# FMCW Radar Phase-Processing for Automotive Application 

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#### Abstract

An unmanned high-speed vehicle requires a high resolution control unit to decide whether the vehicle should break or steer. This paper describes the utilization of a Frequency Modulated Continous Wave (FMCW) radar to detect the distance and angular position of the target relative to the moving vehicle. These informations are calculated from some numbers of data obtained from a radar system that is mounted on the moving vehicle. To obtain a high angular resolution, a phase-processing approach is introduced by extracting the constant phase information from the radar FMCW beat signal. It is found that a very high angular resolution can be obtained by processing the phase information, independent from the change of relative velocity between the radar and the target. However, this method requires a very stable radar system with high phase resolution and accuracy.


Keywords—Angular Position, Continous-Wave, Distance, Frequency-Modulated, Phase, Radar, Synthetic Aperture.

## I. Introduction

Frequency Modulated Continuous Wave (FMCW) radar is a type of radar used to obtain range and other information from a target by using a frequency modulation technique on a continuous signal. The radar transmitter is continuously transmitting this modulated signal as a wave (CW). The frequency modulation used by the radar can take many forms, such as triangular, sawtooth, sinusoidal, or some other shape [1].
This report presents the results of a theoretical study of an FMCW radar system for automotive application. The aim of this study is to develop an FMCW radar system with higher angular resolution, by exploiting the radar measurements information at different positions. This section will introduce the basic sawtooth FMCW radar signal processing to obtain the radar-target range information. The next section will describe a newly proposed method of FMCW radar signal processing for a high-speed vehicle, by using the phase information instead of the frequency information.

## A. Sawtooth FMCW Waveform

For a sawtooth FMCW radar, the sawtooth frequency modulation $f_{t}(t)$ is periodic with $T$ as expressed in eq.(1). $f_{t}(t)=f_{t}(t+T)$ where T is the frequency modulation period. Thus, for a linear frequency modulated sawtooth waveform, the transmit frequency is given by eq.(2) [2] or eq.(3) [3].
$f_{t}(t)=f_{c}-\frac{B W}{2}+\frac{B W}{T} t, 0<t<T$
$f_{t}(t)=f_{c}+\frac{B W}{T} t, \frac{-T}{2}<t<\frac{T}{2}$
where $f_{c}$ is the RF center frequency, $B W$ is the total frequency deviation or the radar Bandwidth, and $T$ is the modulation period or sweep time. As an example, a 30 GHz RF center frequency and a 150 MHz total peak-topeak frequency deviation ( $\mathrm{BW}=150 \mathrm{MHz}$ ) means that the transmit frequency starts at 29.925 GHz and linearly

[^0]increases in frequency at a sweep time $T$ to the maximum frequency of 30.075 GHz .
The transmitted RF phase $\varphi(t)$, assuming that $\varphi(0)=\eta$, is given by:
\[

$$
\begin{align*}
\varphi(t) & =2 \pi \int_{0}^{t} f_{t}(x) d x \\
& =2 \pi\left(f_{c} t+\frac{B W}{2 T} t^{2}\right)+\eta \\
& =2 \pi\left(f_{c} t+\alpha t^{2}\right)+\eta, \frac{-T}{2}<t<\frac{T}{2} \tag{4}
\end{align*}
$$
\]

where $\alpha=B W / 2 T$. Thus, the transmitted signal with amplitude $A$ is given by:
$T(t)=A \cos \varphi(t)=A \cos \left(2 \pi\left(f_{c} t+\alpha t^{2}\right)+\eta\right), \frac{-T}{2}<t<\frac{T}{2}$
The received signal is the transmitted signal delayed in time by the round trip propagation time to the target and back, $\tau$, with reduced amplitude $B$ expressed in eq. (6).
$R(t)=\frac{B}{A} T(t-\tau)$
$\tau=\frac{2 l(t)}{c}$
where $l(t)$ is the radar-target distance, and $c$ is the wave propagation constant ( $c=3 \times 10^{8} \mathrm{~m} / \mathrm{s}$ ).

