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The Chirality Theorem

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Abstract. We show how chirality of the weak interactions stems from string independence in the string-local formalism of quantum field theory.

Σωκράτης – ὁ νοῦν ἔχων γεωργός, ὧν σπερμάτων
κήδοιτο καὶ ἔγκαρπα βούλοιτο γενέσθαι, πότερα
σπουδῇ ἂν θέρους εἰς Ἀδώνιδος κήπους ἀρῶν...;
– Plato (*Phaidros*, 276b) [1]

1. Introduction

Unanswered questions abound in electroweak theory [2]. Only time will tell which ones were prescient, and which born only from theoretical prejudice [3]. A paramount trait of flavourdynamics is the chiral character of the interactions in which fermions and the massive vector bosons participate. A literature search shows that most textbooks dispatch this trait in one word: it is a *fact.* There are a few exceptions. The book by Peskin and Schroeder discusses at some length how left-handed and right-handed components of fermions can come to see (representations of, if you wish) different gauge groups [4, Chap. 19]. The posthumous, reflective book by Bob Marshak [5, Chaps. 1 and 6], discoverer (together with E. C. G. Sudarshan) of the Vector-Axial theory, interestingly elevates the "fact" to a principle, that of chirality invariance, or "neutrino paradigm".

Nevertheless, on the face of it, there is a mystery here, setting flavourdynamics apart from chromodynamics. That cannot be solved by invoking the Glashow–Weinberg–Salam (GWS) model, which introduces chirality by hand from the outset.

The aim of this paper is to tackle this riddle through the theory of string-local quantum fields (SLF). This conceptual framework was introduced in [6,7], improving on old proposals by Mandelstam [8] and Steinmann [9]. It is largely the brainchild of Schroer [10].

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At the considerable price of an extra variable, SL fields appear to offer advantages over the ordinary sort. We summarily list them here.

- The string-local fields evade the theorem that it is impossible to construct on Hilbert space a *vector* field for photons, and more generally for corresponding representations associated with higher fixed-helicity massless particles [11, Sect. 5.9].
  For this reason, the concept of gauge fades into the background.
- Other improved formal properties include a better ultraviolet behaviour for spin and helicity $> \frac{1}{2}$; this turns out to be same for all bosons as for scalar particles, and for fermions as for spin-$\frac{1}{2}$ particles.[1] The upshot is that perturbative renormalization of SLF models should take place without calling upon ghost fields, BRS invariance and the like, since in principle one need not surrender positivity of the energy and of the state spaces for the physical particles. It is fair to say, however, that renormalization of theories with SL field theory is still a work in progress.
- The reach of quantum field theory is enlarged, since the (boson and fermion) Wigner unbounded-helicity particles [14], with Casimirs $P^2 = 0$, $W^2 < 0$, that have no corresponding pointlike fields [15,16], become admitted into the realm of QFT through SL fields [6,7,17].
- Furthermore, SLF proves its worth by shedding light on some phenomenological conundrums of the current theory of fundamental forces and particles. (Chief among them, after chirality, is the observation that "the SM accounts for, but does not explain, electroweak symmetry breaking" [18].)

We are going to show that the physical particle spectrum (charge and mass structure) of the interaction carriers in the electroweak sector, including the scalar particle, determines their relative coupling strengths with the fermion sector entirely, and in particular *forces* the couplings of the massive bosons to fermions to be parity-violating.

In more detail, our input (particle and coupling types) is the experimental datum.

- The *particle types* are the electron, positron, neutrino and antineutrino; the massive vector bosons W_1, W_2 and Z, and the photon; plus a scalar (Higgs) particle.[2]
  Their *masses* obey $m_Z > m_W > 0$, and the photon is massless. The electron and Higgs particle are massive; the masses m_e, m_ν and m_H are otherwise unconstrained, but are assumed to be given.
  The corresponding *electric charges*: the Z and Higgs bosons, the neutrino and antineutrino $\nu, \bar{\nu}$ are neutral; the electron e and W_- boson have charge -1; the positron $\bar{e}$ and W_+ have charge $+1$.

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[1] Arguably, that is inherited from the amazingly good behaviour of the field strengths themselves, beyond naïve power counting, independently of spin, uncovered not long ago [12, 13].

[2] It will be enough here to consider just one generation of leptons: bringing up the full structure of the fermion multiplets only complicates the proof's notation in a way immaterial to the purpose.

- The *couplings* are of two types. For the purely bosonic couplings, see the beginning of Sect. 4.

  For the couplings between bosons and fermions we make the most general Ansatz which respects electric charge conservation, Lorentz invariance and renormalizability (scaling dimension ≤ 4).

  Apart from these general restrictions, our sole assumption is that in photon-fermion couplings, the photon couples only to charged fermions, so it does not couple to the neutrino or antineutrino, and even this could be relaxed. All other coupling constants are left open.

Our powerful tool is the requirement that physical quantities like the $\mathbb{S}$-matrix must be independent of the string direction. This principle is quite restrictive and, as we show here, in fact fixes all coupling constants, bar the overall strength. In particular, it turns out that:

- the neutrino is completely chiral in that only left-handed[3] neutrinos couple;
- the electron also couples in a parity-violating way;
- the Higgs particle couples only to scalar (and not to pseudoscalar) Fermi currents.

This is our **chirality theorem**.

The proof, rigorous within perturbation theory, is achieved entirely within the string-local scheme. It is simple, in that it requires only consideration of tree graphs up to second order. Going *a posteriori* from our framework to the GWS model for fermions is both trivial and almost inconsequential; nevertheless, we indicate how to do it in an appendix.

A valid argument for chirality, with the same outcome as ours, can be made and has indeed been made before, within the conventional framework—see [19–21]; we owe these works a lot. Apparently that proof was scarcely heeded, for reasons not easy to understand. It is certainly couched in the language of (the causal version of) gauge theory, keeping its ungainly retinue of unphysical fields, and there is some circularity in it, since the Kugo–Ojima asymptotic fields invoked *ab initio* have to be derived first. Our method provides a cleaner, more "native" form. Still, theirs was a good case, and we are keen to employ new tools to reclaim it.

The plan of the article is as follows. Section 2 is a précis on free string-local fields. Section 3 reviews the basics of perturbation theory and Epstein–Glaser renormalization, as adapted to SLF, and introduces the simple principle of physical **string independence** governing SLF couplings. The next two sections examine constraints imposed on couplings with fermions by string independence already at the first-order level. Section 6 displays a method, due to one of us, to construct time-ordered products involving SLF for tree diagrams at second order.

Once that has been digested, the rest of the proof, performed in Sect. 7, proceeds by a series of lemmas, of interest in themselves, whose verifications reduce to fairly straightforward calculations, entirely determining the couplings.

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[3] Or right-handed ones—the theory of course cannot tell which.

In particular, chirality of flavourdynamics emerges as an inescapable consequence of string independence, given the mentioned physical spectrum of intermediate vector bosons. Section 8 is the conclusion.

The supplementary sections deal with a few relevant side questions. "Appendices A and B" furnish computational details. "Appendix C" verifies locality for the stringy fields. "Appendix D" manufactures the GWS model from the ascertained chiral coupling constants.

2. String-Local Fields

To define the SLF, we start from free Faraday tensor fields on Minkowski space $\mathbb{M}_4$. These can be built from Wigner's spin 1 or helicity ± 1 unitary, irreducible representations of the restricted Poincaré group [14], by use of appropriate creation operators $\alpha_r^\dagger(p)$ and polarization dreibein or zweibein $e_r^\mu(p)$, under the form:

$$F_a^{\mu\nu}(x) := \sum_r \int \mathrm{d}\mu(p) \left[\mathrm{e}^{i(px)} \left(ip^\mu e_r^\nu(p) - ip^\nu e_r^\mu(p) \right) \alpha_{r,a}^\dagger(p) \right.$$
$$\left. + \mathrm{e}^{-i(px)} \left(-ip^\mu e_r^\nu(p)^* + ip^\nu e_r^\mu(p)^* \right) \alpha_{r,a}(p) \right], \tag{2.1}$$

where $\mathrm{d}\mu(p) := (2\pi)^{-3/2} \, d^3\boldsymbol{p}/2E(\boldsymbol{p})$; we use the notation $(ab) := g_{\lambda\kappa}a^\lambda b^\kappa = a^0 b^0 - \boldsymbol{a}\cdot\boldsymbol{b}$ for Minkowski inner products. Such fields are of the Lorentz transformation type $(1,0) \oplus (0,1)$—see [11, Sect. 5.6]. Consult also [22] in this respect. Free string-local *potential* fields are determined from the F_a:

$$A_a^\mu(x,l) := \int_0^\infty \mathrm{d}t \, F_a^{\mu\lambda}(x+tl) \, l_\lambda \,, \tag{2.2}$$

with $l = (l^0, \boldsymbol{l})$ a null vector. By [half-]string we understand the set of points $\{x + tl\}$, with $t \geq 0$. Each of the A_a lives on the same Fock space as F_a.

The main properties of the potential fields are as follows:

- Transversality: $\left(l \, A_a(x,l) \right) = 0$; and $\left(\partial A_a(x,l) \right) = 0$ in the massless boson case.[4]
- Pointlike differential: $\partial^\mu A_a^\lambda(x,l) - \partial^\lambda A_a^\mu(x,l) = F_a^{\mu\lambda}(x)$, or $dA_a = F_a$ for short.
- Covariance: let U denote the second quantization of the mentioned unitary representations of the restricted Poincaré group on the one-particle states. Then

$$U(c,\Lambda)A_a^\mu(x,l)U^\dagger(c,\Lambda) = A_a^\lambda(\Lambda x + c, \Lambda l) \, \Lambda_\lambda{}^\mu$$
$$= \left(\Lambda^{-1} \right)_\lambda^\mu A_a^\lambda(\Lambda x + c, \Lambda l).$$

- Locality (causality): $[A_a^\mu(x,l), A_a^\lambda(x',l')] = 0$ when the strings $\{x + tl\}$ and $\{x' + t'l'\}$ are causally disjoint.

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[4] Here and later, $(\partial A) = \partial_\mu A^\mu$ denotes a divergence.

The first three properties are nearly obvious. The last one is subtler. It follows from (an easy variant of) the powerful argument in [23], based on modular localization theory, spelled out in "Appendix C".

