Aceh International Journal of Science and Technology, 2 (1): 1-7
April 2013
ISSN: 2088-9860

TECHNICAL PAPER

Vectorial Formalism of Polyphase Synchronous Machine With Permanents Magnets

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Abstract- This paper presents a mathematical model that transforms the real machine to fictitious machines and our goal is to simulate these and see the behavior of these machines in load. The polyphase machines are developed mainly in the field of variable speed drives of high power because increasing the number of phases on the one hand allows to reduce the dimensions of the components in power modulators energy and secondly to improve the operating safety. By a vector approach (vector space), it is possible to find a set of single-phase machine and / or two-phase fictitious equivalent to polyphase synchronous machine. These fictitious machines are coupled electrically and mechanically but decoupled magnetically. This approach leads to introduce the concept of the equivalent machine (multimachine multiconverter system MMS) which aims to analyze systems composed of multiple machines (or multiple converters) in electric drives. A first classification multimachine multiconverter system follows naturally from MMS formalism. We present an example of a pentaphase (polyphase) synchronous machine for a simulation and study the behavior of the machine load.

Keywords: Polyphase machines, multimachine concept, vector space, eigenvectors, eigenvalues, pentaphase machine.

Introduction

A vectorial multimachine modelling splits the multiphase machine in a set of magnetically independent fictitious machines, this approach allows to determine a pertinent control topology and that is the goal principal. Through the many advances in technology, the power applications high and average at speed variable are increasingly made on the based on the whole electrical machinery-static converters. For applications of high power density, low rotor losses and reduced inertia, the permanent magnet synchronous machines (Semail, 2000) are best suited. However with the traditional structures of static converters and high power machines, the power transmitted between the power source and the mechanically receiver can not be treated appropriately. The use of current switches associated with machine double-star (Hadiouche, 2001) on the one hand allows to reduce the power transmitted by each converter and, secondly, to reduce the torque ripple of the machine. Despite this improvement, the torque ripples are important, especially for low speeds. The polyphase machines are an interesting alternative to reducing constraints applied to the switches and coils. Indeed, the increase in the number of phases allows a fractionated of power, and therefore a reduction in switched voltages at a given current. In addition, these machines can reduce the amplitude and increasing the frequency of the torque ripple, which allows at the mechanical loading of filter them more easily. Finally, increasing the number of phases provides increased reliability by allowing run, one or more faulted phases. The polyphase machines are found (Bouscayrol and Verhille, 2002) in areas such as marine, railway, petrochemical industry, avionics, automotive, military ships (Sheng and Cheng, 2011).

Materials and Methods

Principle of modelling vector

- The effects of skin, shock absorbers, saturation, and variation of reluctance of the magnetic circuit are neglected.
- The emf induced in the stator windings are solely due to the rotor magnets which have a shape that is due only to the magnets and the structure of the windings. Armature reaction magnetic (due to stator currents) does not change the form of the emf.
- The phases are the same and offset by an $\alpha = \frac{2\pi}{n}$, n is the number of phase of the machine

The emf induced in-phase depends only on the speed of the rotor and structural parameters such as: $e_k = f_k(\theta)\cdot\Omega$, $f_k(\theta)$ is a function of the form dependent of the rotor position θ and rotational speed Ω.

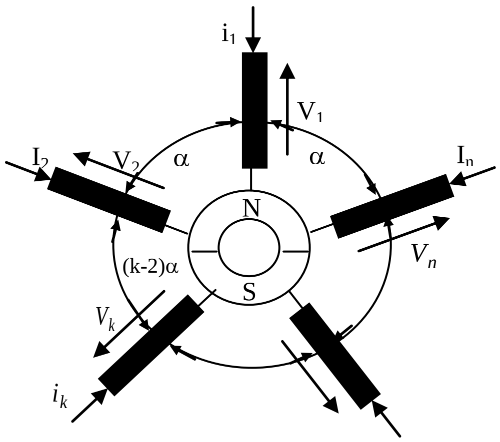

Figure 1. Representation of the polyphase synchronous machine

Figure 1 shows a bipolar machine where the magnitude g is a voltage, current or flux on the phase k is denoted g_k.