From eq. (6), it can be seen that the round trip propagation time, $\tau$, should be much less than the sweep time, $T$. Thus, $\tau \ll T$ or $\tau_{\max } \cong T / 10$.
$\tau_{\text {max }}=\frac{2 R_{\max }}{c} \simeq \frac{T}{10}$
From eq. (8), the maximum distance between the radar system and the target is expressed in (9).
$R_{\text {max }} \simeq \frac{c T}{20}$

## B. Signal Processing

The transmitted wave from the FMCW radar is reflected by the target, and the reflected signal is received and mixed with a portion of the transmitted signal. The mixer output signal is then altered to obtain a beat signal $S(t)$ expressed in (10).
$S(t)=C \cos \left(2 \pi\left(f_{c} \tau+2 \alpha \tau t-\alpha \tau^{2}\right)\right)$
If the target is moving, $l(t)=l_{0}+v t$ and $\tau=2 l(t) / c$, then the expression of beat signal $S(t)$ can be manipulated into:

$$
\begin{gather*}
S(t)=S \cos \left\{2 \pi \left(4 \alpha l_{0} t\left(\frac{1}{c}-\frac{2_{v}}{c^{2}}\right)+\frac{2 f_{e} v t}{c}+4 \alpha v t^{2}\left(\frac{1}{c}-\frac{v}{c^{2}}\right)+\right.\right. \\
\left.\left.\frac{2 l_{0}}{c}\left(f_{c}-\frac{2 \alpha l_{0}}{c}\right)\right)\right\} \tag{11}
\end{gather*}
$$

The expression is eq.(11) contains frequency terms,
which are time varying, and phase terms, which are not. The first frequency term is the range frequency, $f_{r}$, which is proportional to the range of the target $l_{0}$. Since $v$ is assumed to be much less than $c, v \ll c, f_{r}$ can be simplified as in eq.(12). The second frequency term is the Doppler frequency shift, $f_{D}$, expressed in eq.(13). The third frequency term is a cross-term which may either be interpreted as chirp on the range frequency due to the changing distance, or as chirp on the Doppler frequency due to the changing transmitter frequency [4] expressed in eq.(14). The summation of all these frequency terms is known as the beat frequency, $f_{b}$. The final term is a constant phase term expressed in eq.(15).
$f_{r}=4 \alpha l_{0}\left(\frac{1}{c}-\frac{2 v}{c^{2}}\right) \simeq \frac{4 \alpha l_{0}}{T c}$
$f_{D}=\frac{2 f_{c} v}{c}$
$C(t)=8 \pi \alpha v t^{2}\left(\frac{1}{c}-\frac{v}{c^{2}}\right)=4 \pi v t^{2} \frac{B W}{T}\left(\frac{1}{c}-\frac{v}{c^{2}}\right)$
$\phi=\frac{4 \pi l_{0}}{c}\left(f_{c}-\frac{2 \alpha l_{0}}{c}\right)=\frac{4 \pi l_{0}}{c}\left(f_{c}-\frac{B W l_{0}}{T c}\right)$
The transmitted and received frequencies as a function of time, showing beat frequency and Doppler frequency is shown in Figure 1 [5].

## C. Range Resolution

Extrapolating from (12), it can be seen that the range resolution, Res, is linearly proportional to the beat frequency resolution, $\delta f_{b}$, as expressed in (16).
$l_{0}=\frac{T c}{2 B W} f_{b} \rightarrow$ Res $=\frac{T c}{2 B W} \delta f_{b}$
It is shown in [6] that $\delta f_{b} \cong 1 / T$, which when substituted into (16) results in (17).
Res $=\frac{c}{2 B W}$
However, this range resolution is also a function of chirp linearity. If the chirp is not linear, then the range frequency for a target will not be constant and the range resolution will degrade linearly with range [6].

