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Chapter 6

THE BOMB PARTY PROBABILITY ILLUSION

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Imagine that you are invited to a party where all six guests are invited to pick a cracker from a bran tub. Your host tells you that there are six crackers buried in the bran tub and that five of the crackers each contain checks for extremely large sums of money but the remaining cracker contains a bomb that will kill the person that pulls it. This is the predicament that the characters in Graham Greene's (1980) novel *Doctor Fischer of Geneva or The Bomb Party* find themselves in.

Fascinated by the seemingly limitless greed of the rich, Dr. Fischer, who had made a fortune by inventing a toothpaste, invites extremely wealthy guests to a series of dinner parties where he derives huge amusement from observing that they appear consistently willing to suffer all manner of indignities in order to receive expensive presents that they, being so rich, could easily afford to buy for themselves. The bomb party is his final ultimate test of the greed that, so far, had proved greater than any humiliation he had been able to invent.

On a previous occasion the guests had been obliged – on pain of forfeiting their presents – to consume two servings of unsweetened cold porridge. On another occasion they were presented with live lobsters and bowls of boiling water – each guest had to catch and cook a lobster. The horrific details of another dinner are, somewhat ominously, only hinted at: one guest refers to it as the "quail party" and admits it rather upset another of the guests who was very attached to birds.

The presents are usually tailored to each guest and are worth a substantial amount of money. However, the rules include complete submission to the humiliations of Dr. Fischer, which always include barbed verbal taunts that focus on each guest's failings or insecurities. Sometimes the prize will also turn out to be a humiliation – thus at one dinner a particularly vain guest receives a framed photograph of himself; at another dinner Mr. Kips – who suffered a physical disability – receives a leather-bound copy of a cartoon book depicting Mr. Kips as the main character, and cruelly mocking his affliction, that Dr. Fischer had specially commissioned to be produced and also arranged to be published.

It is easy to see Dr. Fischer's dinner parties as behavioral experiments – indeed Dr. Fischer even refers to the dinners as experiments and the whole enterprise as "research". The guests are the experimental subjects who are invited to engage in humiliating activities in order to establish a measure of their valuation of their avarice – or, more neutrally, their willingness to pay for the incentives.

At the bomb party one guest, Monsieur Belmont, shocked by what he has been told, says that if anyone were to be killed then it would be murder. But Doctor Fischer argues that it would not be murder – or even suicide – but more like Russian roulette. He adds that anyone who does not wish to play must leave at once. At this point another guest, Mr. Kips, announces that he will not play and prepares to leave. In spite of Doctor Fischer telling him that there are five chances to one in his favour, Mr. Kips departs saying that he considers gambling for money highly immoral.

Then one of the five remaining guests stands and, while pausing to gather courage to approach the barrel is beaten to it by another – an American woman with blue hair called Mrs. Montgomery – who yells, "Ladies first," runs to the barrel and pulls a cracker. A check for two million Swiss francs pops out. Greene writes: "… Perhaps she had calculated that the odds would never be as favorable again. Belmont had probably been thinking along the same lines, for he protested, 'We should have drawn for turns.'" Belmont presumably felt cheated because, as now there are only five crackers left, the chance of being blown up is increased to one fifth – whereas before it was one sixth.

Nonetheless, after hesitating, Belmont pulls a cracker, and also gets a check. Greene writes: "He had calculated the odds – he had been right to bet." However, the odds on being destroyed by the next cracker have increased to one quarter causing further consternation among those yet to take their turn. Dr. Fischer teases the remaining guests, warning them that the odds are narrowing on their survival.

The story continues: "'What about you Jones?' Doctor Fischer said. 'The odds are narrowing.'

"'I prefer to watch your damned experiment to the end. Greed is winning isn't it?'

"'If you watch you must eventually play – or leave like Mr. Kips.'

"'Oh I'll play I promise you that. I'll bet on the last cracker. That gives better odds to the Divisionaire.'"

(Jones has been contemplating suicide for several days and also pities the Divisionaire, who, despite his high military rank, has never heard a shot fired in anger and has been cruelly taunted by Doctor Fischer for having no record of any act of bravery. Earlier in the story the Divisionaire had complained that he still bore the scar from when a lobster nipped his finger at the lobster party. According to Dr. Fischer this was the only wound in action the Divisionaire ever received.)

