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Application of modified least squares method for order reduction of commensurate higher order fractional systems

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ABSTRACT

The paper related to the reduction and investigation of family of commensurate fractional order systems. The fractional order system is first transformed to integer order and then a hybrid method is applied as a model reduction scheme. In this scheme the reduced denominator is acquired by least square method and the numerator is achieved by time moment matching. This formulated reduced integer model is reconverted to reduced fractional model. Three examples are conferred to authenticate and emphasize the efficacy of the proposed approach. The proposed method is verified by MATLAB simulation, and shows better performance in the estimation process.

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1. INTRODUCTION

Control systems are a multidisciplinary section of engineering and mathematics. The concept of non-integer order operators was deliberated first in the year 1974, in New Haven, Connecticut, USA [1].Many honored mathematicians notably Euler, Laplace, Fourier, Abel, Liourille, Riemann and Lauret focused on fractional calculus [2]. The dynamics of Fractional systems include differential or intergral operators of non integers systems [3].It handles with the adaptation of dynamic systems to achieve a desired reference model having its performance with a set of specifications.In the last decades of the last century there was continuing growth of the applications of fractional calculus mainly promoted by the engineering applications in the field of feedback control, systems theory and singal processing.Among the many applications of fractional controller to engineering problem Position control of a single link flexible robot , Automatic control of a hydraulic canal , Mechtronics , control of Power Electronic Buck converter applying fractional order control can be mentioned [4,5,6,7] . Any linear time invariant systems can be classified as integer and non -integer systems.The non integer systems further classified as Commensurate and non –commensurate systems. Systems in which the non-integer powers of integrators and differentiators are multiple of real numbers such separate class systems are called as commensurate type of fractional order systems .These type of systems can be converted into integer order systems with simple assumptions [8]. The concepts of classical and modern control systems can be adapted easily and the study of commensurate fractional order system becomes effortless as it can be transferred to integer order (IO) domain. Keeping this fact in mind we focus on considering the commensurate fractional order (FO) systems for research. The reduction theory is a well-known and long back adopted to integer systems [9], but little work carried out for fractional order systems [10,11]. The author has come up with a new reduction mode for the commensurate fractional order system. The proposed scheme conveys two techniques, namely modified least square method and time moment matching to determine the denominator and numerator of the diminished FO model respectively. Approximating of higher order system with its reduced model copes with more intricate problems in the field of control engineering. It also facilitates the understanding of system and reduces the computational burden in the simulation studies and controller design [12, 13, 14]. The present study demonstrates through three examples that the proposed method approaches well for stable non-minimum phase class of commensurate FO systems. The author systematized the paper as follows; section II demonstrates the fundamental components. Section III explains the algorithm of the recommended method and determine the corresponding Relative integral square errors (RISE) of step and impulse response [15,16] of the reduced models .Section IV verifies and compares the outcome with familiar techniques [17, 18, 19, 20, 21] and finally section V gives the main conclusions.

2. RELATED WORK

This segment explains the models and their representation a continuous time dynamic fractional order systems which can be drafted as

$$H(D^{\alpha_0\alpha_1\alpha_2\cdots\alpha_m})(y_1, y_2, \cdots, y_l) = G\left(D^{\beta_0\beta_1\beta_2\cdots\beta_n}\right)(u_1, u_2, \cdots, u_k) \quad (1)$$

where, y_i, u_j are functions of time (i = 1,2,3…..l and j =1,2,3…..k) and H (.), G (.) are the combinational laws of the fractional order derivative operator. The above linear time invariant single variable system is denoted with the following equation as

$$H(D^{\alpha_0\alpha_1\alpha_2\cdots\alpha_n})y(t) = G\left(D^{\beta_0\beta_1\beta_2\cdots\beta_m}\right)u(t) \quad (2)$$

with

$$H(D^{\alpha_0\alpha_1\alpha_2\cdots\alpha_n}) = \sum_{k=0}^{n} a_k D^{\alpha_k}$$

$$G(D^{\beta_0\beta_1\beta_2\cdots\beta_m}) = \sum_{k=0}^{m} b_k D^{\beta_k}$$

Where a_k, $b_k \in \mathbb{R}$. Or, explicitly,

$$a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \cdots + a_0 D^{\alpha_0} y(t) = b_m D^{\beta_m} u(t) + b_{m-1} D^{\beta_{m-1}} u(t) + \cdots + b_0 D^{\beta_0} u(t) \quad (3)$$