## II. Description of Work

In the proposed method, the basic idea is to use the data obtained from every $d=\lambda / 4$ radar measurement as can be seen in Figure 2. The radar wavelength, $\lambda$, corresponds to the radar center frequency $\left(f_{c}\right)$ expressed as:
$\lambda=\frac{c}{f_{c}}$
Where, $c$ is the wave propagation constant $\left(c=3 \times 10^{8}\right.$ $\mathrm{m} / \mathrm{s}$ ).
To make these measurements feasible for a moving object with maximum speed $v_{\text {max }}$, the time between two successive radar measurements, $\Delta t$, should be equal to the sweep time ( $T$ ) of the FMCW. Thus, it is obtained that:
$T=\Delta t=\frac{d}{v_{\max }}=\frac{\lambda}{4 v_{\max }}$

## A. Angle Measurement

In the proposed method, the information extracted from the beat signal $S(t)$ is not the frequency, but the constant phase term, $\phi$. Assuming that $f_{c} \gg\left(B W l_{0}\right) /(T c)$, the contant phase term $\phi$ can be expressed as:

$$
\begin{equation*}
\phi \frac{4 \pi l_{0}}{c}\left(f_{c}-\frac{B W l_{0}}{T c}\right) \simeq \frac{4 \pi l_{0} f_{c}}{c} \tag{20}
\end{equation*}
$$

From eq.(20), the difference of $\phi$ at the $(n-1)^{\text {th }}$ position and at the $n^{\text {th }}$ position can be described as follows:
$\Delta \phi_{n}=\phi_{n-1}-\phi_{n}=\frac{4 \pi f_{c}}{c}\left(l_{n-1}-l_{n}\right)=\frac{4 \pi}{\lambda}\left(l_{n-1}-l_{n}\right)$
To calculate the angle $\theta$, it is assumed that $\theta_{n}=\theta_{n-1}=\overline{\theta_{n}}$, where $\overline{\theta_{n}}$ is the average value of $\theta_{n}$ and $\theta_{n-1}$. Substituting $d=\lambda / 4$, the expression in eq.(21) can be simplified as:
$\Delta \phi_{n}=\frac{4 \pi}{\lambda}\left(d \cos \overline{\theta_{n}}\right)=\pi \overline{\theta_{n}}(22)$
Thus, it is observed that the value of $\Delta \phi$ is not related to Doppler effect, since this information is extracted from the constant phase of the beat signal $S(t)$. The change of $\Delta \phi$ value in every radar position, $n$, is simply caused by the change of angle $\theta$ and is independent from the change of relative velocity between the radar and the target, $v$. When $\theta=0$, or when the target is located right in front of the radar system, the value of $\Delta \phi$ is constant regardless of the change in velocity $v$.
From eq.(22), the average angle $\overline{\theta_{n}}$ can be calculated as follows:
$\overline{\theta_{n}}=\cos ^{-1}\left(\frac{4 \varphi_{n}}{\pi}\right)$

## B. Angle Accuracy

The value of the measured angle ranged between $\theta_{n-1}$ and $\theta_{n}$. Thus, the angle measurement should be represented as:
$\theta_{n}=\overline{\theta_{n}} \pm \delta \theta_{n}$
where the angle deviation between two successive measurements, $\delta \theta_{n}$, can be expressed as:
$\delta \theta_{n}=\frac{1}{2}\left(\theta_{n}-\theta_{n-1}\right)$
By using the law of sine:
$\frac{d}{\sin \left(\theta_{n}-\theta_{n-1}\right)}=\frac{l_{n-1}}{\sin \left(\pi-\theta_{n}\right)}=\frac{l_{n}}{\sin \left(\theta_{n-1}\right)}$
we can obtain:

$$
\begin{align*}
\delta \theta_{n} & =\frac{1}{2}\left(\theta_{n}-\theta_{n-1}\right) \\
& =\frac{1}{2} \sin ^{-1}\left\{\frac{d \sin \left(\theta_{n}\right)}{l_{n-1}}\right\}=\frac{1}{2} \sin ^{-1}\left\{\frac{d \sin \left(\theta_{n-1}\right)}{l_{n}}\right\} \tag{27}
\end{align*}
$$

For near range ( $\theta$ is not small), $\delta \theta_{n}$ in eq.(25) can be simplified as:
$\delta \theta_{n}=\frac{1}{2} \sin ^{-1}\left\{\frac{d \sin \overline{\theta_{n}}}{l_{n}}\right\}$
whereas for far range ( $\theta$ is small), $\delta \theta_{n}$ in eq.(25) can be simplified as:
$\delta \theta_{n}=\frac{1}{2} \frac{d \overline{\theta_{n}}}{l_{n}}$
From (29), it can be observed that the angle deviation $\delta \theta_{n}$ for far range, which represents the accuracy of angle measurement, is:

- proportional to the distance between two subsequent radar measurement, $d$;
- proportional to the measured angle $\overline{\theta_{n}}$, and
- inversely proportional to the distance between the radar system and the object, $l_{n}$.