Explicitly, in terms of (2.1), one finds that:

$$A_a^\mu(x,l) = \sum_r \int d\mu(p) \big[e^{i(px)} u_r^\mu(p,l) \alpha_{r,a}^\dagger(p) + e^{-i(px)} u_r^\mu(p,l)^* \alpha_{r,a}(p) \big],$$

with

$$u_r^\mu(p,l) := \int_0^\infty dt \, e^{it(pl)} i \big(p^\mu e_r^\lambda(p) - p^\lambda e_r^\mu(p) \big) l_\lambda = e_r^\mu(p) - p^\mu \frac{(e_r(p)\, l)}{(pl)} . \quad (2.3)$$

Note that in the massless case, the denominator (pl) may vanish; nonetheless, $(e_r(p)\,l)/(pl)$ is locally integrable with respect to the Lorentz-invariant measure $d\mu(p)$. In keeping with the nomenclature of [6,7], the quantities $u_r^\mu(p,l)$, $u_r^\mu(p,l)^*$, and similar ones for stringlike or pointlike fields, are here called *intertwiners*.

In this paper the set $\{F_a\}$ above includes one such field for each of the physical particles, universally denoted $W^\pm$, Z, γ. For the massive ones, it does prove useful to consider the spinless string-local *escort* fields:

$$\phi_b(x,l) := \sum_r \int d\mu(p) \left[e^{i(px)} \frac{i(e_r(p)\,l)}{(pl)} \alpha_{r,b}^\dagger(p) + e^{-i(px)} \frac{-i(e_r(p)\,l)^*}{(pl)} \alpha_{r,b}(p) \right]. \quad (2.4)$$

We remark that

$$A_b^\mu(x,l) - \partial^\mu \phi_b(x,l) =: A_b^{\mathrm{P},\mu}(x) \quad (2.5)$$

defines pointlike *Proca* fields, so that $dA_b^\mathrm{P} = F_b$. All these fields live on the same Fock spaces as the F_b and have the same mass. Moreover:

$$\phi_b(x,l) = \int_0^\infty A_b^{\mathrm{P},\lambda}(x+sl) l_\lambda \, ds.$$

Note the relations $(l\,\partial\phi_b) = -(lA_b^\mathrm{P})$ and

$$\partial_\mu A_b^\mu(x,l) + m_b^2 \phi_b(x,l) = 0.$$

The last relation follows directly from (2.3) and (2.4), since $(p\,e_r(p)) = 0$.

Let now $d_l := \sum_\sigma dl^\sigma (\partial/\partial l^\sigma)$ denote the differential with respect to the string coordinate. We may introduce the (form-valued in the string variable) field:

$$\begin{aligned}
d_l \phi_b(x,l) &= w_b(x,l) \\
&:= \sum_r \int d\mu(p) \left[e^{i(px)} \left(\frac{i e_{r,\sigma}(p)}{(pl)} - \frac{i p_\sigma (e_r(p)l)}{(pl)^2} \right) \alpha_{r,b}^\dagger(p) \right. \\
&\qquad\qquad \left. + e^{-i(px)} \left(\frac{i e_{r,\sigma}(p)}{(pl)} - \frac{i p_\sigma (e_r(p)l)}{(pl)^2} \right)^* \alpha_{r,b}(p) \right] dl^\sigma; \quad (2.6)
\end{aligned}$$

and one obtains

$$\partial_\mu w_b = -\sum_r \int \mathrm{d}\mu(p) \left[\mathrm{e}^{i(px)} \left(\frac{p_\mu e_{r,\sigma}(p)}{(pl)} - \frac{p_\mu p_\sigma (e_r(p)l)}{(pl)^2} \right) \alpha^\dagger_{r,b}(p) \right.$$

$$\left. + \mathrm{e}^{-i(px)} \left(\frac{p_\mu e_{r,\sigma}(p)}{(pl)} - \frac{p_\mu p_\sigma (e_r(p)l)}{(pl)^2} \right)^* \alpha_{r,b}(p) \right] \mathrm{d}l^\sigma = d_l A^\mu_b \,;$$

as well as $d_l w_b := d_l^2 \phi_b = 0$. In the case that A^μ_a describes a massless field, we just take the second equality in (2.6) as *definition* of w_a and $d_l A^\mu_\gamma = \partial^\mu w_\gamma$ still holds.[5]

We hasten now to exhibit a family of (Wightman) two-point functions for our fields, of the general form

$$\langle\!\langle \varphi(x,l)\,\psi(x',l) \rangle\!\rangle = \frac{1}{(2\pi)^3} \int \mathrm{d}^4 p\, \mathrm{e}^{-i(p(x-x'))} \delta_+(p^2 - m^2)\, M^{\varphi\psi}(p,l)\,;$$

where any of the two fields φ, ψ, belong to the collection

$$\{ F^{\mu\nu}_a(x),\ A^\mu_a(x,l),\ \phi_b(x,l),\ \partial^\mu \phi_b(x,l),\ w_a(x,l),\ \partial^\mu w_a(x,l) \}$$

with a running over $(1,2,3,4)$ and b over $(1,2,3)$. We shall suppress the subindex notation a, b in the rest of this section. Here $\delta_+(p^2 - m^2) = \delta(p_0 - \sqrt{|\boldsymbol{p}|^2 + m^2})/2\sqrt{|\boldsymbol{p}|^2 + m^2}$ and $\langle\!\langle - \rangle\!\rangle$ denotes a vacuum expectation value of the included operator.

The respective $M^{\varphi\psi}$ are computed from the definitions of the fields. It is enough to note that:

$$M^{\varphi\psi}_{\alpha\beta} := \sum_r u^{(\varphi)}_{r,\alpha}(p,l)^*\, u^{(\psi)}_{r,\beta}(p,l),$$

in terms of intertwiners $u^{(\varphi)}, u^{(\psi)}$ already given. We get, to begin with,

$$M^{AA}_{\mu\nu} = -g_{\mu\nu} + \frac{p_\mu l_\nu + p_\nu l_\mu}{(pl)}\,. \tag{2.7a}$$

The noteworthy and truly valuable fact here is that this is of order 0 as $p^2 \to \infty$, while the two-point function of a Proca field goes like p^2. The formula is analogous to that which comes out of lightcone gauge-fixing [25]. However, the meaning is quite different; in particular, our formalism is fully covariant. On configuration space, therefore, $\langle\!\langle A(x,l)\,A(x',l) \rangle\!\rangle$ essentially scales like λ^{-2} under $x \mapsto \lambda x$, whereas $\langle\!\langle A^\mathrm{p}(x)\,A^\mathrm{p}(x') \rangle\!\rangle$ goes as λ^{-4}.

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[5] The form-valued w_γ suffers from expected infrared problems. A promising way to deal with them in perturbation theory has come to light recently [24].

Let us fill up a little table of vacuum expectation values of field products, needed further down:

$$M^{FF}_{\mu\nu,\rho\sigma} = -(p_\mu p_\rho \, g_{\nu\sigma} - p_\nu p_\rho \, g_{\mu\sigma} - p_\mu p_\sigma \, g_{\nu\rho} + p_\nu p_\sigma \, g_{\mu\rho}).$$

$$M^{\partial A,A}_{\mu\nu,\lambda} = i\left(p_\mu g_{\nu\lambda} - p_\mu \frac{p_\nu l_\lambda + p_\lambda l_\nu}{(pl)}\right), \qquad M^{F\phi}_{\mu\nu} = \frac{p_\nu l_\mu - p_\mu l_\nu}{(pl)},$$

$$M^{A\phi}_\mu = -\frac{il_\mu}{(pl)}, \qquad M^{A,\partial\phi}_{\mu\nu} = \frac{p_\nu l_\mu}{(pl)}, \qquad M^{\partial A,\phi}_{\mu\nu} = -\frac{p_\mu l_\nu}{(pl)},$$

$$M^{\phi\phi} = \frac{1}{m^2}, \qquad M^{\partial\phi,\phi}_\mu = -\frac{ip_\mu}{m^2}, \tag{2.7b}$$

as well as

$$M^{Aw}_\mu = \frac{i}{(pl)} M^{AA}_{\mu\sigma} \, \mathrm{d}l^\sigma = i\left(\frac{-g_{\mu\sigma}}{(pl)} + \frac{p_\sigma l_\mu}{(pl)^2}\right) \mathrm{d}l^\sigma, \qquad M^{w\phi} = 0,$$

$$M^{ww} = \frac{1}{(pl)^2} M^{AA}_{\sigma\tau} \, \mathrm{d}l^\sigma \wedge \mathrm{d}l^\tau = -\frac{g_{\sigma\tau}}{(pl)^2} \, \mathrm{d}l^\sigma \wedge \mathrm{d}l^\tau,$$

$$M^{Fw}_{\mu\nu} = d_l \, M^{F\phi}_{\mu\nu} = i\left(\frac{p_\nu g_{\mu\sigma} - p_\mu g_{\nu\sigma}}{(pl)} + \frac{p_\mu l_\nu - p_\nu l_\mu}{(pl)^2} p_\sigma\right) \mathrm{d}l^\sigma, \tag{2.7c}$$

using the relation $l_\sigma \, \mathrm{d}l^\sigma = 0$. It is clear that massless bosons do not bear escort quantum fields.[6]

The construction of SLF for spin 2 or helicity ± 2 proceeds in the same way, from the equivalent object to the Faraday tensor F, the linearized Riemann tensor R for spin or helicity 2, towards the string-local replacement for the pointlike (symmetric rank 2 tensor) "potential". Note that physical scalar fields are not stringy.[7]

3. Perturbation Theory for SLF: The Role of String Independence

New theories demand care with the mathematics. We intend to borrow from the Stückelberg–Bogoliubov–Epstein–Glaser (SBEG) "renormalization without regularization" formalism for perturbation theory, both most rigorous and flexible [30, 31]. Since renormalization theory for SLF is in its infancy, it still works partly as a heuristic guide. We only outline what we need here from it.

The method involves the construction of a scattering operator $\mathbb{S}[g; l]$ functionally dependent on a (multiplet of) smooth external fields $g(x)$, which mathematically are test functions. The procedure is natural in view of locality; the functional scattering operator acts on the Fock spaces corresponding to local free fields, of the pointlike or stringlike variety, for a prescribed set of free particles. It is submitted to the following conditions.