If we associate the n-phase machines (Semail *et al.*, 2003) Euclidean vector space E^n of dimension n, an orthonormal basis of the space B^n: $B^n = \left\{\overrightarrow{x_1^n}, \overrightarrow{x_2^n}, \ldots\ldots, \overrightarrow{x_n^n}\right\}$

It is called natural when the vector g can be written as:

$$\vec{g} = g_1 \cdot \overrightarrow{x_1^n} + g_2 \cdot \overrightarrow{x_2^n} + \ldots + g_n \cdot \overrightarrow{x_n^n} \quad (1)$$

$g_1, g_2, \ldots, g_n$: Measurable magnitudes of the stator phases. Consequently in this space can therefore be defined vectors:

- Voltage: $\vec{v} = v_1 \cdot \overrightarrow{x_1^n} + v_2 \cdot \overrightarrow{x_2^n} + \ldots + v_n \cdot \overrightarrow{x_n^n}$
- Current: $\vec{i} = i_1 \cdot \overrightarrow{x_1^n} + i_2 \cdot \overrightarrow{x_2^n} + \ldots + i_n \cdot \overrightarrow{x_n^n}$

The voltage vector of the machine is:

$$\vec{v} = R_s \cdot \vec{i} + \left[\frac{d\overrightarrow{\Phi_s}}{dt}\right]_{/B^n} + \vec{e} \quad (2)$$

The projection of the machine voltage v to a vector k of the voltage of a phase stator gives:

$$v_k = \vec{v} \cdot \overrightarrow{x_k^n} = R_s \cdot \overrightarrow{i_k} + \left[\frac{d\overrightarrow{\Phi_{sk}}}{dt}\right]_{/B^n} + \overrightarrow{e_k} \quad (3)$$

This is the equation of the stator voltage with a phase:

- Φ_{sk} : the flux in the phase *k* created by the stator currents.
- e_k : is the emf induced in the phase k created by rotor magnets. Assumptions of unsaturation and non reluctance variation can define a linear relationship $\overrightarrow{\Phi_s} = \lambda(\vec{i})$ between the current vector and the stator flux more usually written in the form of a matrix with constant coefficients:

$\Phi_S = \left[L_S^n\right] * \left[i_n\right]$

$$\left[L_S^n\right] = \begin{pmatrix} L_{s_1s_1} & L_{s_1s_2} & \cdots & L_{s_1s_n} \\ L_{s_2s_1} & L_{s_2s_2} & \cdots & L_{s_2s_n} \\ \vdots & \vdots & \ddots & \vdots \\ L_{s_ns_1} & L_{s_ns_2} & \cdots & L_{s_ns_n} \end{pmatrix} \quad (4)$$

$L_{s_ks_k}$ Is the inductance of a stator phase

$L_{s_js_k}$ Mutual inductance between stator phases.

The instantaneous power is transiting in the machine:

$$p = \sum_{k=1}^{n} v_k \cdot i_k = \vec{v} \cdot \vec{i} \quad (5)$$

By replacing the expression vector voltage (2), we obtain the following equation:

$$p = R_S \left(\vec{i}\right)^2 + \left[\frac{d\overrightarrow{\Phi_S}}{dt}\right]_{/B^n} \cdot \vec{i} + \vec{e} \cdot \vec{i} \quad (6)$$

- Losses by Joule effect are: $p_j = R_s \left(\vec{i}\right)^2$
- The magnetic power is: $p_w = \left[\frac{d\overrightarrow{\Phi_s}}{dt}\right]_{/B^n} \cdot \vec{i}$
- Electromagnetic power is: $p_{em} = \vec{e} \cdot \vec{i}$
- The electromagnetic torque is: $C_{em} = \frac{\vec{e} \cdot \vec{i}}{\Omega}$

Ω is the instantaneous speed of the rotor.