Then the third cracker is pulled by another guest and also turns out to contain two million francs. This leaves three crackers and two guests, Jones and the Divisionaire. Jones tries to encourage the Divisionaire, telling him: "I suggest you go first, Divisionaire. I'm not in need of money and it gives you the better odds." Doctor Fischer tells the Divisionaire: "If you act quickly the odds are two to one in your favour." However the Divisionaire realizes that he is too afraid – too cowardly – to take the risk: "I haven't the courage. I should have gone to the barrel first, when the odds were better."

The comments of the characters at the bomb party imply that the longer you wait your turn to pull a cracker the more dangerous the risk becomes. Is this correct reasoning? What can be done to maximize the chances of winning two million francs while at the same time minimizing the possibility of being blown up?

While Doctor Fischer is correct when he says that the odds on winning are narrowing after the first two crackers have been pulled, this is conditional on neither of them containing the bomb. If the bomb had been detonated by the first guest to take the risk, the others could have pulled the remaining crackers secure in the knowledge that they would benefit financially rather than be exploded. The longer you wait your turn the odds on survival shorten *providing the bomb cracker is not pulled*. However, a direct corollary is that the longer you wait the more likely it is that someone else will pull the cracker before you.

Doctor Fischer and his guests appear not to have taken this into account in their reasoning. The first guest has a 1/6 chance of pulling the lethal cracker and a 5/6 chance of surviving. Before the first guest pulls her cracker, the second guest has a 1/5 chance of survival if the first guest does not blow up; the chance of the second guest surviving are therefore 1/5 x 5/6 = 1/6. Similarly the chances for the third guest are 1/4 x 4/5 x 5/6 = 1/6. In fact, for any number of guests drawing any number of crackers in which any numbers of bombs have been concealed, the position of a guest in the queue does not affect the probability of survival.

We decided to carry out some investigations to try to establish the extent to which the reasoning shown in the novel might be prevalent in others.

STUDY 1: PERCEPTIONS OF THE BOMB PARTY

How beguiling is the notion that there is something to be gained by choosing a particular position in the queue? We investigated this with a short questionnaire describing the situation to a group of 77 psychology undergraduates and asking them to imagine that they were guests at a bomb party before any crackers had been pulled. Our description of the bomb party deliberately tried to re-create the situationdescribed in the novel:

> In *The Bomb Party by* Graham Greene, Dr. Fischer tests the greed of his six dinner party guests by inviting them to draw a cracker from a barrel and pull it. He tells them that five of the crackers contain checks for two million Swiss francs but the other contains a bomb. If they pull a cracker with a check they can keep the money, but if they pull a cracker with a bomb their life will end.
>
> All of the guests agree to play and promise to pull a cracker but they start to argue about who should go first. Then one guest suddenly rushes to the barrel, pulls out a cracker, pulls it, and gets a check.
>
> One guest complains that now the odds are worse. Originally there was a 1/6 chance of blowing up, now it is 1/5. Then another guest pulls a cracker and gets a check. Now there are only four crackers left making a 1/4 chance of blowing up.
>
> Imagine you are a guest who has agreed to play at a similar party, *before* any crackers have been pulled.

The questionnaire then first asked respondents which position at the bomb party they would prefer and why, and then, which has the best chance of surviving.

Table 6.1. Percentage of Responses to Each Position at the Bomb Party (Study 1)

QUESTION \ POSITION	1st	2nd	3rd	4th	5th	6th	"Any"	Equal	Don't know	«First or last»
Would you rather go 1st, 2nd, 3rd, 4th, 5th or last?	40 %	12 %	4 %	0 %	0 %	42 %	3 %	–	–	–
Which has the best chance of surviving?	48 %	0 %	3 %	0 %	0 %	19 %	–	23 %	4 %	3 %

As the responses shown in table 6.1 indicate, this study confirms that the illusion of an advantage in going first is held by many subjects; last is also a popular choice but the bottom row of the table shows that only a minority (23 %) correctly affirms that all positions are equally likely to draw the bomb. Almost half of the respondents thought that the first position offered the best chance while nearly a fifth thought that the last position was best.

There are also preferences for which position respondents would like to take at the bomb party which, somewhat curiously, do not quite accord with the perceptions of the position with the best chance of survival. For example, more than twice as many respondents want to go last (42 %) than believe that last offers the best chance for survival (19 %); fewer people want to go first (40 %) than see first as offering the best chance (48 %), and 12 % of respondents wanted to go second even though no-one thought that had the best chance of survival. Only 3 % of subjects said that they had no preference for a particular position in the queue.