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[6] Spacelike strings have been more often employed in the literature on SLF. It is, nevertheless, better here to deal with lightlike strings, since then in general the intertwiners are functions, not just distributions; so we need not smear them. Our arguments work either way [26].

[7] Nor are free Dirac fields; SLF for half-integer spin greater than $\frac{1}{2}$ or integer spin greater than 2 are discussed elsewhere [27–29].

- Covariance: $U(a, \Lambda)\mathbb{S}[g; l]U^\dagger(a, \Lambda) = \mathbb{S}[(a, \Lambda)g; \Lambda l]$, with $(a, \Lambda)g(x) = g(\Lambda^{-1}(x - a))$.
- Unitarity: $\mathbb{S}^{-1}[g; l] = \mathbb{S}^\dagger[g; l]$.
- Causality. Let V^+, V^- denote the future and past solid light cones. Then

$$\mathbb{S}[g_1 + g_2; l] = \mathbb{S}[g_1; l]\,\mathbb{S}[g_2; l] \tag{3.1}$$

when $(\operatorname{supp} g_2 + \mathbb{R}^+ l) \cap (\operatorname{supp} g_1 + \mathbb{R}^+ l + V^+) = \emptyset$, or equivalently $(\operatorname{supp} g_1 + \mathbb{R}^+ l) \cap (\operatorname{supp} g_2 + V^- + \mathbb{R}^+ l) = \emptyset$.

In practice one looks for $\mathbb{S}[g; l]$ as a power series in g, of the form

$$\mathbb{S}[g; l] = 1 + \sum_{k=1}^\infty \frac{i^k}{k!} \int_{\mathbb{M}_4^k} S_k(x_1, \ldots, x_k, l)g(x_1)\cdots g(x_k)\,\mathrm{d}x_1\cdots\mathrm{d}x_k. \tag{3.2}$$

Only the first-order term S_1 is postulated. This will be a Wick polynomial in the free fields.[8]

We come back in a moment to the structure of S_1 in the present context. In consonance with (3.1), the $S_k(x_1, \ldots, x_k, l)$ for $k \geq 2$ are *time-ordered products*, which need to be constructed. By locality, the causal factorization

$$S_2(x, x', l) = \mathrm{T}[S_1(x, l)S_1(x', l)] := S_1(x, l)S_1(x', l) \;\text{ or }\; S_1(x', l)S_1(x, l), \tag{3.3}$$

according as $\{x + tl\}$ is later or earlier than $\{x' + tl\}$, fixes S_2 on a large region of $\mathbb{M}_4^2 \times S^2$. Indeed, assuming $l^0 > 0$, a string $\{x + tl\}$ lies to the future of another string $\{x' + t'l\}$ if and only if $((x - x')l) \geq 0$ and the intersection of the strings is empty. That is, x lies to the future of, or on, the hyperplane $x' + l^\perp$, but not on the full line $x' + \mathbb{R}l$ [26]. Consequently, the strings cannot be ordered if and only if x lies on the string $\{x' + t'l\}$ or vice versa, i.e. if and only if $x - x'$ is lightlike and parallel to l. This exceptional set:

$$\mathcal{D} := \{(x, x', l) : (x - x')^2 = 0, \; ((x - x')l) = 0\} \tag{3.4}$$

is of measure zero in $\mathbb{M}_4^2 \times S^2$. The extension of such products to the whole of $\mathbb{M}_4^2 \times S^2$, mainly by upholding string independence, is the SBEG renormalization problem in a nutshell.

Existence of the adiabatic limit is the property that the S_k be integrable distributions, in the sense of Schwartz [32]. In that limit, as g goes to a constant, the covariant $\mathbb{S}[g; l]$ is expected to approach the invariant physical scattering matrix $\mathbb{S}$, so that $U(a, \Lambda)\mathbb{S}\,U^\dagger(a, \Lambda) = \mathbb{S}$, all dependence on the string disappearing.

A lesson of gauge field theory is that couplings of quantum fields should fall out from *a simple underlying principle*. The natural and essential hypothesis of interacting SLF theory is simple enough: physical observables and quantities closely related to them, particularly the $\mathbb{S}$-matrix, cannot depend on the string coordinates. This is the **string independence** principle: colloquially, the

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[8] In many models it looks like an interaction Lagrangian. It should, however, be kept in mind that the building blocks in the procedure are quantum fields; ditto, our starting point is Wigner's theory of quantum Poincaré modules [14] and corresponding field-strength representations of the Lorentz group, rather than a classical Lagrangian that one attempts to "quantize".

string "ought not to be seen". Let S_1 denote a first-order vertex coupling in general. For the physics of the model described by S_1 to be string-independent, one must require that a vector field $Q_1^\mu(x,l)$ exist such that

$$d_l S_1 = (\partial Q_1) \equiv \partial_\mu Q_1^\mu \,, \tag{3.5}$$

so that, regarding the $\mathbb{S}$-matrix as the adiabatic limit of Bogoliubov's functional $\mathbb{S}$-matrix, on applying integration by parts, the contribution from the divergence vanishes. Moreover, (perturbative) string independence should hold at every order in the couplings, surviving renormalization.

Already the condition that $d_l S_1$ be a divergence severely restricts the interaction vertices in S_1; we proceed to throw light on the fermion sector by using it in the next section. Further along, all the time-ordered products S_k in the functional $\mathbb{S}$-matrix ought to be determined from string independence.

4. On the String-Local Boson Sector

It turns out that the string independence principle holds great power both as a heuristic device and a justification tool, dictating *symmetry* (of the Abelian and non-Abelian kind) from interaction[9] down to almost every nut and bolt. A complete account of electroweak theory would start by showing that, when the string independence principle is applied to the physically relevant set of boson SLF, with their known masses and charges, replacing the standard point-like fields, plus one *physical* Higgs particle $\phi_4(x)$,[10] one recovers precisely the phenomenological couplings of flavourdynamics in the Standard Model (SM), with massive bosons mediating the weak interactions, and the $U(2)$ structure constants, as, for instance, in [35] or [36, Ch. 1]. (One cannot quite say that we recover *the* Standard Model picture after spontaneous symmetry breaking has allegedly taken place, since our boson fields are different, and our rule set cares little for Lagrangians. But the coincidence of the couplings ought to be evident—see the discussion at the end of Sect. 7.)

Such a derivation, spelled out in a future paper [37], requires one to examine time-ordered products corresponding to graphs involving boson particles up to third order in the couplings. For want of space, here we can just display its flavour, and foremost the results we need, to build up our derivation for chirality of weak interactions.

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[9] Thus, reversing Yang's *dictum*, restated in the famous terminological discussion on gauge interactions between Dirac, Ferrara, Kleinert, Martin, Wigner, Yang himself and Zichichi [33].

[10] Following Okun [34], and for obvious grammatical reasons, henceforth we refer to a (physical) Higgs boson as a higgs, with a lowercase h. Note also that, in the presence of a massless A_4, the notation ϕ_4 is not meant to purport the higgs as a rogue escort!

- Apart from the higgs particle sector, a string-local theory of interacting bosons at first order in the coupling constant g must be of the form:

$$S_1^B(x,l) = g \sum_{a,b,c} f_{abc} F_a(x) A_b(x,l) A_c(x,l)$$

$$+ g \sum_{a,b,c}{}' f_{abc}(m_a^2 - m_b^2 - m_c^2)\big(A_a(x,l)A_b(x,l)\phi_c(x,l)$$

$$- A_a(x,l)\,\partial\phi_b(x,l)\phi_c(x,l)\big), \tag{4.1}$$

  where we omit the notation $: \!\!-\!\! :$ for Wick products, and the restricted sum $\sum'$ runs over massive fields only. Here the f_{abc} denote the (completely skewsymmetric) structure constants of the (reductive) symmetry group of the model; the mass of the vector boson A_a is denoted m_a, and complete contraction of Lorentz indices is understood. Notice that the escort fields hold a somewhat analogous place to Stückelberg fields.
- Now it is straightforward to check that the 1-form $d_l S_1^B$, measuring the dependence on the string variable of the vertices in (4.1), is a divergence: $d_l S_1^B(x,l) = \big(\partial Q_1^B\big)(x,l)$, where Q_1^B is given by:

$$2g \sum_{a,b,c} f_{abc}(F_a A_c)_\mu w_b + g \sum_{a,b,c}{}' f_{abc}(m_a^2 + m_c^2 - m_b^2)(A_{a\mu} - \partial_\mu \phi_a)\phi_c w_b. \tag{4.2}$$

  We shall need Q_1^B to prove chirality of the couplings to the fermion sector.
- At once we adapt our notation to the one used in the SM. This model has three masses $m_1 = m_2 < m_3$ different from zero and one $m_4 = 0$. Defining the Weinberg angle[11] by $m_1/m_3 =: \cos\Theta$, we employ the basis in which

$$f_{123} = \tfrac{1}{2}\cos\Theta, \quad f_{124} = \tfrac{1}{2}\sin\Theta, \quad f_{134} = f_{234} = 0,$$

  all other f_{abc} following from complete skewsymmetry. They are seen to be the structure constants of (the Lie algebra of) the $U(2)$ determined by the *physical* particle fields. We shall use the standard notations

$$W_\pm \equiv \frac{1}{\sqrt{2}}(W_1 \mp iW_2) := \frac{1}{\sqrt{2}}(A_1 \mp iA_2), \quad Z := A_3, \quad A := A_4$$

  and similarly for $\phi_\pm$, $w_\pm$, ϕ_Z and w_Z, with masses $m_W = m_1$, $m_Z = m_3$ and $m_\gamma = m_4 = 0$.
- With this in hand, we focus on (4.2), keeping in mind that, although an escort field does not exist for the photon, the field w_4 exists at the same title as w_1, w_2 and w_Z. The first summand in (4.2) yields:

---

[11] This makes sense in the renormalized theory [41, Sect. 29.1].