If all the orders of derivation, in the preceding equation are integer multiples of a base order α, that is, α_k, $\beta_k = k\alpha$, $\alpha \in \mathbb{R}+$, then such systems are classified as commensurate order and the above equation becomes

$$\sum_{k=0}^{n} a_k D^{k\alpha} y(t) = \sum_{k=0}^{m} b_k D^{k\alpha} u(t) \quad (4)$$

The transfer function for the above fractional equation is obtained by applying the Laplace transform with initial zero conditions.

3. METHODOLOGY

Consider a continuous commensurate FO system, with transfer function represented as

$$G(s) = \frac{M(s)}{N(s)} = \frac{f_0 + f_1(s^\gamma)^1 + \cdots + f_{n-1}(s^\gamma)^{n-1}}{e_0 + e_1(s^\gamma)^1 + \cdots + e_{n-1}(s^\gamma)^{n-1} + e_n(s^\gamma)^n} \quad (5)$$

On substituting $\lambda = s^\gamma$. The fractional order system becomes integer-order system as

$$G_n(\lambda) = \frac{M(\lambda)}{N(\lambda)} = \frac{f_0 + f_1(\lambda)^1 + \cdots + f_{n-1}(\lambda)^{n-1}}{e_0 + e_1(\lambda)^1 + \cdots + e_{n-1}(\lambda)^{n-1} + e_n(\lambda)^n} \quad (6)$$

The required r^{th} order reduced model for commensurate fractional order system is

$$G_r(\lambda) = \frac{M_r(\lambda)}{N_r(\lambda)} = \frac{p_0 + p_1\lambda + \cdots + p_{r-1}\lambda^{r-1}}{q_0 + q_1\lambda + \cdots + q_{r-1}\lambda^{r-1} + \lambda^r} \quad (7)$$

3.1. The Proposed method follows the consecutive steps

Step 1: Process to determine the denominator coefficients of reduced order model

Enlarging $G_n(\lambda)$ about λ =0, to acquire the time moment proportional (H_i) are given by

$$G_n(\lambda) = H_0 + H_1\lambda + H_2\lambda^2 + \cdots = \sum_{i=0}^{\infty} H_i \lambda^i \quad (8)$$

Similarly enlarging $G_n(\lambda)$ about λ =∞, to obtain Markov parameters (R_j) given by

$$G_n(\lambda) = R_1\lambda^{-1} + R_2\lambda^{-2} + R_3\lambda^{-3} + .. = \sum_{j=0}^{\infty} \frac{R_j}{\lambda^j} \quad (9)$$

Assessing (7) and (8) to maintain't' time moments of the original system in the reduced model provides the consecutive set of equations:

$$\left.\begin{array}{l} p_0 = q_0 H_0 \\ p_1 = q_1 H_0 + q_0 H_1 \\ \ldots.. \\ \ldots.. \\ \ldots.. \end{array}\right\} \quad (10)$$

$$\left.\begin{array}{l} p_{r-1} = q_{r-1} H_0 + \cdots + q_0 H_{r-1} \\ -H_0 = q_{r-1} H_1 + \cdots + q_1 H_{r-1} + q_0 H_r \\ -H_1 = q_{r-1} H_2 + \cdots + q_1 H_r + q_0 H_{r+1} \\ -H_{t-r-1} = q_{r-1} H_{t-r} + \cdots + q_0 H_{t-1} \end{array}\right\} \quad (11)$$

A subsequent set of equations obtained on assessing (7) and (9) to preserve 'v' Markov parameters of the original system in its reduced model.

$$\left.\begin{array}{l} p_{r-1} = R_1 \\ p_{r-2} = R_1 q_{r-1} + R_2 \\ p_0 = R_1 q_1 + R_2 q_2 + \cdots + R_r \\ R_1 q_0 + R_2 q_1 + \cdots + R_r q_{r-1} = -R_{r+1} \\ R_{v-r} q_0 + \cdots + R_{v-1} q_{r-1} = -R_v \end{array}\right\} \quad (12)$$