The questionnaire also included an open-ended question that asked respondents to explain why they had chosen the position that they did. This enabled some scrutiny of the reasons underlying the different choices. Although first and last positions are roughly equally popular choices, different justifications were offered for each. When asked why they had chosen as they had, a common reason for those who preferred to go first was that they had a better chance of surviving. However, this was not a common reason for those who said that they preferred to go last. Those choosing to go last mentioned two main arguments. One was that if the bomb had not gone off, they would always refuse to actually pull a cracker and therefore they could guarantee their survival. The other common reason was that going last would ensure that one knew one's fate – you would know what would happen to you before you pulled the cracker. Note though that this defense overlooks the fact that you wouldn't know what happened to you until other guests pulled their crackers – though it highlights that the perceived *utility* of each position may not be the same and this may influence probability judgments for each position. A few (12 %) of respondents wanted to go second even though no-one thought that this offered the best chance of survival. Among those who chose to go second a common rationale was that they didn't have the courage to go first, but a 1/5 chance was still quite good.

STUDY 2: A DETERRENT FOR RUNNING AWAY

To avoid subjects feeling that they could increase their chances of surviving by going last and then running away if a cracker hadn't exploded, we conducted another study on 70 more psychology undergraduates, none of whom had participated in Study 1. The study was identical to Study 1 except that this time the questionnaire scenario added the sentence: "You *must* pull a cracker (Dr. Fischer will shoot anyone trying to run away)."

Table 6.2. Percentage of Responses to Each Position at the Bomb Party (Study 2)

QUESTION / POSITION	1st	2nd	3rd	4th	5th	6th	"Any"	Equal	Don't know	«First or last»
Would you rather go 1st, 2nd, 3rd, 4th, 5th or last?	50 %	4 %	1 %	0 %	1 %	33 %	12 %	–	–	–
Which has the best chance of surviving?	41 %	0 %	0 %	0 %	0 %	17 %	–	33 %	7 %	1 %

The results are very similar to those for Study 1 except that now the last position is, understandably, somewhat less popular. A rather larger proportion of subjects now correctly affirm that the chances of survival are equal for all positions but this is still a minority of respondents.

STUDY 3: RESPONSES TO A FREQUENTIST BOMB PARTY

Gigerenzer (e.g., 1994) and Gigerenzer and Hoffrage (1995) argue that the difficulties that people have with probabilities vanish when they are invited to consider probability in terms of frequency. According to this argument people are not very well adapted to reasoning about the relative likelihood of single events, but they can deal effectively with uncertainty when invited to consider it in terms of natural frequencies.

To explore this notion in the context of the bomb party, we developed a frequentist version of the bomb party. Subjects were told (*after* completing the original questionnaire used in Study 1) that Dr. Fischer had, over the years, held many bomb parties and that he had kept careful records of the numbers of people that had exploded in each position in the queue. Subjects then saw six different graphical representations of the possible accumulated distributions of outcome and asked to select the graph that would be most like Dr. Fischer's (See figure 6.1). Finally subjects were asked to think again about which position they thought would have the best chance of surviving if they were at a bomb party.

Table 6.3 shows that the initial presentation of the bomb party (as in Study 1) produces very similar results to those in Studies 1 and 2. The popular positions are first and last and a minority of subjects (26 %) considers that all positions are equally likely to survive prior to subjects contemplating the frequency distributions; 41 % think that the first person has the best chance to survive and 27 % think the last person is best placed. On viewing the frequency graphs, few subjects (11 %) chose the uniform distribution (41 % chose E; 22 % chose F).

The effects of contemplating the graphs were modest; although 11 % fewer subjects thought that first position was best, there was only a slight increase in the numbers perceiving that all positions were equally likely to survive (from 26 % to 30 %). Krauss and Wang (2003) also reported difficulties in debiasing people's illusions with a frequency version of the Monty Hall problem – only combining of interventions produced a significant improvement – as with the present study, they only obtained modest effects of individual variations.