$$2g \sum f_{abc}(\partial_\mu A_{a\lambda} - \partial_\lambda A_{a\mu}) A_c^\lambda w_b$$
$$= ig \sin \Theta \big[(\partial_\mu A_\lambda - \partial_\lambda A_\mu)(w_- W_+^\lambda - w_+ W_-^\lambda)$$
$$+ (\partial_\mu W_{-\lambda} - \partial_\lambda W_{-\mu})(w_+ A^\lambda - w_4 W_+^\lambda)$$
$$+ (\partial_\mu W_{+\lambda} - \partial_\lambda W_{+\mu})(w_4 W_-^\lambda - w_- A^\lambda) \big]$$
$$+ ig \cos \Theta \big[(\partial_\mu Z_\lambda - \partial_\lambda Z_\mu)(w_- W_+^\lambda - w_+ W_-^\lambda)$$
$$+ (\partial_\mu W_{-\lambda} - \partial_\lambda W_{-\mu})(w_+ Z^\lambda - w_Z W_+^\lambda)$$
$$+ (\partial_\mu W_{+\lambda} - \partial_\lambda W_{+\mu})(w_Z W_-^\lambda - w_- Z^\lambda) \big]. \tag{4.3}$$

- Our $Q_{1\mu}^B$ above is not complete, since bosonic couplings involving the higgs sector have not been included. They are also derived from the string independence principle.[12] Of those, for our purposes in this paper we need only:

$$\frac{g}{2 \cos \Theta} m_Z (\phi_4 (\partial_\mu \phi_Z - Z_\mu) - \partial_\mu \phi_4 \, \phi_Z) w_Z; \tag{4.4}$$

  actually these play a pivotal role in our problem. Clearly, terms of this type are suggested by the last group of summands in (4.2).
- By the way, the expected $g^2 AAAA$ terms and thus the indications of the classical geometrical gauge approach are recovered in our formalism from string independence at the level of S_2.

5. The First-Order Constraints

Our framework for electroweak theory is outlined next. This both exemplifies the principle and contributes to the core of this paper.

- The couplings between interaction carriers and matter currents in a theory with massive or massless vector bosons $A_{a\mu}$ must be of the form

$$g\big(b^a A_{a\mu} J_V^\mu + \tilde{b}^a A_{a\mu} J_A^\mu + c^a \phi_a S + \tilde{c}^a \phi_a S_5 \big); \tag{5.1}$$

  where

$$J_V^\mu = \overline{\psi} \gamma^\mu \psi, \quad J_A^\mu = \overline{\psi} \gamma^\mu \gamma^5 \psi, \quad S = \overline{\psi} \psi, \quad S_5 = \overline{\psi} \gamma^5 \psi,$$

  with electric charge conserved in the interaction vertices. Our key assumption point is that these A_a^μ and ϕ_a above are now given as string-local quantum fields, thus satisfying renormalizability by power counting. There exist *no other scalar couplings which comply with renormalizability*. To wit, Lorentz invariance requires that all cubic terms be of the above form, and renormalizability forbids quartic terms.[13]
- The ψ in (5.1) are ordinary fermion fields—we should not assume chiral fermions *ab initio*, and we do not.

---

[12] There again, SLF theory goes one better: the "negative squared mass" in the higgs' self-potential, not accounted for in the SM [18], is derived from string independence. We refer to the forthcoming [38] in this respect.

[13] Since the two Fermi fields required by Lorentz invariance already have scaling dimension 3, any two further fields would give 5, exceeding the power-counting limit.

- The coefficients b^a, $\tilde{b}^a$, c^a, $\tilde{c}^a$ in (5.1) are to be determined from string independence.

The proof of chirality in the couplings of electroweak bosons to the fermion sector of the SM from string independence develops in two stages. In the first stage, we need not invoke the Q_1-vector of the boson sector. For these couplings, we make the most general Ansatz, as explained after (5.1), again omitting the notation :—: for the Wick products:

$$
\begin{aligned}
S_1^F(x,l) := g\big(&b_1 W_{-\mu}\bar{e}\gamma^\mu\nu + \tilde{b}_1 W_{-\mu}\bar{e}\gamma^\mu\gamma^5\nu + b_2 W_{+\mu}\bar{\nu}\gamma^\mu e + \tilde{b}_2 W_{+\mu}\bar{\nu}\gamma^\mu\gamma^5 e \\
&+ b_3 Z_\mu\bar{e}\gamma^\mu e + \tilde{b}_3 Z_\mu\bar{e}\gamma^\mu\gamma^5 e + b_4 Z_\mu\bar{\nu}\gamma^\mu\nu + \tilde{b}_4 Z_\mu\bar{\nu}\gamma^\mu\gamma^5\nu \\
&+ b_5 A_\mu\bar{e}\gamma^\mu e + \tilde{b}_5 A_\mu\bar{e}\gamma^\mu\gamma^5 e + b_6 A_\mu\bar{\nu}\gamma^\mu\nu + \tilde{b}_6 A_\mu\bar{\nu}\gamma^\mu\gamma^5\nu \\
&+ c_1\phi_-\bar{e}\nu + \tilde{c}_1\phi_-\bar{e}\gamma^5\nu + c_2\phi_+\bar{\nu}e + \tilde{c}_2\phi_+\bar{\nu}\gamma^5 e \\
&+ c_3\phi_Z\bar{e}e + \tilde{c}_3\phi_Z\bar{e}\gamma^5 e + c_4\phi_Z\bar{\nu}\nu + \tilde{c}_4\phi_Z\bar{\nu}\gamma^5\nu \\
&+ c_0\phi_4\bar{e}e + \tilde{c}_0\phi_4\bar{e}\gamma^5 e + c_5\phi_4\bar{\nu}\nu + \tilde{c}_5\phi_4\bar{\nu}\gamma^5\nu\big).
\end{aligned}
\tag{5.2}
$$

All the boson fields here are string-local, except for the pointlike higgs field ϕ_4. Here e stands for an electron, or muon, or τ-lepton pointlike field or for (a suitable combination of) quark fields d, s, b; and ν for the neutrinos or for the quarks u, c, t.[14] Charge is conserved in each term. Unitarity of the $\mathbb{S}$-matrix, in the light of (3.2) dictates that S_1 be Hermitian. Thus, for instance, $b_2 = b_1^*$ and $\tilde{b}_2 = \tilde{b}_1^*$; and we may choose phases so that both b_1 and $\tilde{b}_1$ are real. Moreover, b_3, b_4, b_5, b_6 and $\tilde{b}_3, \tilde{b}_4, \tilde{b}_5, \tilde{b}_6$ are all real; $c_2 = c_1^*$ and $\tilde{c}_2 = -\tilde{c}_1^*$; c_3, c_4, c_0, c_5 are real, whereas $\tilde{c}_3, \tilde{c}_4, \tilde{c}_0, \tilde{c}_5$ are imaginary. We may assume that the photon should not couple to neutrinos, which are uncharged, and drop the corresponding terms, with coefficients $b_6, \tilde{b}_6$, right away.[15]

As indicated, the ψ-fields e and ν are ordinary pointlike fermion fields. Let us use the Dirac equation to handle them; we could employ Weyl equations as well. The important feature is that the SBEG procedure is thoroughly an on-shell construction:

$$
\overrightarrow{\slashed{\partial}}\,\psi = -im_\psi\psi, \qquad \bar{\psi}\,\overleftarrow{\slashed{\partial}} = im_\psi\bar{\psi}.
\tag{5.3}
$$

String independence at this order demands that there be a $Q_\mu^F(x,l)$ such that

$$
d_l S_1^F(x,l) = \partial^\mu Q_\mu^F(x,l).
\tag{5.4}
$$

---

[14] As already indicated, we consider just one generation of leptons.
[15] Were we not to do so, electric charge would appear as the *difference* between the couplings of the photon to the electron and the neutrino.

Proposition 1. *The string independence requirement* (5.4) *can be satisfied if and only if*

$$2c_1 = i(m_e - m_\nu)b_1, \qquad\qquad c_3 = 0,$$
$$c_2 = i(m_\nu - m_e)b_1, \qquad\qquad c_4 = 0,$$
$$\tilde{c}_1 = i(m_e + m_\nu)\tilde{b}_1, \qquad\qquad \tilde{c}_3 = 2im_e\tilde{b}_3,$$
$$\tilde{c}_2 = i(m_\nu + m_e)\tilde{b}_1, \qquad \tilde{c}_4 = 2im_\nu\tilde{b}_4, \quad and \quad \tilde{b}_5 = 0. \qquad (5.5)$$

The corresponding $Q_1^{F\mu}$ is unique and is of the form

$$Q_1^{F\mu} := g\big(b_1 w_- \bar{e}\gamma^\mu\nu + \tilde{b}_1 w_- \bar{e}\gamma^\mu\gamma^5\nu + b_1 w_+ \bar{\nu}\gamma^\mu e + \tilde{b}_1 w_+ \bar{\nu}\gamma^\mu\gamma^5 e$$
$$+ b_3 w_Z \bar{e}\gamma^\mu e + \tilde{b}_3 w_Z \bar{e}\gamma^\mu\gamma^5 e + b_4 w_Z \bar{\nu}\gamma^\mu\nu + \tilde{b}_4 w_Z \bar{\nu}\gamma^\mu\gamma^5\nu$$
$$+ b_5 w_4 \bar{e}\gamma^\mu e\big). \qquad (5.6)$$

Note that there are no restrictions at this stage on the set $\{c_0, \tilde{c}_0, c_5, \tilde{c}_5\}$, since the corresponding vertices are pointlike.