## III. Result and Discussion

In this section, the application of this theory for a relatively slow vehicle ( $v_{\max }=10 \mathrm{~m} / \mathrm{s}$ ) and high-speed vehicle ( $v_{\max }=80 \mathrm{~m} / \mathrm{s}$ ) is studied by several simulations.

## A. Simulation for Slow Vehicle

The parameters used in the case of slow vehicle are as follows:

- $v=10 \mathrm{~m} / \mathrm{s}$
- $f_{c}=30 \mathrm{GHz}$
- $\lambda=1 \mathrm{~cm}$
- $d=0.25 \mathrm{~cm}$
- $\Delta t=T=0.25 \mathrm{~ms}$
- $\mathrm{BW}=150 \mathrm{MHz}$
- Res $=1 \mathrm{~m}$
- $N=400$

The radar system is moving towards the object along the $x$-axis with a velocity $(v)$ of $10 \mathrm{~m} / \mathrm{s}$. The center frequency $\left(f_{c}\right)$ of the FMCW radar utilized in this case is 30 GHz . Thus, the distance between two successive radar measurements, $d$, is 0.25 cm , which results in a time interval $(\Delta t)$ of 0.25 ms . The radar bandwidth (BW) is 150 MHz , which corresponds to the resolution of 1 m as in eq.(17). The number of radar measurements for this case is $400(N=400)$, which equal to a one-meter movement of radar system.

From the expression of $R_{\max }$ as in eq.(9), the maximum distance that can be measured by this radar system is:
$R_{\text {max }}=\frac{c T}{20}=\frac{3\left(10^{8}\right)(0.25)\left(10^{-3}\right)}{20}=3750 \mathrm{~m}=3,75 \mathrm{~km}$
Near Range Simulation
The purpose of this simulation is to find if the radar angular resolution can be improved (Res $<1 \mathrm{~m}$ ) for a near range measurement. In simulation $I$, the object's initial position is assumed to be $(x, y)$ where $x=10 \mathrm{~m}$ and $y=[0,1] \mathrm{m}$ and $[4,5] \mathrm{m}$.
Figure 3 shows the phase difference as expressed in eq.(21) for $y=0$ and $y=1 \mathrm{~m}$. From the figure, it can be observed that the phase difference for every sweep is 0.0173 rad. From this value and the values of $\Delta \phi$, it can be seen that the radar can have a higher resolution with a factor of $m$, where:
$m=\frac{0.0173}{(0+0.0036 / 2}=9.61111111$
$\Delta \phi$ as a function of $n$ for $y=[0,1] \mathrm{m}$, with an improved resolution factor of approximately 9 (Res $=0.1111 \mathrm{~m})$, is plotted in Figure 4. From the figure, it can be seen that there is no overlapping value of $\Delta \phi$. This means that these positions of $y$ can be distinguished properly. This range resolution corresponds to an angular resolution of approximately $0.1111 / 10 \mathrm{rad} \cong 10^{-2} \mathrm{rad} \cong 0.6^{\circ}$.

Figure 5 shows the phase difference as expressed in eq.(21) for $y=4 \mathrm{~m}$ and $y=5 \mathrm{~m}$. From the figure, it can be observed that the phase difference for every sweep is 0.1154 rad. Thus, the improved resolution factor, $m$, can be derived as:
$m=\frac{0.1154}{(0.0636+0.0459) / 2}=2.108$
Thus, the resolution for $y=[4 ; 5] \mathrm{m}$ can be improved by a factor of approximately $2(\operatorname{Res}=0.5) . \Delta \phi$ as a function of $n$ for $y=[4,5] \mathrm{m}$, with an improved resolution factor of approximately 2 (Res $=0.5 \mathrm{~m}$ ), is plotted in Figure 6. From the figure, it can be seen that there is no overlapping value of $\Delta \phi$ for any measurement. Thus, in this case, the resolution of 0.5 m can be obtained.