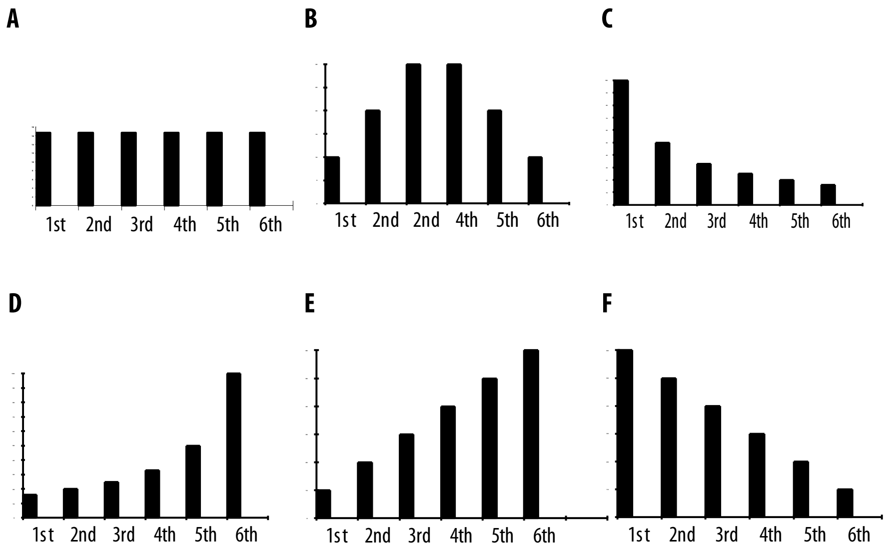

Figure 6.1. Possible Frequency Distributions for Dr. Fischer's Bomb Parties as Presented to Participants in Study 3.

Table 6.3. Percentage of Responses to Each Position at the Bomb Party

QUESTION / POSITION	1st	2nd	3rd	4th	5th	6th	"Any"	Equal	Don't know	«First or last»
Would you rather go 1st, 2nd, 3rd, 4th, 5th or last?	46 %	9 %	1 %	0 %	1 %	38 %	4 %	–	–	–
Which has the best chance of surviving?	41 %	1 %	1 %	0 %	0 %	27 %	–	26 %	2 %	1 %
Which has the best chance of surviving? (After selecting frequency graph)	30 %	1 %	3 %	1 %	2 %	29 %		30 %	4 %	0 %

WHAT IS THE CAUSE OF THE EFFECT?

Given the evidence for the misconception, the question arises as to its cause. One possibility is that the description of the problem engenders a kind of misunderstanding. Our description of the problem asks subjects to consider which position they would prefer and which has the best chance of survival *before* any crackers are pulled. However one possibility is that subjects assume that the experimenter is asking them which they would prefer – a party where no bomb has exploded and there are 6, 5, 4, 3, 2 or 1 cracker left. In this case the probabilities are 1/6; 1/5; 1/4; 1/3; 1/2 and 1. Although the text of the problem does not encourage or, strictly, permit this representation, given that there might well be a tendency to focus on the situation where you are actually pulling your cracker, subjects might perceive the problem in this way.

Another possibility resulting in the same perception is that subjects fail to consider the possibility of the bomb exploding early in the sequence. Konold (1989) has suggested that when reasoning with uncertainty, people tend to be outcome oriented and convert probabilities into YES or NO. Probabilities greater than 50 % are assumed to refer to events that occur and probabilities less than 50 % refer to events that will not occur. 50/50 chances are interpreted as "don't know." If people do reason in this way then they might be able to perceive the bomb party as a gradual escalation of risk. This also suggests that if subjects were told about a bomb party in which there were a much larger number – say 100 crackers – and only one bomb, they might find the illusion even more compelling. The chances of the first cracker exploding are very small and yet the chances for the next person would clearly be worse if it does not do so.

In this respect the bomb party illusion bears some similarity with one explanation of the so-called surprise test paradox (see Margalit & Bar-Hillel, 1983). The surprise test paradox emerges when a teacher tells her class that they will have a surprise test one day next week. The pupils go home feeling miserable until one pupil explains that it cannot happen. The pupil explains that the surprise test couldn't happen on a Friday as they would know on Thursday night, if the test hadn't already happened, that it would have to be the next day and so couldn't be a surprise. Hence the last day the test could be is Thursday. However, knowing this the students would know on Wednesday night, if it hadn't already occurred, that the test was going to be on Thursday and hence it couldn't be on Thursday either. Applying this logic the pupils satisfy themselves that the test cannot occur on any day next week and then are doubly astonished when the test happens on Tuesday – both by the occurrence of the test and the palpable fact that they could indeed be surprised!