Proof. The string differential $d_l S_1^F$ with the Ansatz (5.2) for S_1^F is expressed with the help of the form-valued fields defined in (2.6):

$$d_l S_1^F(x,l)$$
$$= g\big(b_1 \partial_\mu w_- \bar{e}\gamma^\mu\nu + \tilde{b}_1 \partial_\mu w_- \bar{e}\gamma^\mu\gamma^5\nu + b_1 \partial_\mu w_+ \bar{\nu}\gamma^\mu e + \tilde{b}_1 \partial_\mu w_+ \bar{\nu}\gamma^\mu\gamma^5 e$$
$$+ b_3 \partial_\mu w_Z \bar{e}\gamma^\mu e + \tilde{b}_3 \partial_\mu w_Z \bar{e}\gamma^\mu\gamma^5 e + b_4 \partial_\mu w_Z \bar{\nu}\gamma^\mu\nu + \tilde{b}_4 \partial_\mu w_Z \bar{\nu}\gamma^\mu\gamma^5\nu$$
$$+ b_5 \partial_\mu w_4 \bar{e}\gamma^\mu e + \tilde{b}_5 \partial_\mu w_4 \bar{e}\gamma^\mu\gamma^5 e$$
$$+ c_1 w_- \bar{e}\nu + \tilde{c}_1 w_- \bar{e}\gamma^5\nu + c_2 w_+ \bar{\nu}e + \tilde{c}_2 w_+ \bar{\nu}\gamma^5 e$$
$$+ c_3 w_Z \bar{e}e + \tilde{c}_3 w_Z \bar{e}\gamma^5 e + c_4 w_Z \bar{\nu}\nu + \tilde{c}_4 w_Z \bar{\nu}\gamma^5\nu\big).$$

Using the Dirac equations (5.3) and $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$ and *defining* Q_1^F as in Eq. (5.6), one finds:

$$d_l S_1^F(x,l)$$
$$= \partial_\mu\big[Q_1^{F\mu} + \tilde{b}^5 w_4 \bar{e}\gamma^\mu\gamma^5 e\big]$$
$$+ g\big[(c_1 - i(m_e - m_\nu)b_1)w_- \bar{e}\nu + (\tilde{c}_1 - i(m_e + m_\nu)\tilde{b}_1)w_- \bar{e}\gamma^5\nu$$
$$+ (c_2 - i(m_\nu - m_e)b_1)w_+ \bar{\nu}e + (\tilde{c}_2 - i(m_\nu + m_e)\tilde{b}_1)w_+ \bar{\nu}\gamma^5 e$$
$$+ (\tilde{c}_3 - 2im_e\tilde{b}_3)w_Z \bar{e}\gamma^5 e + (\tilde{c}_4 - 2im_\nu\tilde{b}_4)w_Z \bar{\nu}\gamma^5\nu - 2im_e\tilde{b}_5 w_4 \bar{e}\gamma^5 e$$
$$+ c_3 w_Z \bar{e}e + c_4 w_Z \bar{\nu}\nu\big].$$

The last four lines cannot be expressed as divergences, and by linear independence of the cubic operators, the corresponding terms must vanish separately. This implies the claims. $\qquad\square$

Notice also that the argument for $\tilde{b}_5 = 0$ would have failed if the electron were massless, whereas the axial terms for massive vector bosons in the original Ansatz have survived. They will keep surviving, as we shall see.

It is pertinent to substitute expressions (5.5) into (5.2), which we do now for convenience later on:

$$
\begin{aligned}
S_1^F(x,l) & \\
&= g\big(b_1 W_{-\mu}\bar{e}\gamma^\mu\nu + \tilde{b}_1 W_{-\mu}\bar{e}\gamma^\mu\gamma^5\nu + b_1 W_{+\mu}\bar{\nu}\gamma^\mu e + \tilde{b}_1 W_{+\mu}\bar{\nu}\gamma^\mu\gamma^5 e \\
&\quad + b_3 Z_\mu \bar{e}\gamma^\mu e + \tilde{b}_3 Z_\mu \bar{e}\gamma^\mu\gamma^5 e + b_4 Z_\mu \bar{\nu}\gamma^\mu\nu + \tilde{b}_4 Z_\mu \bar{\nu}\gamma^\mu\gamma^5\nu + b_5 A_\mu \bar{e}\gamma^\mu e \\
&\quad + i(m_e - m_\nu)b_1\phi_-\bar{e}\nu + i(m_e + m_\nu)\tilde{b}_1\phi_-\bar{e}\gamma^5\nu - i(m_e - m_\nu)b_1\phi_+\bar{\nu}e \\
&\quad + i(m_e + m_\nu)\tilde{b}_1\phi_+\bar{\nu}\gamma^5 e + 2im_e\tilde{b}_3\phi_Z\bar{e}\gamma^5 e + 2im_\nu\tilde{b}_4\phi_Z\bar{\nu}\gamma^5\nu \\
&\quad + c_0\phi_4\bar{e}e + \tilde{c}_0\phi_4\bar{e}\gamma^5 e + c_5\phi_4\bar{\nu}\nu + \tilde{c}_5\phi_4\bar{\nu}\gamma^5\nu\big).
\end{aligned}
\tag{5.7}
$$

6. Time-Ordered Products for Tree Graphs

Recall that the causal factorization (3.3) fixes the time-ordered product $\mathrm{T}[S_1(x,l)S_1(x',l)]$ only outside the set $\mathcal{D}$. The possible extensions across $\mathcal{D}$ are restricted by the requirement that the Wick expansion, valid outside $\mathcal{D}$, hold everywhere: we require that the time-ordered product of Wick polynomials $U = U(x,l)$, $V' = V(x',l)$ satisfy

$$
\mathrm{T}[UV'] = {:}UV'{:} + \underbrace{\left[\sum_{\varphi,\chi'} {:}\frac{\partial U}{\partial \varphi}\langle\!\langle \mathrm{T}\,\varphi\,\chi'\rangle\!\rangle\frac{\partial V'}{\partial\chi'}{:}\right]}_{\mathrm{T}[UV']_{\mathrm{tree}}} + \cdots + \langle\!\langle \mathrm{T}[UV']\rangle\!\rangle,
\tag{6.1}
$$

where the sum in the brackets goes over all free fields, and we have employed formal derivation within the Wick polynomial. The terms in brackets are called the *tree graphs*. Thereby, the extension problem is reduced to the extension of numerical distributions.

In particular, at the tree graph level, it only remains to extend the time-ordered two-point functions $\langle\!\langle \mathrm{T}\,\varphi\,\chi'\rangle\!\rangle$ of free fields. One such extension is given by

$$
\langle\!\langle \mathrm{T}_0\,\varphi(x,l)\,\chi(x',l)\rangle\!\rangle := \frac{i}{(2\pi)^4}\int \mathrm{d}^4 p\,\frac{\mathrm{e}^{-i(p(x-x'))}}{p^2 - m^2 + i0}\,M^{\varphi\chi}(p,l).
\tag{6.2}
$$

It has the nice feature that it preserves all off-shell relations between the fields.[16]

If the scaling degree of the two-point function $\langle\!\langle \varphi\,\chi'\rangle\!\rangle$ with respect to $\mathcal{D}$ and to the diagonal $\{x = x'\}$ is lower than the respective codimensions 3 and 4, then the time-ordered two-point function is unique, $\langle\!\langle \mathrm{T}\,\varphi\,\chi'\rangle\!\rangle = \langle\!\langle \mathrm{T}_0\,\varphi\,\chi'\rangle\!\rangle$. Otherwise, it admits the addition of a distribution with support on $\mathcal{D}$.

A look at tables (2.7) shows that this happens only in the cases $\langle\!\langle \mathrm{T}\,\partial_\lambda A_\mu\,A'_\kappa\rangle\!\rangle$ and $\langle\!\langle \mathrm{T}\,\partial_\lambda A_\mu\,\partial'_\kappa w'\rangle\!\rangle$. These have scaling degree 3 with respect

---

[16] The string derivative d_l fulfils the Leibniz rule with T_0 unconditionally. As long as no on-shell relations are involved, ∂_μ can be exchanged with T_0 as well, e.g.:

$$
\partial_\mu\langle\!\langle \mathrm{T}_0\,A_\nu\chi'\rangle\!\rangle - \partial_\nu\langle\!\langle \mathrm{T}_0\,A_\mu\chi'\rangle\!\rangle = \langle\!\langle \mathrm{T}_0\,F_{\mu\nu}\chi'\rangle\!\rangle.
$$

to both $\mathcal{D}$ and the diagonal $\{x = x'\}$, and therefore admit a renormalization by adding a numerical distribution supported on $\mathcal{D}$ and with the same scaling degree. Any such distribution is of the form

$$\delta_l(x' - x) := \int_0^\infty \mathrm{d}s \, \delta(x' - x - sl), \qquad (6.3)$$

multiplied by some well-behaved function $f(x' - x, l)$. Thus, in these cases the most general two-point functions are

$$\langle\!\langle \mathrm{T}\, \partial_\lambda A_\mu \, A'_\kappa \rangle\!\rangle = \langle\!\langle \mathrm{T}_0 \, \partial_\lambda A_\mu \, A'_\kappa \rangle\!\rangle + c_{\lambda\mu\kappa} \, \delta_l \,, \qquad (6.4a)$$

$$\langle\!\langle \mathrm{T}\, \partial_\lambda A_\mu \, \partial'_\kappa w' \rangle\!\rangle = \langle\!\langle \mathrm{T}_0 \, \partial_\lambda A_\mu \, \partial'_\kappa w' \rangle\!\rangle + b_{\lambda\mu\kappa} \,, \qquad (6.4b)$$

where $c_{\lambda\mu\kappa}$ and $b_{\lambda\mu\kappa}$ are some well-behaved function and one-form, respectively, as yet undetermined.

We now seek to enforce string independence of time-ordered products at second order in the coupling constant. String independence at first order (3.5) plus the factorization (3.3) implies that the relation

$$d_l \, \mathrm{T}[S_1(x,l)S_1(x',l)] = \partial_\mu \, \mathrm{T}[Q_1^\mu(x,l)S_1(x',l)] + \partial'_\mu \, \mathrm{T}[S_1(x,l)Q_1^\mu(x',l)] \quad (6.5)$$

holds for all $(x - x', l)$ outside $\mathcal{D}$. The **string independence principle** forces us to require that this relation be valid everywhere. It turns out that this requirement fixes all coefficients in (5.2).