## Far Range Simulation

The purpose of this simulation is to find if the radar angular resolution can be improved (Res $<1 \mathrm{~m}$ ) for a far range measurement. In simulation II, the object's initial position is assumed to be $(x, y)$ where $x=100 \mathrm{~m}$ and $y=$ $[0,1] \mathrm{m}$ and $[4,5] \mathrm{m}$.

Figure 7 shows the phase difference as expressed in eq.(21) for $y=0$ and $y=1 \mathrm{~m}$. From the figure, it can be
observed that the phase difference for every sweep is $15.8647 \times 10^{-5}$ rad. From this value and the values of $\Delta \phi$, it can be seen that the radar can have a higher resolution with a factor of $m$, where:
$m=\frac{15.8647}{(0+0.3181) / 2}=99.7466$
Thus, for this case, the resolution can be improved by a factor of approximately 100 into 1 cm . This range resolution corresponds to an angular resolution of approximately $10^{-2} / 100 \mathrm{rad} \cong 10^{-4} \mathrm{rad} \cong 0.00573^{\circ}$.

Figure 8 shows the phase difference as expressed in eq.(21) for $y=4 \mathrm{~m}$ and $y=5 \mathrm{~m}$. From the figure, it can be observed that the phase difference for every sweep is $15.235 \times 10^{-5}$ rad. Thus, the improved factor of resolution, $m$, can be derived as:
$m=\frac{14.235}{(0.7923+0.5078) / 2}=21.8983(34)$
In other words, the resolution can be improved by a factor of approximately 22 into 4.6 cm , which corresponds to an angular resolution of approximately $4.6 \times 10^{-2} / 100 \mathrm{rad} \cong 4.6 \times 10^{-4} \mathrm{rad} \cong 0.026^{\circ}$.

## B. Simulation for High-Speed Vehicle

The parameters used in the case of slow vehicle are as follows:

- $v=80 \mathrm{~m} / \mathrm{s}$
- $f_{c}=30 \mathrm{GHz}$
- $\lambda=1 \mathrm{~cm}$
- $d=0.25 \mathrm{~cm}$
- $\Delta t=T=31.25 \mu \mathrm{~s}$
- $\mathrm{BW}=150 \mathrm{MHz}$
- Res $=1 \mathrm{~m}$
- $N=50$

In this case, the radar system is assumed to be moving towards the target along the $x$-axis with a velocity $(v)$ of $80 \mathrm{~m} / \mathrm{s}$. The center frequency $\left(f_{c}\right)$ of the FMCW radar utilized in this case is 30 GHz . Thus, the distance between two successive radar measurements, $d$, is 0.25 cm, which results in a time interval $(\Delta t=T=d / v)$ of 0.03125 ms or $31.25 \mu \mathrm{~s}$. The radar bandwidth (BW) is 150 MHz , which corresponds to the resolution of 1 m as in eq.(17). The number of radar measurements for this case is 50 .
From the expression of $R_{\max }$ as in eq.(9), the maximum distance that can be measured by this radar system is:
$R_{\max }=\frac{c^{2}}{80 f_{c} v_{\max }}=\frac{\left(3\left(10^{8}\right)\right)^{2}}{80(30)\left(10^{9}\right) 80}=468.75 \mathrm{~m}$
Resolution Calculation
The calculation of resolution between $y_{1}$ and $y_{2}$, where $y_{2}-y_{1}=1 \mathrm{~m}$, can be seen in Figure 9. With the same approach as in eq.(31), eq.(32), eq.(33), and eq.(34), the resolution can be improved by a factor of $m$ as expressed in (36)
$m=\frac{2\left(\Delta \phi_{1_{N / 2}}\right)}{\left(\Delta \phi_{1_{0}}-\Delta \phi_{1_{N}}\right)+\left(\Delta \phi_{2_{0}}-\Delta \phi_{2_{N}}\right)}$
Substituting eq.(22) to eq.(36), we obtain:
$m=\frac{2\left(\cos \left(\overline{\theta_{1_{N / 2}}}\right)-\cos \left(\overline{\theta_{2_{N / 2}}}\right)\right)}{\left(\cos \left(\overline{\theta_{1_{0}}}\right)-\cos \left(\overline{\theta_{1_{N}}}\right)\right)+\left(\cos \left(\overline{\theta_{2_{0}}}\right)-\cos \left(\overline{\theta_{2_{0}}}\right)\right)}$