Step 2: Omission of p_j (j=0, 1, 2… r-1) in (12) by substituting (10) turn over the reduced denominator coefficients as the solution of

$$\underbrace{\begin{bmatrix} H_{t-1} & H_{t-2} & \cdots & H_{t-r+1} & H_{t-r} \\ H_{t-2} & H_{t-3} & \cdots & \cdots & H_{t-r-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ H_{r-1} & H_{r-2} & \cdots & H_1 & H_0 \\ H_{r-2} & H_{r-3} & \cdots & H_0 & -R_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -R_{v-r} & -R_{v-r-1} & \cdots & -R_{v-2} & -R_{v-1} \end{bmatrix}}_{W} * \underbrace{\begin{bmatrix} q_0 \\ q_1 \\ \vdots \\ \vdots \\ \vdots \\ q_{r-2} \\ q_{r-1} \end{bmatrix}}_{Q} = \underbrace{\begin{bmatrix} -H_{t-r-1} \\ -H_{t-r-2} \\ \vdots \\ R_1 \\ R_2 \\ \vdots \\ R_v \end{bmatrix}}_{u} \quad (13)$$

(or) W*Q= u in matrix vector form.

Equation (13) is comparable to compare all the significant time moments (H_i) and Markov parameters (R_j) of $G_n(\lambda)$. Where $i = 0, 1, \ldots, t-1$ and $j = 1, 2, \ldots, v$.

Step 3: The coefficients of 'Q' vector of equation (13) can only be estimated in least square sense, i.e.

$$Q = [W^T \ W]^{-1} W^T u \quad (14)$$

Step 4: Finally the reduced denominator is obtained as

$$N_r(\lambda) = q_0 + q_1\lambda + \cdots + q_{r-1}\lambda^{r-1} + \lambda^r \quad (15)$$

Step 5: After obtaining the reduced denominator coefficients the (7) can be replaced about λ =0. Then the time moments proportionals, 'Ti' given as

$$G_r(\lambda) = \sum_{i=0}^{\infty} T_i \lambda^i \quad (16)$$

Equate (8) with (16) to obtain the unknown reduced numerator coefficients

Thus the numerator coefficients $p_0, p_1, \ldots p_{r-1}$ of the reduced model obtained as

$$M_r(\lambda) = p_0 + p_1\lambda + \cdots + p_{r-1}\lambda^{r-1} \quad (17)$$

By re-substituting $\lambda = s^{\gamma}$ the Fractional order reduced model obtained as

$$G_r(s) = \frac{p_0+p_1(s^{\gamma})^1+\cdots+p_{n-1}(s^{\gamma})^{n-1}}{q_0+q_1(s^{\gamma})^1+\cdots+q_{n-1}(s^{\gamma})^{n-1}+q_n(s^{\gamma})^n} \quad (18)$$

The relative mapping errors of the original system related to its reduced model are expressed by means of the relative integral square error (RISE) criterion, which are given by [15,16]:

$$I = \int_0^{\infty}[X(t) - X_r(t)]^2.\,dt \,/ \int_0^{\infty} X^2(t).\,dt \quad (19)$$

$$J = \int_0^{\infty}[Y(t) - Y_r(t)]^2.\,dt \,/ \int_0^{\infty}[Y(t) - Y(\infty)]^2\,.\,dt \quad (20)$$

Where
I = Relative impulse Integral square error
J = Relative step Integral square error
$X(t)$ and $Y(t)$ are the impulse and step response of original system, respectively and $X_r(t)$and $Y_r(t)$ are their approximations.

4. RESULTS AND DISCUSSION

To show the flexibility and accuracy, the author applied the proposed procedure to some of the numerical examples that are taken from the literature and correlated with the responses of existing methods.

Example 1

Let us consider a commensurate fraction order transfer function [8]

$$G(s) = \frac{s^{2.4}+7s^{1.6}+24s^{0.8}+24}{s^{3.2}+10s^{2.4}+35s^{1.6}+50s^{0.8}+24} \quad (21)$$

Substitute λ=$s^{0.8}$, an integer fourth order transfer function in terms of λ obtained as

$$G_4(\lambda) = \frac{24 + 24\lambda + 7\lambda^2 + \lambda^3}{24 + 50\lambda + 35^2 + 10\lambda^3 + \lambda^4} \quad (22)$$