Modelling of the machine with n phases in a base ensuring decoupling magnetic: the relation $\overrightarrow{\Phi_s} = \lambda(\vec{i})$, which is one of the morphism, between the current vector and stator flux remains true whatever the base of the space E^n chosen (Toliyat *et al.*, 1991). The base where exist the magnetic decoupling is that in which one coordinated stator flux vector can be expressed as a function of a single coordinate of the current vector (matrix diagonal inductance).

Diagonalization of an inductance matrix requires research of the eigenvalues and eigenvectors associated with them. We define the eigenvalues Λ_k the morphism λ as being solutions of the characteristic equation: $\det(\Lambda[I_n]-[L_s^n])=0$

$[I_n]$ is the identity matrix of dimension n.

The eigenvalues are real because the inductance matrix is symmetric. The hypothesis of regularity spatial of phases construction, allows us to affirm that inductance matrix is circulant. Circularity property allows us to calculate analytically the eigenvalues by using the formula for circulant determinant. These two conditions are respected; the complex eigenvalues are given by the solutions of the equation:

$$\prod_{l=1}^{n}\left(\Lambda-\sum_{k=1}^{n}\left(L_{s_1 s_k}\cdot e^{\frac{2j\pi(l-1)(k-1)}{n}}\right)\right)=0 \tag{7}$$

j is the complex operator.

Equation (7) is divided into n equations each having a specific value as a solution of the morphism λ. These n eigenvalues are given in complex forms which are associated eigenvectors. These complex coordinate vectors form an orthonormal basis of the Hermitian space associated to machine. we want, as with the transform Concordia, work with real coordinates eigenvectors associated with real eigenvalues. The inductance matrix is symmetrical, therefore the values Λ_k are real we notice that: $\Lambda_k=\Lambda_{n-k+2}$ then there exists an eigenvector associated with the eigenvalue Λ_k. It is therefore in the plane spanned by the vectors of an infinite orthonormal bases generated by the eigenvectors. The property $e^{j(n-k)2\pi/n}=e^{-jk2\pi/n}$ allows determining an orthonormal basis composed of eigenvectors with real coefficients such that:

The new matrix inductance $[L_s^d]$, characteristic morphism in the new base $B^d=\left\{\overrightarrow{x_1^d},\overrightarrow{x_2^d},\ldots,\overrightarrow{x_n^d}\right\}$ becomes:

$$[L_s^d]=\begin{pmatrix}\Lambda_1 & 0 & \ldots & 0\\ 0 & \Lambda_2 & \ldots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \ldots & \Lambda_{n'}\end{pmatrix} \tag{8}$$

This matrix is diagonal and we recall that the inductances of the matrix are at least equal in pairs (eigenvalue of multiplicity of order 2). This research eigenvalues associated to the inductance matrix can formulate a generalized Concordia transformation with transition matrix as the natural base to base decoupling (Lyra and Lipo, 2001).

Bases of departure and arrival are orthonormal. This transformation has the property to preserve the instantaneous power regardless of the base in which it is expressed. The equations of the machine in a base decoupling:

A vector $\vec{g}$ the initial space decomposes into:

$$\vec{g}=\sum_{g=1}^{g=N}\overrightarrow{g_g} \tag{9}$$

Let N be subspaces each associated to an eigenvalues Λ_g, $\overrightarrow{g_g}$ is the projection of the vector $\vec{g}$ on the subspace E^g .