$$
=\frac{2\left(\frac{x_{N / 2}}{\sqrt{x_{N / 2}^{2}+y_{1}^{2}}}-\frac{x_{N / 2}}{\sqrt{x_{N / 2}^{2}+y_{1}^{2}}}\right)}{\left(\frac{x_{0}}{\sqrt{x_{0}^{2}+y_{1}^{2}}}-\frac{x_{N}}{\sqrt{x_{N}^{2}+y_{1}^{2}}}\right)+\left(\frac{x_{N / 2}}{\sqrt{x_{0}^{2}+y_{2}^{2}}}-\frac{x_{N / 2}}{\sqrt{x_{N}^{2}+y_{2}^{2}}}\right)}
$$

$$
\begin{equation*}
=\frac{2 x_{N / 2}\left(\frac{1}{\sqrt{x_{N / 2}^{2}+y_{1}^{2}}}-\frac{1}{\sqrt{x_{N / 2}^{2}+y_{1}^{2}}}\right)}{x_{0}\left(\frac{1}{\sqrt{x_{0}^{2}+y_{1}^{2}}}+\frac{1}{\sqrt{x_{0}^{2}+y_{2}^{2}}}\right)-x_{N}\left(\frac{1}{\sqrt{x_{N}^{2}+y_{1}^{2}}}-\frac{1}{\sqrt{x_{N}^{2}+y_{2}^{2}}}\right)} \tag{37}
\end{equation*}
$$

From eq.(37), the radar resolutions for several values of $y_{1}$ and $y_{2}$ when $x_{0}=100 \mathrm{~m}$ are listed in Table 1, whereas the radar resolutions for $x_{0}=200 \mathrm{~m}$ and $x_{0}=400 \mathrm{~m}$ are listed in Table 2 and Table 3 respectively.

## IV. Conclusion

A very high angular resolution can be obtained by processing the phase information, independent from the change of relative velocity between the radar and the target. However, this method requires a very stable radar system with high phase resolution and accuracy. The maximum detectable distance in this method compromises with the maximum velocity of the vehicle and the FMCW center frequency. It is also found that larger radar-target distance results in higher angular resolution but lower range resolution.


Figure. 1. Transmitted and received frequencies of a sawtooth FMCW radar system as a function of time.

This trade-off requires further investigations both for FMCW and CW radar. It is possible that the implementation of this idea for CW radar is better than that for FMCW radar.

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Figure. 2. Basic idea of angle measurement.


Figure. 3. Phase difference in near-range, slow-vehicle radar measurements for $y=0$ and $y=1 \mathrm{~m}$.


Figure. 4. Phase difference in near-range, slow-vehicle radar measurements, $y=[0 ; 1] \mathrm{m}$ (Res $=0.1111 \mathrm{~m})$.


Figure. 5. Phase difference in near-range, slow-vehicle radar measurements for $y=4 \mathrm{~m}$ and $y=5 \mathrm{~m}$.


Figure. 6. Phase difference in near-range, slow-vehicle radar measurements, $y=[4 ; 5] \mathrm{m}($ Res $=0.5 \mathrm{~m})$.


Figure. 7. Phase difference in far-range, slow-vehicle radar measurements for $y=0$ and $y=1 \mathrm{~m}$.


Figure. 8. Phase difference in far-range, slow-vehicle radar measurements for $y=4$ and $y=5 \mathrm{~m}$.
TABLE 1.