Where $H_0 = 1$; $H_1 = -1.0833$; $H_2 = 1.0902$; $H_3 = -1.0665$ are the four time moments of G(λ).

$$\begin{bmatrix} 1.09028 & -1.08333 \\ -1.06655 & 1.09028 \end{bmatrix} * \begin{bmatrix} q_0 \\ q_1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1.0833 \end{bmatrix}$$

Using (14) the denominator coefficients of the reduced model

$$Q = \begin{bmatrix} 2.5034 \\ 3.4425 \end{bmatrix} \quad (23)$$

Equate (8) with (16) the numerator coefficients of the reduced model can be obtained as

$$p_0 = 2.5034\,;\; p_1 = 0.7304 \quad (24)$$

From the above coefficients of denominator and numerator the reduced model transfer function is obtained as

$$G_2(\lambda) = \frac{0.7304\lambda+2.5034}{\lambda^2+3.4425\lambda+2.5034} \quad (25)$$

Re-substitute $\lambda = s^{0.8}$, the reduced fractional model obtained using proposed method

$$G_r(s) = \frac{0.7304s^{0.8}+2.5034}{s^{0.16}+3.4425s^{0.8}+2.5034} \quad (26)$$

Reduced model fractional order transfer function obtained using Saxena method

$$G_r(s) = \frac{0.28389s^{0.8}+1}{0.3986s^{1.6}+1.3744s^{0.8}+1} \quad (27)$$

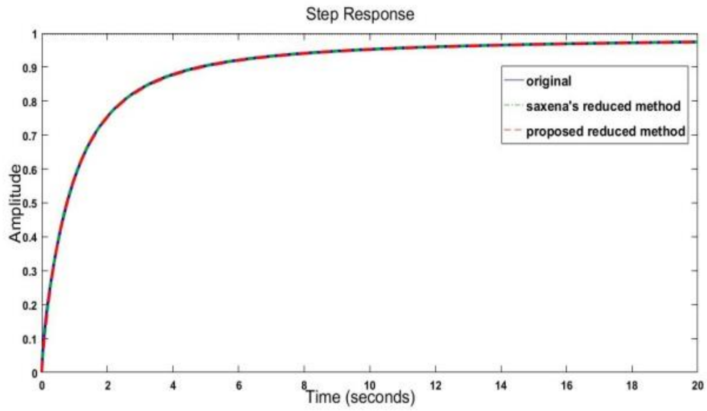

Figure 1. Comparison of Step responses of the original system with approximate models

Example 2

Consider a commensurate fraction system [11] having the form described in (5) where

$$M(s) = 18s^{8.4} + 514s^{7.2} + 5982s^{6.0} + 36380s^{4.8} + 122664s^{3.6} + 222088s^{2.4} + 185760s^{1.2} + 40320$$
$$N(s) = s^{9.6} + 36s^{8.4} + 546s^{7.2} + 4536s^{6.0} + 22449s^{4.8} + 67284s^{3.6} + 118124s^{2.4} + 109584s^{1.2} + 40320 \quad (28)$$

Substitute $\lambda = s^{1.2}$ in the above equation (28) the corresponding integer model given as

$$M(\lambda) = 18\lambda^7 + 514\lambda^6 + 5982\lambda^5 + 36380\lambda^4 + 122664\lambda^3 + 222088\lambda^2 + 185760\lambda + 40320$$
$$N(\lambda) = \lambda^8 + 36\lambda^7 + 546\lambda^6 + 4536\lambda^5 + 22449\lambda^4 + 67284\lambda^3 + 118124\lambda^2 + 109584\lambda + 40320 \quad (29)$$

The second order reduced model obtained from the proposed method is

$$G_2(\lambda) = \frac{18\lambda + 5.714764}{\lambda^2 + 7.203178\lambda + 5.714764} \quad (30)$$

Now re-substitute the $\lambda = s^{1.2}$ in above equation the reduced fractional model obtained by proposed method

$$G_r(s) = \frac{18s^{1.2} + 5.714764}{s^{2.4} + 7.203178s^{1.2} + 5.714764} \quad (31)$$