The new equation of the flux vector and current:

$$\overrightarrow{\Phi_s}=\sum_{g=1}^{g=N}\overrightarrow{\Phi_{sg}}=\sum_{g=1}^{g=N}\Lambda_g\overrightarrow{i_g} \tag{10}$$

Allows writing in each sub-space, a new voltage equation:

$$\overrightarrow{v_g}=R_s\overrightarrow{i_g}+\left[\frac{d\overrightarrow{\Phi_{sg}}}{dt}\right]_{/E^n}+\overrightarrow{e_g} \tag{11}$$

Using the property (10) into (11):

$$\overrightarrow{v_g}=R_s\overrightarrow{i_g}+\left[\frac{d\left(\Lambda_g\overrightarrow{i_g}\right)}{dt}\right]_{/E^g}+\overrightarrow{e_g}$$

$$\overrightarrow{v_g}=R_s\overrightarrow{i_g}+\Lambda_g\left[\frac{d\overrightarrow{i_g}}{dt}\right]_{/E^g}+\overrightarrow{e_g} \tag{12}$$

The electrical power which transits into the real machine is expressed by [6]:

$$p=\vec{v}\cdot\vec{i}=\sum_{g=1}^{N}\overrightarrow{v_g}\cdot\overrightarrow{i_g} \tag{13}$$

By replacing the expression of voltage (12) in the power equation (13) we obtain:

$$p=\sum_{g=1}^{N}\left(R_s\left(\vec{i_g}\right)^2+\Lambda_g\left[\frac{d\vec{i_g}}{dt}\right]_{/E^g}\cdot\vec{i_g}+\vec{e_g}\cdot\vec{i_g}\right) \tag{14}$$

The previous equation shows that the energy transits through N fictitious machines, independent magnetically associated with N eigenspaces. Consequently, the actual torque is written:

$$C=\sum_{g=1}^{N}C_g \tag{15}$$

With:

$$C_g\cdot\Omega=\vec{e_g}\cdot\vec{i_g}\,.$$

Remark:

Equation (14) shows that each fictive machine produces a torque participant in the creation of a total torque. These N fictitious machines are mechanically coupled: they rotate at the same speed and are rigidly coupled to the same mechanical shaft.

Example for Pentaphase machine application:

We model, for the application, a synchronous machine with permanent magnets pentaphase. This machine is represented symbolically in Figure 2.

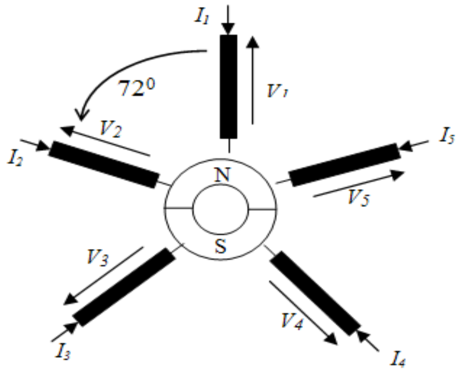

Figure 2. Representation of the pentaphase machine

Interpretation: this is a real machine to stage 5 phase shifted 72^0.

Modelling of the machine in the natural basis:

We associate the five phases a Euclidean vector space E^5 of dimension 5. We write in an orthonormal base B^n the voltage equation of the machine:

$$\vec{v}=R_s\cdot\vec{i}+\left[\frac{d\overrightarrow{\Phi_s}}{dt}\right]_{/B^n}+\vec{e} \tag{16}$$

$B^n=\left\{\vec{x_1},\vec{x_2},\vec{x_3},\vec{x_4},\vec{x_5}\right\}$ Orthonormal basis

In linear mode, there exists morphism between vectors stator flux and current, such as:

$$\overrightarrow{\Phi_s}=\lambda(\vec{i}) \tag{17}$$

$$\left[L_s^n\right]=\begin{pmatrix} L & M_1 & M_2 & M_2 & M_1\\ M_1 & L & M_1 & M_2 & M_2\\ M_2 & M_1 & L & M_1 & M_2\\ M_2 & M_2 & M_1 & L & M_1\\ M_1 & M_2 & M_2 & M_1 & L \end{pmatrix} \tag{18}$$

With:

- L the inductance of a phase $\left(L=L_p+l_f\right)$;
- M_1 The mutual inductance between two phases shifted from $\pm\frac{2\pi}{5}$;
- M_2 The mutual inductance between two phases shifted from $\pm\frac{4\pi}{5}$.

emf vector :

$$\vec{e}=e_1\cdot\vec{x_1}+e_2\cdot\vec{x_2}+e_3\cdot\vec{x_3}+e_4\cdot\vec{x_4}+e_5\cdot\vec{x_5}$$

With:

$$e_k=e_k=E_{\max}\sin(\omega t-\frac{2(k-1)\pi}{5}),k=1,\ldots,5$$

$E_{\max e}=k\cdot\Omega$ is the maximum value of the emf with k emf coefficient and Ω the rotational speed of the rotor.