As advertised, to this end we shall only need to examine tree graphs in S_2. We reckon that the tree graph contribution to the obstruction (6.5) is given by

$$\sum_{\varphi,\chi'} \left[d_l \frac{\partial S_1}{\partial \varphi} \langle\!\langle \mathrm{T}\, \varphi\chi' \rangle\!\rangle \frac{\partial S'_1}{\partial \chi'} + \frac{\partial S_1}{\partial \varphi} \, d_l \langle\!\langle \mathrm{T}\, \varphi\chi' \rangle\!\rangle \frac{\partial S'_1}{\partial \chi'} + \frac{\partial S_1}{\partial \varphi} \langle\!\langle \mathrm{T}\, \varphi\chi' \rangle\!\rangle \, d_l \frac{\partial S'_1}{\partial \chi'} \right]$$

$$- \sum_{\psi,\chi'} \left(\partial_\mu \frac{\partial Q^\mu}{\partial \psi} \langle\!\langle \mathrm{T}\, \psi\chi' \rangle\!\rangle + \frac{\partial Q^\mu}{\partial \psi} \partial_\mu \langle\!\langle \mathrm{T}\, \psi \, \chi' \rangle\!\rangle \right) \frac{\partial S'_1}{\partial \chi'} - [x \leftrightarrow x'], \quad (6.6)$$

where we have written Q for Q_1. This expression expands to

$$\sum_{\varphi,\chi'} \left[d_l \frac{\partial S_1}{\partial \varphi} \langle\!\langle \mathrm{T}\, \varphi\chi' \rangle\!\rangle \frac{\partial S'_1}{\partial \chi'} + \frac{\partial S_1}{\partial \varphi} \langle\!\langle \mathrm{T}\, d_l\varphi\chi' \rangle\!\rangle \frac{\partial S'_1}{\partial \chi'} \right.$$

$$\left. + \frac{\partial S_1}{\partial \varphi} \langle\!\langle \mathrm{T}\, \varphi d_l\chi' \rangle\!\rangle \frac{\partial S'_1}{\partial \chi'} + \frac{\partial S_1}{\partial \varphi} \langle\!\langle \mathrm{T}\, \varphi\chi' \rangle\!\rangle \, d_l \frac{\partial S'_1}{\partial \chi'} \right]$$

$$- \sum_{\psi,\chi'} \left(\partial_\mu \frac{\partial Q^\mu}{\partial \psi} \langle\!\langle \mathrm{T}\, \psi\chi' \rangle\!\rangle + \frac{\partial Q^\mu}{\partial \psi} \langle\!\langle \mathrm{T}\, \partial_\mu\psi \, \chi' \rangle\!\rangle \right) \frac{\partial S'_1}{\partial \chi'} - [x \leftrightarrow x']$$

$$+ \sum_{\varphi,\chi'} \frac{\partial S_1}{\partial \varphi} \left(d_l \langle\!\langle \mathrm{T}\, \varphi\chi' \rangle\!\rangle - \langle\!\langle \mathrm{T}\, d_l\varphi\chi' \rangle\!\rangle - \langle\!\langle \mathrm{T}\, \varphi d_l\chi' \rangle\!\rangle \right) \frac{\partial S'_1}{\partial \chi'} \qquad (6.7a)$$

$$- \sum_{\psi,\chi'} \frac{\partial Q^\mu}{\partial \psi} \left(\partial_\mu \langle\!\langle \mathrm{T}\, \psi\chi' \rangle\!\rangle - \langle\!\langle \mathrm{T}\, \partial_\mu\psi\chi' \rangle\!\rangle \right) \frac{\partial S'_1}{\partial \chi'} - [x \leftrightarrow x']. \qquad (6.7b)$$

The first, second, fifth and sixth terms reduce to a tree graph contribution:

$$\sum_{\chi'}\left[\sum_{\varphi}\left(d_l\frac{\partial S_1}{\partial\varphi}\langle\!\langle\mathrm{T}\,\varphi\chi'\rangle\!\rangle + \frac{\partial S_1}{\partial\varphi}\langle\!\langle\mathrm{T}\,d_l\varphi\chi'\rangle\!\rangle\right)\right.$$
$$\left.-\sum_{\psi}\left(\partial_\mu\frac{\partial Q^\mu}{\partial\psi}\langle\!\langle\mathrm{T}\,\psi\chi'\rangle\!\rangle + \frac{\partial Q^\mu}{\partial\psi}\langle\!\langle\mathrm{T}\,\partial_\mu\psi\,\chi'\rangle\!\rangle\right)\right]\frac{\partial S_1'}{\partial\chi'}$$
$$= \mathrm{T}[(d_l S_1)S_1']_{\mathrm{tree}} - \mathrm{T}[(\partial_\mu Q^\mu)S_1']_{\mathrm{tree}}\,, \qquad (6.8)$$

which vanishes by construction; we refer to "Appendix A" for the proof of that equality. The other four terms in the first two summations vanish similarly.

Thus, the whole expression (6.6) reduces to the sum (6.7) of the last two lines above, which we may call the "obstruction to string independence".

We now seek to determine this quantity. Its vanishing, even admitting the most general time-ordering prescription T, will provide the correct couplings, and in the occasion chirality of the interaction of the fermions with the massive intermediate vector bosons.

We distinguish three types of 2-point obstructions. For terms φ, χ in S_1 and ψ, C^μ in Q_1^μ, we label them as follows:

$$\widehat{\mathcal{O}}(\varphi, \chi') := d_l\langle\!\langle\mathrm{T}\,\varphi\chi'\rangle\!\rangle - \langle\!\langle\mathrm{T}\,d_l\varphi\chi'\rangle\!\rangle - \langle\!\langle\mathrm{T}\,\varphi\,d_l\chi'\rangle\!\rangle, \qquad (6.9a)$$

$$\mathcal{O}_\mu(\psi, \chi') := \langle\!\langle\mathrm{T}\,\partial_\mu\psi\chi'\rangle\!\rangle - \partial_\mu\langle\!\langle\mathrm{T}\,\psi\chi'\rangle\!\rangle, \qquad (6.9b)$$

$$\mathcal{O}(C, \chi') := \langle\!\langle\mathrm{T}\,\partial_\mu C^\mu\chi'\rangle\!\rangle - \partial_\mu\langle\!\langle\mathrm{T}\,C^\mu\chi'\rangle\!\rangle. \qquad (6.9c)$$

Since the T_0 ordering preserves all off-shell relations between the fields, the first two types only occur for $\mathrm{T} \neq \mathrm{T}_0$. More specifically, the only obstructions of these types that we meet are

$$\widehat{\mathcal{O}}(F_{\mu\nu}, A_\kappa') = d_l(c_{[\mu\nu]\kappa}\,\delta_l), \qquad (6.10a)$$

$$\mathcal{O}_\mu(w, F_{\alpha\beta}') = b_{[\alpha\beta]\mu}\,, \qquad (6.10b)$$

with skewsymmetrization $c_{[\mu\nu]\kappa} \equiv c_{\mu\nu\kappa} - c_{\nu\mu\kappa}$ and similarly for $b_{[\alpha\beta]\mu}$. These are numerical 1-forms in the l variable. On the other hand, all obstructions of type (6.9c) are 0-forms, since the only candidate field C^μ for a 1-form is $\partial^\mu w$—but this does not appear in Q_1^μ, see (4.3) and (4.4). We conclude that the terms in (6.7b) which involve two-point obstructions of the third type must cancel separately, i.e. cannot be cancelled by terms involving the first two types of two-point obstructions.

We now examine 2-point obstructions of the third type (6.9c). First of all, there are two that vanish:

$$\mathcal{O}(A, \phi') := \langle\!\langle\mathrm{T}_0\,\partial_\mu A^\mu\phi'\rangle\!\rangle - \partial_\mu\langle\!\langle\mathrm{T}_0\,A^\mu\phi'\rangle\!\rangle = 0, \qquad (6.11a)$$

$$\mathcal{O}(\partial_\lambda A, \phi') := \langle\!\langle\mathrm{T}_0\,\partial_\mu\partial_\lambda A^\mu\phi'\rangle\!\rangle - \partial_\mu\langle\!\langle\mathrm{T}_0\,\partial_\lambda A^\mu\phi'\rangle\!\rangle = 0. \qquad (6.11b)$$

Indeed, the left-hand side of (6.11a) is $-m^2\langle\!\langle\mathrm{T}_0\,\phi\phi'\rangle\!\rangle - \partial_\mu\langle\!\langle\mathrm{T}_0\,A^\mu\phi'\rangle\!\rangle$, which vanishes because

$$\partial_\mu\langle\!\langle\mathrm{T}_0\,A^\mu\phi'\rangle\!\rangle = \frac{-i}{(2\pi)^4}\int\mathrm{d}^4 p\,\frac{\mathrm{e}^{-i(p(x-x'))}}{p^2 - m^2 + i0} \equiv -i\,D_F(x - x') = -m^2\langle\!\langle\mathrm{T}_0\,\phi\phi'\rangle\!\rangle,$$

in view of (2.7b). Thus, (6.11a) holds, and a similar calculation yields (6.11b). Note that, by definition,

$$D_F(x) := \frac{1}{(2\pi)^4} \int \mathrm{d}^4 p \, \frac{e^{-i(px)}}{p^2 - m^2 + i0}, \quad \text{so that} \quad (\Box + m^2) D_F(x) = -\delta(x).$$

Next, we consider

$$\mathcal{O}(A, A'_\kappa) := \langle\!\langle \mathrm{T}_0 \, \partial_\mu A^\mu A'_\kappa \rangle\!\rangle - \partial_\mu \langle\!\langle \mathrm{T}_0 \, A^\mu A'_\kappa \rangle\!\rangle.$$

Using (2.7), we get

$$\mathcal{O}(A, A'_\kappa) = \frac{1}{(2\pi)^4} \int \mathrm{d}^4 p \, \frac{e^{-i(p(x-x'))}}{p^2 - m^2 + i0} \, \frac{(m^2 - p^2) l_\kappa}{(pl)}$$

$$= -\frac{l_\kappa}{(2\pi)^4} \int \mathrm{d}^4 p \, \frac{e^{-i(p(x-x'))}}{(pl)} \, .$$

On bringing in the distributions $1/(pl) = -i \int_0^\infty \mathrm{d}s \, e^{is(pl)}$ and δ_l of (6.3), we may rewrite the obstruction as

$$\mathcal{O}(A, A'_\kappa) = \frac{i l_\kappa}{(2\pi)^4} \int_0^\infty \mathrm{d}s \int \mathrm{d}^4 p \, e^{-i(p(x-x'-sl))} = i l_\kappa \, \delta_l(x - x'). \qquad (6.12)$$