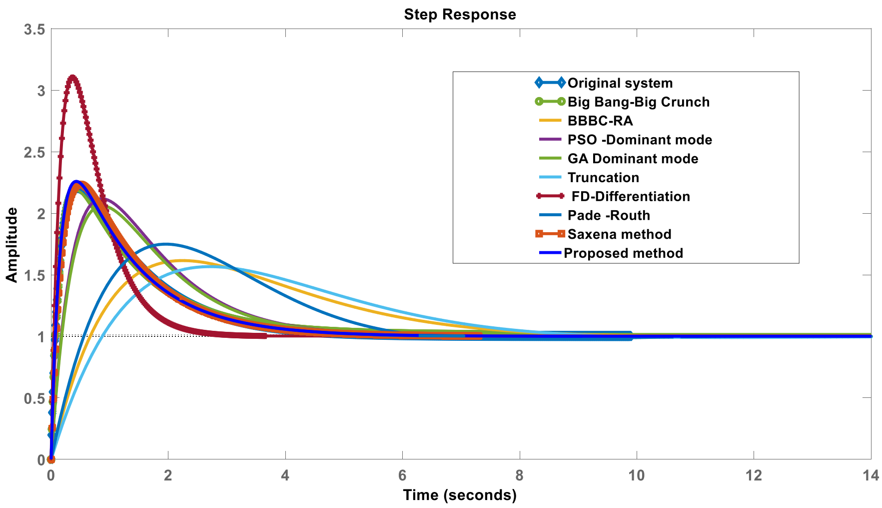

Figure 2. Comparison of Step responses of the original system with reduced models obtained from other methods

The Figure 2 depict the comparision of the step response of original system and the reduced model obtained by various conventional and mixed heuristic techniques.They are Big Bang Big Crunch(BBBC), BBBC-Routh Approximation (BBBC-RA) [17], Particle swarm optimization–Dominant mode (PSO-dominant mode,Genetic Algorithm(GA) dominant mode)[18],Truncation method[19] ,Factor Division (FD) differentiation method [20] ,Pade Routh Method[21] and Saxena Method .The performance of the proposed technique can be clearly seen superior from the time response plots wherein the response of the reduced model obatained via proposed approach exactly matches the response of original system.In addition to the time response plot the transfer functions obtained from various model reduction techniques and their corresponding Relative integral square error (RISE) of step and Impulse responses are tabulated in Table 1.The values in Table 1 clearly shows the superiority of the proposed method with minimum values of performance Indices.

Table 1. Performance observation of the proposed method with other existing methods using Relative integral square error (RISE) of step and Impulse responses.

Method	Reduced Transfer function	RISE step	RISE impulse
Proposed method	$\frac{18s^{1.2}+5.714764}{s^{2.4}+7.203178s^{1.2}+5.714764}$	0.001681	0.001506
Big Bang Big Crunch	$\frac{150s^{1.2}+48.94}{8.995s^{2.4}+61.49s^{1.2}+48.28}$	0.001967	0.002376
BBBC-RA	$\frac{2.06774s^{1.2}+0.43184}{s^{2.4}+1.17368s^{1.2}+0.43184}$	1.313403	0.754134
PSO –Dominant mode	$\frac{3.657s^{1.2}+1}{0.5s^{2.4}+1.5s^{1.2}+1}$	0.188651	0.273606
GA Dominant mode	$\frac{7.076s^{1.2}+2}{s^{2.4}+3s^{1.2}+2}$	0.185438	0.284902
Truncation method	$\frac{185760s^{1.2}+40320}{118124s^{2.4}+109584s^{1.2}+40320}$	1.632963	0.819787
FD Differentiation	$\frac{24.5506s^{1.2}+9.557}{s^{2.4}+6.494s^{1.2}+9.557}$	0.340766	0.262660
Pade-Routh	$\frac{151772.544s^{1.2}+40320}{65520s^{2.4}+75600s^{1.2}+40320}$	1.127123	0.731906
Saxena method	$\frac{3.10845s^{1.2}+1.005}{0.2075s^{2.4}+1.2434s^{1.2}+1}$	0.002678	0.014689

Example 3

Let us consider a high order commensurate fraction system [8] represented in equation (5) where

$$M(s) = -4000s^{4.8} - 26000s^{3.6} + 240000s^{2.4} - 690000s^{1.2} + 750000$$
$$N(s) = s^{12} + 75s^{10.8} + 2193s^{9.6} + 31914s^{8.4} + 251620s^{7.2} + 1167000s^{6} + 3357000s^{4.8} + 6032000s^{3.6} + 6433000s^{2.4} + 3563000s^{1.2} + 750000 \quad (32)$$