Modelling in a base decoupling:

There exists an orthonormal basis $B^d=\left\{\vec{x_z},\vec{x_{p\alpha}},\vec{x_{p\beta}},\vec{x_{s\alpha}},\vec{x_{s\beta}}\right\}$ in which the inductance matrix is diagonal:

$$[L_s^d] = \begin{pmatrix} \Lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \Lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \Lambda_5 & 0 & 0 \\ 0 & 0 & 0 & \Lambda_3 & 0 \\ 0 & 0 & 0 & 0 & \Lambda_4 \end{pmatrix} \quad (19)$$

It appears then double values:

$\Lambda_1 = L + 2(M_1 + M_2)$

$\Lambda_2 = \Lambda_5 = L - 2 \cdot \left(M_1 \cdot \cos\left(\frac{3\pi}{5}\right) + M_2 \cdot \cos\left(\frac{\pi}{5}\right)\right)$

$\Lambda_3 = \Lambda_4 = L - 2 \cdot \left(M_1 \cdot \cos\left(\frac{\pi}{5}\right) + M_2 \cdot \cos\left(\frac{3\pi}{5}\right)\right)$

Inductors are associated with eigenvectors.

There is a single eigenvalue and two double eigenvalues. This property allows us to decompose the vector space E^5 at three orthogonal subspaces, namely:

- A subspace E^z generated by the eigenvector $(\overrightarrow{x_z})$ associated with the eigenvalue $\Lambda_z = \Lambda_1$.

This subspace is a straight line called homopolar

- A subspace E^p generated by the eigenvectors $(\overrightarrow{x_{p\alpha}}, \overrightarrow{x_{p\beta}})$ associated to the eigenvalue:

$\Lambda_p = \Lambda_2 = \Lambda_5$.

This subspace is called the primary plan.

- A subspace E^s generated by the eigenvectors $(\overrightarrow{x_{s\alpha}}, \overrightarrow{x_{s\beta}})$ associated with the eigenvalue $\Lambda_s = \Lambda_3 = \Lambda_4$. This subspace is called the secondary plan.

the vector $\vec{g} = \overrightarrow{g_h} + \overrightarrow{g_p} + \overrightarrow{g_s}$

A fictive machine may be associated with each subspace, respectively

- A machine associated with the two-phase principal plan possessing the time constant and emf induced the most important; electrical
- A machine associated with the plan secondary at two-phase possessing the electrical constant time lowest and emf induced the less important;
- A machine phase associated with the straight line with the homopolar electric time constant and emf induced weaker.

Simulation of the pentaphase machine:

We implement the model of the machine on the software numerical simulation Matlab Simulink. We apply a load at startup resistant C_r=1N.m we take the speed loop fixed to the rated speed 157 rad / s.

The parameters of simulation are:

Λ_z=0.001H homopolar inductance (homopolar fictious machine)
Λ_p=0.0081H principal inductance (principal fictious machine)
Λ_s=0.001H secondary inductance (secondary fictious machine)
R=6.2 Ω restance
K=0.61 Emf coefficient

emf:

$$e_k = E_{\max} \sin(\omega t - \frac{2(k-1)\pi}{5}), k = 1,\ldots,5$$

The currents are sinusoidal and in phase with the emf

$$i_k = I_{\max} \sin(\omega t - \frac{2(k-1)\pi}{5}), k = 1,\ldots,5$$

K means the number of the phase. For our machine there are 5 phases (k phases).

Results and Discussion

emf in the base of Concordia fictitious machines (Figure 3).