We next determine

$$\mathcal{O}(\partial\phi, A'_\kappa) := \langle\!\langle \mathrm{T}_0 \, \partial_\mu \partial^\mu \phi A'_\kappa \rangle\!\rangle - \partial_\mu \langle\!\langle \mathrm{T}_0 \, \partial^\mu \phi A'_\kappa \rangle\!\rangle$$

$$= -(\Box + m^2) \langle\!\langle \mathrm{T}_0 \, \phi A'_\kappa \rangle\!\rangle = -\frac{1}{(2\pi)^4} \int \mathrm{d}^4 p \, e^{-i(p(x-x'))} \, \frac{l_\kappa}{(pl)} = i l_\kappa \, \delta_l \, .$$

Since $\mathcal{O}$ is bilinear in its arguments, this yields a useful result: $\mathcal{O}(A - \partial\phi, A'_\kappa) = 0$. Likewise,

$$\mathcal{O}(\partial A_\lambda, \phi') := \langle\!\langle \mathrm{T}_0 \, \partial_\mu \partial^\mu A_\lambda \phi' \rangle\!\rangle - \partial_\mu \langle\!\langle \mathrm{T}_0 \, \partial^\mu A_\lambda \phi' \rangle\!\rangle$$

$$= -(\Box + m^2) \langle\!\langle \mathrm{T}_0 \, A_\lambda \phi' \rangle\!\rangle = -i l_\lambda \, \delta_l \, .$$

We now tackle the obstruction $\mathcal{O}(\partial_\lambda A, A'_\kappa)$, which involves $\langle\!\langle \mathrm{T} \, \partial_\lambda A_\mu A'_\kappa \rangle\!\rangle$ that is not unique but admits the renormalization (6.4a). To wit,

$$\mathcal{O}(\partial_\lambda A, A'_\kappa) := \langle\!\langle \mathrm{T} \, \partial_\mu \partial_\lambda A^\mu A'_\kappa \rangle\!\rangle - \partial_\mu \langle\!\langle \mathrm{T} \, \partial_\lambda A^\mu A'_\kappa \rangle\!\rangle$$

$$= \partial_\lambda \left(-m^2 \langle\!\langle \mathrm{T}_0 \, \phi A'_\kappa \rangle\!\rangle - \partial^\mu \langle\!\langle \mathrm{T}_0 \, A_\mu A'_\kappa \rangle\!\rangle \right) - \partial^\mu \left(c_{\lambda\mu\kappa} \, \delta_l \right)$$

$$= i l_\kappa \, \partial_\lambda \delta_l - \partial^\mu \left(c_{\lambda\mu\kappa} \, \delta_l \right). \qquad (6.13)$$

Next, we find, using (2.7a) and (6.12), that

$$\mathcal{O}(\partial A_\lambda, A'_\kappa) := \langle\!\langle \mathrm{T} \, \partial_\mu \partial^\mu A_\lambda A'_\kappa \rangle\!\rangle - \partial_\mu \langle\!\langle \mathrm{T} \, \partial^\mu A_\lambda A'_\kappa \rangle\!\rangle$$

$$= -(\Box + m^2) \langle\!\langle \mathrm{T}_0 \, A_\lambda A'_\kappa \rangle\!\rangle - \partial^\mu (c_{\mu\lambda\kappa} \, \delta_l)$$

$$= -i g_{\lambda\kappa} \, \delta + i(l_\lambda \partial_\kappa + l_\kappa \partial_\lambda) \, \delta_l - \partial^\mu (c_{\mu\lambda\kappa} \, \delta_l). \qquad (6.14)$$

On subtracting (6.13) from (6.14), we arrive at

$$\mathcal{O}(F_{\bullet\lambda}, A'_\kappa) \equiv \mathcal{O}(\partial_\lambda A - \partial A_\lambda, A'_\kappa) = -i g_{\lambda\kappa} \, \delta + i l_\lambda \partial_\kappa \, \delta_l + \partial^\mu (c_{[\lambda\mu]\kappa} \, \delta_l).$$

Finally, we take note of

$$\mathcal{O}(\partial\phi_a, \phi_a') := \langle\!\langle T_0\, \partial_\mu \partial^\mu \phi_a \phi_a'\rangle\!\rangle - \partial_\mu \langle\!\langle T_0\, \partial^\mu \phi_a \phi_a'\rangle\!\rangle = \frac{i}{m_a^2}\, \delta \quad \text{for} \quad a = 1, 2, 3;$$

$$\mathcal{O}(\partial\phi_4, \phi_4') := \langle\!\langle T_0\, \partial_\mu \partial^\mu \phi_4 \phi_4'\rangle\!\rangle - \partial_\mu \langle\!\langle T_0\, \partial^\mu \phi_4 \phi_4'\rangle\!\rangle = i\, \delta.$$

To sum up: the obstructions of the third bosonic type are:

$$\mathcal{O}(A, \phi') = 0, \qquad \mathcal{O}(A, A_\kappa') = il_\kappa\, \delta_l, \qquad \mathcal{O}(\partial\phi_a, \phi_a') = (i/m_a^2)\, \delta,$$

$$\mathcal{O}(\partial_\lambda A, \phi') = 0, \qquad \mathcal{O}(\partial\phi, A_\kappa') = il_\kappa\, \delta_l, \qquad \mathcal{O}(\partial\phi_4, \phi_4') = i\, \delta,$$

$$\mathcal{O}(A - \partial\phi, A_\kappa') = 0, \qquad \mathcal{O}(\partial A_\lambda, \phi') = -il_\lambda\, \delta_l,$$

$$\mathcal{O}(\partial_\lambda A, A_\kappa') = il_\kappa\, \partial_\lambda \delta_l - \partial^\mu\big(c_{\lambda\mu\kappa}\, \delta_l\big),$$

$$\mathcal{O}(\partial A_\lambda, A_\kappa') = -ig_{\lambda\kappa}\, \delta + i(l_\lambda \partial_\kappa + l_\kappa \partial_\lambda)\, \delta_l - \partial^\mu\big(c_{\mu\lambda\kappa}\, \delta_l\big),$$

$$\mathcal{O}(F_{\bullet\lambda}, A_\kappa') = -ig_{\lambda\kappa}\, \delta + il_\lambda \partial_\kappa\, \delta_l + \partial^\mu\big(c_{[\lambda\mu]\kappa}\, \delta_l\big). \qquad (6.15)$$

The fermionic obstructions, which do not involve stringlike fields, are much simpler. They are of two kinds, where ψ, ψ' denote two fermions of the same type:

$$\mathcal{O}(\gamma\psi, \bar\psi') := \langle\!\langle T_0\, \gamma^\mu \partial_\mu \psi\, \bar\psi'\rangle\!\rangle - \gamma^\mu \partial_\mu \langle\!\langle T_0\, \psi\, \bar\psi'\rangle\!\rangle = -\delta,$$

$$\mathcal{O}(\psi', \bar\psi\gamma) := \langle\!\langle T_0\, \psi'\, \partial_\mu \bar\psi\rangle\!\rangle \gamma^\mu - \partial_\mu \langle\!\langle T_0\, \psi'\, \bar\psi\rangle\!\rangle \gamma^\mu = +\delta. \qquad (6.16)$$

Indeed, using (5.3), we obtain

$$\mathcal{O}(\gamma\psi, \bar\psi') = -(\partial\!\!\!/ + im_\psi)\langle\!\langle T_0\, \psi\, \bar\psi'\rangle\!\rangle = -i(\partial\!\!\!/ + im_\psi)S^F(x - x') = -\delta(x - x'),$$

and the second case follows similarly.

7. Computing the Second-Order Constraints

A priori, in equation (6.5) there may be three kinds of contractions pertinent to our problem of the type (6.7b), coming from the *crossing* of the, respectively, bosonic and fermionic couplings S_1^B and S_1^F with the Q_1^B and Q_1^F vector operators. These crossings contain information about the fermionic vertices. Happily, the bosonic interaction set S_1^B and the fermionic Q_1^F-vertex turn out an inert combination, because there are no obstructions involving the form-valued fields w_a.

Our goal in this section is to determine the couplings, as far as possible, from the vanishing of obstructions in (6.7b) of the third type (6.9c)—which have to vanish separately from the other two types as remarked after Eq. (6.10). Firstly, we seek the $\tilde b_3$ and $\tilde b_4$ coefficients of the Z-boson, which are determined together with the higgs couplings c_0 and c_5. Secondly, we shall be able to determine the quotient $b_1/\tilde b_1$, thereby obtaining chirality of the charged boson interactions in the SM; the value of b_1 is trivially determined afterwards. Thirdly, we shall look for the electromagnetic coupling b_5. At the end, we find the missing terms for the neutral current and show vanishing of the other higgs couplings.

In what follows, we consider two types of crossings. The first involves a Q_1^B vector ψ, namely a summand taken from the formulas (4.3) and (4.4), and a S_1^F coupling χ' that is a summand of (5.7); these we call (Q_1^B, S_1^F)-type crossings. The second type pairs a Q_1^F vector summand ψ of (5.6) with a term χ' in (5.7); these will be (Q_1^F, S_1^F)-type crossings. (The possible fermionic crossings are listed in "Appendix B".) Each such crossing yields a single term in the total obstruction (6.7b), consisting of a 2-point obstruction combined with certain (Wick) products of fields. Different individual crossings may, and will, turn out to have the same field content—which give opportunities for cancellation of their obstructions.

For convenience and readability, we shall omit the factor g^2 in all crossings in this section, reinstating it in the final result.

7.1. Step 1: Impact of Higgs Couplings

Lemma 2. *The crossings with field content $w_Z(x,l)\phi_Z(x,l)\bar{e}(x)e(x)$ yield no obstruction to string independence, if and only if the higgs and Z-boson coupling coefficients c_0 and $\tilde{b}_3$ are related as follows:*

$$c_0 = \frac{8\tilde{b}_3^2\, m_e \cos^2\Theta}{m_W}. \tag{7.1}$$

Proof. One such crossing, of the (Q_1^B, S_1^F)-type, arises from the last term $-\frac{1}{2\cos\Theta}\, m_Z\, \partial_\mu\phi_4\, \phi_Z w_Z$ in (4.4) with the term $c_0\phi_4\bar{e}e$ in (5.7). From table (6.15), this contributes to the total obstruction the term:

$$-ic_0 \frac{m_Z}{\cos\Theta}\, w_Z(x,l)\phi_Z(x,l)\bar{e}(x)e(x)\, \delta(x-x').$$

A factor of 2 comes from appending the identical second contribution in (6.5); we do likewise from now on without further notice.