Substitute $\lambda = s^{1.2}$ in the above equation (33) the corresponding integer model obtained as follows:

$$M(\lambda) = -4000\lambda^{4} - 26000\lambda^{3} + 240000\lambda^{2} - 690000\lambda + 750000$$
$$N(\lambda) = \lambda^{10} + 75\lambda^{9} + 2193\lambda^{8} + 31914\lambda^{7} + 251620\lambda^{6} + 1167000\lambda^{5} + 3357000\lambda^{4} + 6032000\lambda^{3} + 6433000\lambda^{2} + 3563000\lambda + 750000 \quad (33)$$

The second order reduced model obtained from the proposed method is

$$G_2(\lambda) = \frac{-0.246330\lambda+0.177883}{\lambda^2+0.762385\lambda+0.177883} \quad (34)$$

Now re-substitute the λ=$s^{1.2}$ in above equation the reduced fractional model obtained by proposed method

$$G_r(s) = \frac{-0.246330s^{1.2}+0.177883}{s^{2.4}+0.762385s^{1.2}+0.177883} \quad (35)$$

Reduced model fractional order transfer function obtained using Saxena method

$$G_r(s) = \frac{1.27s^{2.4}-0.898s^{1.2}+1}{5.622s^{2.4}+4.2859s^{1.2}+1} \quad (36)$$

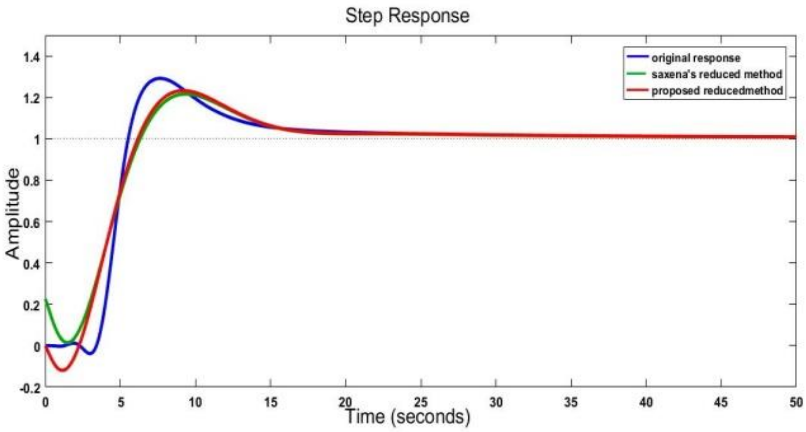

Figure 3. Comparison of Step responses of the original system with approximate models

Table2. Time Domain Specifications

Specifications	Tr	Tp	Ts
Example 1			
Original	4.5759	47.8319	21.5300
Proposed	4.5769	50.7867	21.5829
Saxena [compared paper method]	4.6070	45.2944	21.7320
Example 2			
Original	650.5678	0.1162	1.0912e+03
Proposed	610.2386	0.1151	1.0867e+03
Saxena [compared paper method]	813.9728	0.1151	1.0802e+03
Example 3			
Original	1.5263	7.6547	25.6965
Proposed	2.9597	9.1416	26.6941
Saxena [compared paper method]	2.5247	9.4064	27.5625

From the figures 1, 2 and 3 it is noticed that the steady state error of the two fractional systems i.e original and reduced model is zero and the overall performance sounds to be akin with each other. The approximate model co-efficient and their corresponding RISE values for step and impulse are given in table1 and the time domain specifications are shown in table 2 respectively. The table 1 shows the superioty of the proposed method with minimum vales of I and J when compared with the other familiar techniques .The table 2 clearly shows the reduced fractional model obtained exactly approximates the response of the Original fractional system.

5. CONCLUSION AND FUTURE SCOPE

In this paper, the author proposed a method of reduced order modeling of commensurate type FO system. A blended model order reduction tool is adopted to extract low order system which constitute a modified least squares and time moment matching algorithms. As the method associated with matrix vector algebra, its estimation is clear, efficient and also develops a precise stable model. Simulations results validate the dominance of this scheme where the original system may be retrieved with the approximate model thereby shorten the design procedure.

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