Interpretation: We notice (Figure 3) in the basis of the emf Concordia Machine pentaphase become a machine fictitious two-phase and this is the principal.

- emf in the principal machine:

$$e_{\alpha p} = \sqrt{5/2}E_{\max}\sin(\omega t)$$
$$e_{\beta p} = -\sqrt{5/2}E_{\max}\cos(\omega t)$$

- emf in the secondary machine :

$$e_{\alpha s} = 0$$
$$e_{\beta s} = 0$$

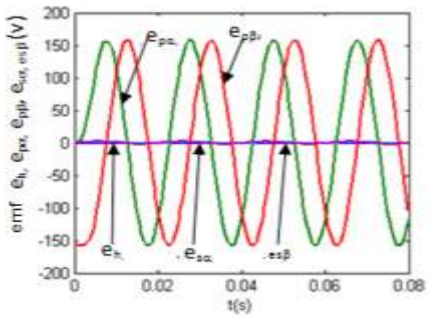

Figure 3. emf in fictitious machines (the basis of Concordia).

- emf in homopolar machine:

$$e_h = 0$$

We find that although in this case only the principal machine can provide the couple. Currents in the base of Concordia fictitious machines (Figure 4).

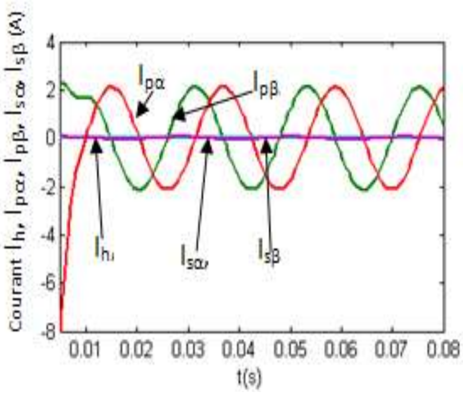

Figure 4. Current in fictitious machines (the basis of Concordia)

Interpretation: We notice (Figure 4) in the basis of the current Concordia machine pentaphase become a machine fictitious two-phase and this is the principal.

- Current in the principal machine :

$$i_{\alpha p} = \sqrt{5/2} I_{\max} \sin(\omega t)$$
$$i_{\beta p} = -\sqrt{5/2} I_{\max} \cos(\omega t)$$

- Current in the secondary machine:

$$i_{\alpha s} = 0 \text{ et } i_{\beta s} = 0$$

- Current in the homopolar machine :

$$i_h = 0$$

The emf in the fictitious machine (the base of Park): Figure 5

$$e_h = 0 \text{ , } e_{pd} = 0 \text{ , } e_{sd} = 0 \text{ et } e_{sq} = 0$$

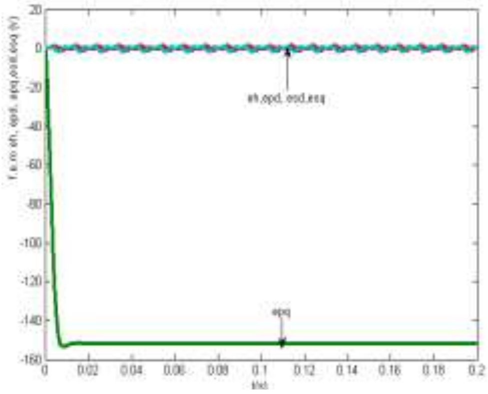

Figure 5. emf in the fictitious machine (Base of Park)

Interpretation: We notice (Figure 5) in the basis of Park, the emf components are zero except the emf principal quadratic which is a constant.

