined occur, our model is a good first step, and a considerable improvement on making no adjustment. This is demonstrated in Section 5 where we simulated the loss of individuals from the German data set, and compared the resulting survival curves from our models with those from the original data.

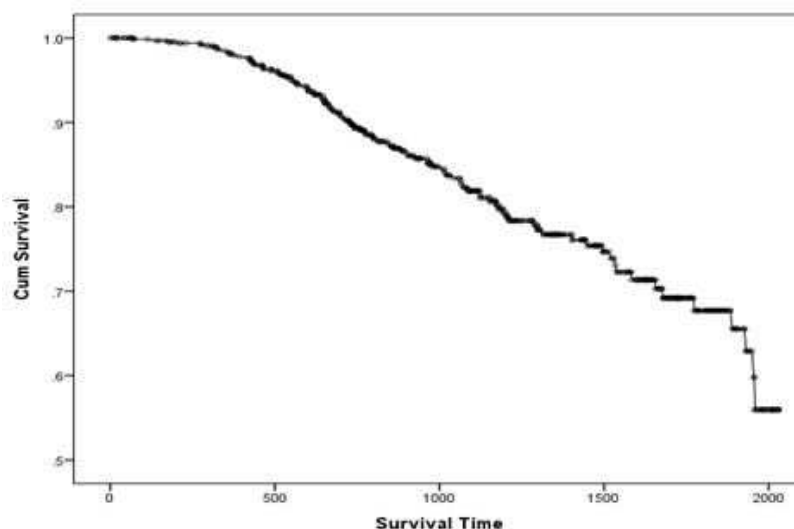


FIGURE 10. An adjusted survival curve for the German data with simulated loss following an exponential distribution with mean 2000 using the method with censoring from Section 3.2.

This leads on to the question, how prevalent will the problems that we have described be? With sufficiently accurate records and follow-up of individuals they will not occur, and of course a better solution than applying our methods is to have these processes in place. Nevertheless, in reality they often will not be. This is particularly the case in regions with a history of upheaval and developing medical services. It can be argued that these are precisely the regions which most need accurate survival models and so the application of our methods can be of significant value.

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