Quine (1953) has offered an explanation of this paradox in terms of neglect of the possibility that the test has already occurred. In the surprise test paradox, people seem to reason conditionally backwards from the end of the week but, overlooking the possibility that the critical event could have occurred earlier in the week, they thereby require it to occur later. The bomb party also sees people apparently reasoning conditionally – though here people are reasoning forwards in time – but also apparently finding it easy to overlook the possibility that the critical event has occurred earlier in time. In each case there seems to be a problem with finding a particular scenario so alluring that it becomes difficult to consider alternatives.

Although the bomb party illusion may have similarities with the surprise test paradox, it can be distinguished from some other notoriously illusory probability problems. In the three prisoner problem (Falk, 1992), Tom, Dick and Harry are imprisoned in separate cells awaiting execution. One of them has been pardoned by a lottery but none of them knows which. The warden is forbidden to tell any of them of their fate. Dick argues that at least one of Tom and Harry must be executed and convinces the warden that by naming one of them he will not be violating his instructions. The warden names Harry. Dick immediately cheers up, reasoning: "Before, my chances of a pardon were 1/3; now only Tom and myself are candidates for a pardon, and since we are both equally likely to receive it, my chances of being freed have increased from 1/3 to 1/2." A paradox arises because if the warder had named Tom the same conclusion could have been drawn – in fact merely *imagining* the conversation could lead to this conclusion; indeed, one need only merely imagine the existence of the warder to come to the same conclusion. In fact, Dick's chances are unchanged at 1/3 – but Tom's are improved to 2/3.

The Monty Hall problem – a better known isomorph of the three prisoner problem (see Rosenhouse, 2009) – offers game show contestants a choice of opening three doors to win a prize. Behind one is a sports car; behind each of the other two is a goat. After the contestant has chosen one door, Monty always opens one of the other doors to reveal a goat. He then asks if the contestant would like to change their mind. Should the contestant stick or switch? The counterintuitive answer is that the contestant should switch because their original choice of door only has a 1/3 chance of revealing the car whereas the other unopened door has a 2/3 chance.

The difficulties that these two problems causes have been attributed to the operation of "subjective theorems" (cf. Shimojo & Ichikawa, 1989) or heuristics. One suggestion is that people overuse or misapply a uniformity heuristic (Falk & Lann, 2008) to probability problems. The uniformity heuristic effectively assumes that when there are a number of uncertain alternatives, one should, in the absence of indications to the contrary, assume that they are all equally likely. The idea can be seen in writings by Leibniz (1678) and Laplace (1776) and seems to be at the heart of numerous other probability problems (see Falk & Lann, 2008). However, the bomb party illusion is distinct from the fallacies triggered in these problems. The bomb party illusion *violates* the uniformity principle inasmuch as people erroneously believe that the set of alternatives identified are *not* equally likely when they are – quite the opposite to the typical error made with the three prisoners and Monty Hall problems.

A number of every day events follow the bomb party pattern of serially sampling without replacement. When waiting for a bus, one is entitled to believe that it is more likely to arrive in the next five minutes if it didn't arrive in the previous five minutes (see Teigen & Keren (2007) for evidence that people fail to reason appropriately about such situations). However, if it is late at night, I may also start to feel that with every passing minute it is more likely that I have missed the last bus. Similarly when I am looking for a lost paper on my desk I may feel that I am more likely to find it as I proceed. However, where there is a possibility that the paper is not there I may also increasingly feel that I am searching in vain.

Falk, Lipson, and Konold (1994) investigated people's reasoning about such situations and found that only about a fifth of their subjects correctly divined that short term hope should increase as long term hope declines. For example, according to Falk et al. (1994), marriage patterns in several societies are such that, for individuals between 18 and 30, although their long term probability of ever getting married decreases with every passing year, their short-term probabilities of marrying within the next six months continue to rise. If people have schemas for such serial searches with the possibility of failure, then might subjects be confusing the long-term and short term probabilities in these common situations when they come to reason about the bomb party?

Before conducting the empirical studies described here I wrote a short article (Ayton & McClelland, 1987) that discussed the illusion and also commented on Graham Greene's autobiographical description of his own experiences playing Russian roulette which also evidences misconceptions of probability, and may well have been an inspiration for his later novel. I sent the article to him and to my amazement received a letter in reply which acknowledged receipt of the article and said: "It amused me, but I am afraid it is far too mathematical for me to follow."

It is now too late to remedy my failure to explain the illusion to Graham Greene but I still have hope that future studies will provide insight into the causes of and remedies for the bomb party illusion.

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