Current in the fictitious machine (the base of Park): Figure 6

$$i_{sd} = 0 \quad i_{sq} = 0$$

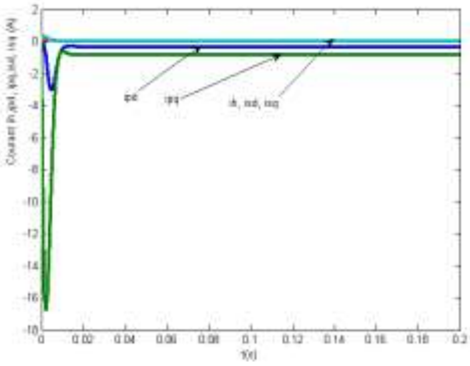

Figure 6. Current in the fictitious machines

Interpretation: We notice (Figure 6) in the basis of Park, the currents electrical of the homopolar and secondary machine are zero but the currents of the principal machine are constants nonzero. The electromagnetic torque: Figure 7

$$C_n = \frac{1}{\Omega}(e_z i_z + \overrightarrow{e_{\alpha\beta p} i_{\alpha\beta p}} + \overrightarrow{e_{\alpha\beta p} i_{\alpha\beta p}})$$

$$C_n = \frac{1}{\Omega}(e_{pq} i_{pq}) = \frac{5}{2}\frac{E_{\max} I_{\max}}{\Omega} = \frac{5}{2} k I_{\max}$$

$$k = \frac{E_{\max e}}{\Omega}$$

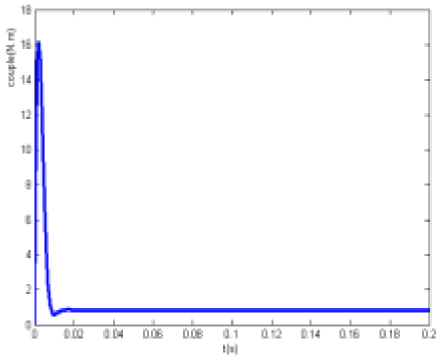

Figure 7. Electromagnetic torque

Interpretation: we remark (Figure 7) that the torque is constant is equal to the load torque. The speed of the pentaphase machine**:** Figure 8

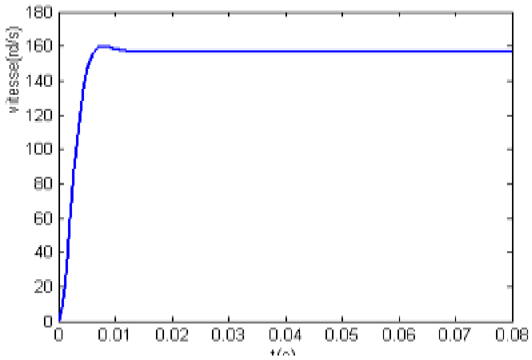

Figure 8. The speed of the pentaphase machine

Interpretation: we remark (Figure 8) that speed is constant is equal to 157 r / min, which corresponds well to the real speed of the machine.

Interpretation of the simulation:

The simulations allow us to observe the emf in fictitious machines and currents in the base of Park. The transformation of Park applied to these magnitudes indicates that only the component (q) of the emf of the principal machine is a nonzero constant (Fig 5). One can observe that the homopolar emf is zero, which is coherent for a machine sinusoidal, balanced and non-electrically coupled. Accordingly, the homopolar machine produces no torque (C_h = 0) as well as for the secondary machine (C_s=0).So the principal machine (two-phase) alone produces torque. We find that the torque is constant $C = C_h + C_p + C_s = C_p$ (Fig 7).

With: $C_h = 0 \; and \; C_s = 0$

The simulation shows that the load has no effect on homopolar machines and secondary fictitious and only the main machine is operational fictitious

Conclusions

A polyphase machine is composed of n windings spatially *2□ /n* and powered by the voltage phase-shifted temporally *2□ /n*. These machines are characterized by a magnetic coupling between phases. The generalization of the method of space vector allows defining a base change of dimension n, implying a simplification of the study of the machine by diagonalization of the matrix inductance. This change leads to subspaces orthogonal base of dimension 2 and 1, each subspace can be independent. Association the vector space modelling and concept multimachine allows us to consider a polyphase machine as equivalent to a set of fictitious machines mechanically coupled. The study of a complex machine turns into several studies of simple machines.

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