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Comment

# "Brainland" vs. "flatland": How many dimensions do we need in brain dynamics? Comment on the paper "The unreasonable effectiveness of small neural ensembles in high-dimensional brain" by Alexander N. Gorban et al.

Roberto Barrio

*Departmento de Matemática Aplicada and IUMA, University of Zaragoza, E50009 Zaragoza, Spain*

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In their review article (this issue) [1], Gorban, Makarov and Tyukin develop a successful effort to show in biological, physical and mathematical problems the relevant question of how high-dimensional brain can organise reliable and fast learning in the high-dimensional world of data using reduction tools. In fact, this paper, and several recent studies, focuses on the crucial problem of how the brain manages the information it receives, how it is organized, and how mathematics can learn about this and use dimension related techniques in other fields. Moreover, the opposite problem is also relevant, that is, how we can recover high-dimensional information from low-dimensional ones, the relevant problem of embedding dimensions (the other side of reducing dimensions).

The human brain is a real open problem and a great challenge in human knowledge. The way the memory is codified is a fundamental problem in Neuroscience. As mentioned by the authors, the idea of blessing the dimensionality (and the opposite curse of dimensionality), are becoming more and more relevant in machine learning.

## "Brainland": managing high-dimensional inputs

For years, the *curse of dimensionality*, that is, that many problems become much harder as the dimension increases, has been the paradigm in machine learning. And so, a common approach is to transform the data, reducing the dimension, and finally going back to the original space. Recently, another paradoxical situation has been discovered, that some problems become much easier when the dimension increases. This is the *blessing of dimensionality* [2]. A prototypical mathematical example is the Johnson-Lindenstrauss dimension reduction lemma [3]. The authors, in this review article, focus on some manifestations of the blessing of dimensionality phenomenon. By using techniques of measure theory, like the theory of measure concentration phenomena and stochastic separation theorems, they provide some examples and theorems. It is remarkable the link of several philosophical ideas and the opinions of well known physicists, like Stephen Hawking, Einstein, ... and how the use of the blessing of dimensionality is

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*E-mail address:* rbarrio@unizar.es.

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connected with the brain pattern assimilation problem (the detection of visual patterns, like the idea of "grandmother cells" –Jennifer Aniston's cell– commented in the article). In this case, it is shown in the paper that this idea, that some neurons react selectively to some patterns, illustrates how some brain processes are managed. There are deep reasons for the appearance of small neuronal ensembles (like the idea of "grandmother's cell") with certain functions in the multidimensional brain system. Understanding these problems of visual patterns in the brain can be useful in computer and mathematical techniques of pattern recognition. Nowadays, the role of techniques of manifold learning methods is important [4,5] in understanding the structure of multidimensional patterns. To manipulate, organize and use high-dimensional data can be difficult. One approach is to use spectral methods for dimensionality reduction, and in particular to assume that the data lies on an embedded non-linear manifold of lower dimension, and hence the data can be visualised in the low-dimensional space. This is another example of how high-dimension is useful in selecting more suitable low-dimensional spaces to represent and organize the data.

In fact, the use of small neuronal ensembles is basic in several areas of Neuroscience, as not all the neuronal processes are generated in the brain. A lot of physiological phenomena are controlled by small groups of neurons, the Central Pattern Generators (CPG) that produce rhythmic outputs in the absence of input [6,7], like in walking, breathing, flying, and swimming movements.

On the way of understanding how the brain processes high-dimensional information, recently in a remarkable paper (related with the Blue Brain Project) it has been used powerful Mathematical machinery, Algebraic Topology, to unreveal in the brain neural network how cliques of neurons provide a link between structure and function [8]. This fact is related with the above idea of "grandmother's cell" and the blessing of dimensionality, giving the impression that the brain operates in many dimensions and makes use of the blessing of dimensionality principle.

## "Flatland": using reduced dimensional inputs

On the other hand, once we are just studying small group of neurons, or even just one, we are now in a reduced dimensional space. Here, the Computational and Mathematical Neuroscience has managed to obtain results about different particularities of neuron dynamics, like the existence of chaotic dynamics [9,10]. These are reduced models, and as indicated in the article, some simple models of several neuronal mechanisms of memory provide useful insights into the process.

The question now is that the brain activity takes place in three spatial-plus time dimensions. But indeed, it has been detected that various brain activities can be better analyzed as trajectories embedded in phase spaces of dimensions higher than the trivial ones. That is, we are living in "flatland" [11], but we see some "brainland" phenomena which needs more dimensions to be explained [12], the multidimensional brain.

In fact, this is a manifestation of another common fact: we usually observe low dimensional data that live in a higher dimensional space. A typical example is in the analysis of a scalar time series (or of small dimension $n$), that are our observable data, that correspond to a higher dimensional object ("flatland" vision of reality). Nowadays there are several mathematical techniques of embedding [13] such a time series in an $m$-dimensional space, with $n < m$. Obviously, the choice of the observable is relevant in order to extract useful information from the embedded attractor. Here, a relevant result is the delay embedding theorem of Floris Takens [14]. It gives the conditions under which a smooth attractor can be reconstructed from observations.

Therefore, a crucial problem nowadays is the "game" of moving from "brainland" to "flatland" and backward. Here, theoretical and experimental results on blessing the dimensionality are relevant.

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