Figure. 9. Calculation of resolution for high-speed vehicle radar measurements

RESOLUTIONS FOR HIGH-SPEED VEHICLE RADAR MEASUREMENTS WHEN $X=100$ METER

| $\mathbf{y 1}$ | $\mathbf{y 2}$ | $\mathbf{m}$ | Range <br> Resolution (cm) | Angular Resolution <br> (degree) |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 799.559 | 0.12506888 | 0.000716584 |
| 1 | 2 | 479.764 | 0.208435621 | 0.001194115 |
| 2 | 3 | 307.544 | 0.325156599 | 0.00186243 |
| 3 | 4 | 223.893 | 0.446642515 | 0.002557509 |
| 4 | 5 | 175.526 | 0.569716797 | 0.00326094 |
| 5 | 6 | 144.193 | 0.693513124 | 0.003967544 |
| 6 | 7 | 122.294 | 0.817698305 | 0.004675204 |
| 7 | 8 | 106.144 | 0.942116864 | 0.005382819 |
| 8 | 9 | 93.7483 | 1.066686497 | 0.006089709 |
| 9 | 10 | 83.9377 | 1.191359562 | 0.006795398 |
| 10 | 11 | 75.9817 | 1.316106574 | 0.007499514 |
| 11 | 12 | 69.4007 | 1.440908309 | 0.008201748 |
| 12 | 13 | 63.8671 | 1.565751699 | 0.008901828 |
| 13 | 14 | 59.1496 | 1.690627553 | 0.00959951 |
| 14 | 15 | 55.0804 | 1.815529221 | 0.010294566 |
| 15 | 16 | 51.5344 | 1.94045177 | 0.010986784 |

TABLE2.
RESOLUTIONS FOR HIGH-SPEED VEHICLE RADAR MEASUREMENTS
WHEN $X=200$ METER

| $\mathbf{y y y y y}$ | WHEN $X=$ 200 METER |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{y 1}$ | $\mathbf{y 2}$ | $\mathbf{m}$ | Range <br> Resolution (cm) | Angular Resolution <br> (degree) |
| 0 | 1 | 1599.53 | 0.062518377 | 0.000179102 |
| 1 | 2 | 959.732 | 0.104195731 | 0.000298491 |
| 2 | 3 | 615.214 | 0.162544966 | 0.000465621 |
| 3 | 4 | 447.876 | 0.223275899 | 0.000639541 |
| 4 | 5 | 351.123 | 0.284800734 | 0.000815688 |
| 5 | 6 | 288.445 | 0.346686436 | 0.000992809 |
| 6 | 7 | 244.638 | 0.408766451 | 0.001170413 |
| 7 | 8 | 212.331 | 0.470963057 | 0.001348263 |
| 8 | 9 | 187.535 | 0.533235107 | 0.00152623 |
| 9 | 10 | 167.91 | 0.595558777 | 0.00170423 |
| 10 | 11 | 151.994 | 0.657919322 | 0.001882209 |
| 11 | 12 | 138.83 | 0.720307124 | 0.002060127 |
| 12 | 13 | 127.76 | 0.782715644 | 0.00223795 |
| 13 | 14 | 118.324 | 0.84514028 | 0.002415654 |
| 14 | 15 | 110.183 | 0.9075777 | 0.002593214 |
| 15 | 16 | 103.09 | 0.970025431 | 0.002770612 |

Table 3.
RESOLUTIONS FOR HIGH-SPEED VEHICLE RADAR MEASUREMENTS
WHEN $X=400$ METER

| $\mathbf{y 1}$ | $\mathbf{y 2}$ | $\mathbf{m}$ | Range <br> Resolution (cm) | Angular Resolution <br> (degree) |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 3199.51 | 0.031254741 | 0.000044769 |
| 1 | 2 | 1919.72 | 0.052091037 | 0.000074614 |
| 2 | 3 | 1230.59 | 0.08126197 | 0.000116397 |
| 3 | 4 | 895.868 | 0.111623564 | 0.000159883 |
| 4 | 5 | 702.336 | 0.142382047 | 0.000203935 |
| 5 | 6 | 576.964 | 0.173320933 | 0.000248241 |
| 6 | 7 | 489.34 | 0.20435696 | 0.000292681 |
| 7 | 8 | 424.716 | 0.235451271 | 0.0003372 |
| 8 | 9 | 375.117 | 0.266583295 | 0.000381767 |
| 9 | 10 | 335.862 | 0.297741124 | 0.000426363 |
| 10 | 11 | 304.028 | 0.328917384 | 0.000470978 |
| 11 | 12 | 277.695 | 0.360107504 | 0.000515603 |
| 12 | 13 | 255.553 | 0.391307504 | 0.000560234 |
| 13 | 14 | 236.678 | 0.4225157994 | 0.000604865 |
| 14 | 15 | 220.395 | 0.453730471 | 0.000649495 |
| 15 | 16 | 206.207 | 0.484950299 | 0.00069412 |


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