On the other hand, there is a crossing of type (Q_1^F, S_1^F), matching $\tilde{b}_3 w_Z\bar{e}\gamma^\mu\gamma^5 e$ in (5.6) and $2im_e\tilde{b}_3\phi_Z\bar{e}\gamma^5 e$ in (5.7). Here there are two $\bar{e}$-e contractions of equal value, see table (B.1), for a total contribution of

$$8im_e\tilde{b}_3^2\, w_Z(x,l)\phi_Z(x,l)\bar{e}(x)e(x)\, \delta(x-x').$$

String independence therefore demands cancellation of the last two expressions; since there are no more crossings with this field content, this yields (7.1). □

Lemma 3. *The crossings with field content $w_Z(x,l)\phi_4(x)\bar{e}(x)\gamma^5 e(x)$ yield no obstruction to string independence, if and only if $c_0 = m_e/2m_W$. Hence*

$$\tilde{b}_3 = \pm\frac{1}{4\cos\Theta} =: \varepsilon_1 \frac{1}{4\cos\Theta} \tag{7.2}$$

where the sign $\varepsilon_1 = \pm 1$ is yet to be determined.

Proof. There is one crossing of type (Q_1^B, S_1^F), of $\frac{1}{2\cos\Theta}m_Z w_Z\phi_4\, \partial_\mu\phi_Z$ from (4.4) with the term $2im_e\tilde{b}_3\phi_Z\bar{e}\gamma^5 e$ from (5.6). For this one, (6.15) yields

$$-2\tilde{b}_3 \frac{m_e}{m_W}\, w_Z(x,l)\phi_4(x)\bar{e}(x)\gamma^5 e(x)\, \delta(x-x').$$

Now there are two relevant (Q_1^F, S_1^F)-type crossings: $\tilde{b}_3 w_Z \bar{e} \gamma^\mu \gamma^5 e$ with $c_0 \phi_4 \bar{e} e$ and $b_3 w_Z \bar{e} \gamma^\mu e$ with $\tilde{c}_0 \phi_4 \bar{e} \gamma^5 e$. The second vanishes—see (B.1) again—and the first yields

$$4 \tilde{b}_3 c_0 \, w_Z(x, l) \phi_4(x) \bar{e}(x) \gamma^5 e(x) \, \delta(x - x').$$

Cancellation of these crossings requires $c_0 = m_e / 2 m_W$, as claimed. Comparing that with the relation (7.1), we arrive at $\tilde{b}_3^2 = 1/(16 \cos^2 \Theta)$, and (7.2) follows. □

Lemma 4. *The vanishing of obstructions implies similar relations between the higgs and Z-boson coupling coefficients c_5 and $\tilde{b}_4$:*

$$c_5 = \frac{8 \tilde{b}_4^2 \, m_\nu \cos^2 \Theta}{m_W} = \frac{m_\nu}{2 m_W}$$

and thereby leads to a determination of $\tilde{b}_4$ with another unspecified sign ε_2:

$$\tilde{b}_4 = \pm \frac{1}{4 \cos \Theta} =: \varepsilon_2 \frac{1}{4 \cos \Theta}. \tag{7.3}$$

Proof. In much the same way as before, we look now for crossings of either type with field content $w_Z(x, l) \phi_Z(x, l) \bar{\nu}(x) \nu(x)$. There are just two of these: $-\frac{1}{2 \cos \Theta} m_Z w_Z \, \partial_\mu \phi_4 \, \phi_Z$ with $c_5 \phi_4 \bar{\nu} \nu$ and $\tilde{b}_4 w_Z \bar{\nu} \gamma^\mu \gamma^5 \nu$ with $2 i m_\nu \tilde{b}_4 \phi_Z \bar{\nu} \gamma^5 \nu$. These cancel provided that c_5 and $\tilde{b}_4$ satisfy the first relation above.

On the other hand, the field content $w_Z(x, l) \phi_4(x) \bar{\nu}(x) \gamma^5 \nu(x)$ can arise from four crossings: $\frac{1}{2 \cos \Theta} m_Z w_Z \phi_4 (\partial_\mu \phi_Z - Z_\mu)$ with both $2 i m_\nu \tilde{b}_4 \phi_Z \bar{\nu} \gamma^5 \nu$ and $\tilde{b}_4 Z_\kappa \bar{\nu} \gamma^\kappa \gamma^5 \nu$; and moreover the (Q_1^F, S_1^F)-type ones $\tilde{b}_4 w_Z \bar{\nu} \gamma^\mu \gamma^5 \nu$ with $c_5 \phi_4 \bar{\nu} \nu$, and $b_4 w_Z \bar{\nu} \gamma^\mu \nu$ with $\tilde{c}_5 \phi_4 \bar{\nu} \gamma^5 \nu$. The second and fourth of these again vanish. Cancellation of the first and third leads to $c_5 = m_\nu / 2 m_W$; and $\tilde{b}_4^2 = 1/(16 \cos^2 \Theta)$ follows at once. □

Note that the higgs couplings c_0 and c_5 come out, respectively, proportional to the electron and neutrino masses, with the same proportionality constant—as it should be.[17]

7.2. Step 2: The Road to Chirality

The signs ε_1 and ε_2 turn out to be related. This is the main step in the proof.

Lemma 5. *The coefficients $\tilde{b}_3$ and $\tilde{b}_4$ have opposite signs: $\varepsilon_2 = -\varepsilon_1$.*

Proof. Consider together obstructions with field contents $w_- W_{+\kappa} \bar{e} \gamma^\kappa \gamma^5 e$ and $w_+ W_{-\kappa} \bar{e} \gamma^\kappa \gamma^5 e$. They may come from crossings of type (Q_1^F, S_1^F):

$$b_1 w_- \bar{e} \gamma^\mu \nu \quad \text{with} \quad \tilde{b}_1 W_{+\kappa} \bar{\nu} \gamma^\kappa \gamma^5 e \quad \text{and} \quad \tilde{b}_1 w_- \bar{e} \gamma^\mu \gamma^5 \nu \quad \text{with} \quad b_1 W_{+\kappa} \bar{\nu} \gamma^\kappa e,$$

$$b_1 w_+ \bar{\nu} \gamma^\mu e \quad \text{with} \quad \tilde{b}_1 W_{-\kappa} \bar{e} \gamma^\kappa \gamma^5 \nu \quad \text{and} \quad \tilde{b}_1 w_+ \bar{\nu} \gamma^\mu \gamma^5 e \quad \text{with} \quad b_1 W_{-\kappa} \bar{e} \gamma^\kappa \nu.$$

Each line gives rise to two identical obstructions, with total value

$$-4 b_1 \tilde{b}_1 (w_- W_{+\kappa} - w_+ W_{-\kappa}) \bar{e} \gamma^\kappa \gamma^5 e \, \delta(x - x').$$

---

[17] We have left aside the possibility that $\tilde{b}_3$, c_0, $\tilde{b}_4$ and c_5 all vanish; this will soon be refuted.

Such a term also arises from the (Q_1^B, S_1^F)-type crossing of the term $i\cos\Theta\,(w_-W_+^\lambda - w_+W_-^\lambda)\,F_{\mu\lambda}^Z$ in (4.3) with $\tilde{b}_3 Z_\kappa \bar{e}\gamma^\kappa\gamma^5 e$. As we saw in Sect. 6, this is a "dangerous" crossing, yielding

$$2\tilde{b}_3\cos\Theta\,(w_-W_{+\kappa} - w_+W_{-\kappa})\bar{e}\gamma^\kappa\gamma^5 e\,\delta(x-x')$$
$$+ 2i\tilde{b}_3\cos\Theta\,(w_-W_+^\lambda - w_+W_-^\lambda)\bar{e}\gamma^\kappa\gamma^5 e\,\partial^\mu\big(c_{[\lambda\mu]\kappa}\,\delta_l(x-x')\big).$$

The term $il_\lambda\,\partial_\kappa\delta_l$ in $\mathcal{O}(F_{\bullet\lambda}^Z, Z_\kappa')$ does not contribute, since $l_\lambda W_\pm^\lambda = 0$ by transversality (see Sect. 2). We obtain, in all:

$$(2\tilde{b}_3\cos\Theta - 4b_1\tilde{b}_1)(w_-W_{+\kappa} - w_+W_{-\kappa})\bar{e}\gamma^\kappa\gamma^5 e\,\delta(x-x')$$
$$- 2i\tilde{b}_3\cos\Theta\,(w_-W_+^\lambda - w_+W_-^\lambda)\bar{e}\gamma^\kappa\gamma^5 e\,\partial^\mu\big(c_{[\lambda\mu]\kappa}\,\delta_l(x-x')\big).$$

Here string independence dictates that $c_{[\lambda\mu]\kappa} = 0$.[18] The end result is

$$2b_1\tilde{b}_1 = \tilde{b}_3\cos\Theta. \tag{7.4a}$$

A completely parallel computation, for obstructions with the field contents $w_\mp W_{\pm\kappa}\bar{\nu}\gamma^\kappa\gamma^5\nu$, gives the relation

$$2b_1\tilde{b}_1 = -\tilde{b}_4\cos\Theta. \tag{7.4b}$$

In view of (7.2) and (7.3), this says that $\varepsilon_2 = -\varepsilon_1$. □

Corollary 6. *The interactions with fermions of the charged vector bosons must be fully chiral, because* $\tilde{b}_1 = \varepsilon_1 b_1$.

Proof. We now observe that $w_-\phi_Z\bar{e}\nu$ is produced either by the term from (4.2) of the form $\frac{i}{2}m_W^2\sec\Theta\,w_-\,\partial_\mu\phi_+\,\phi_Z$, crossed with $i(m_e-m_\nu)b_1\phi_-\bar{e}\nu$ from (5.7), or by purely fermionic crossings, between $\tilde{b}_1 w_-\bar{e}\gamma^\mu\gamma^5\nu$ and the terms $2im_e\tilde{b}_3\phi_Z\bar{e}\gamma^5 e + 2im_\nu\tilde{b}_4\phi_Z\bar{\nu}\gamma^5\nu$. This, together with (7.2) and (7.3), leads to

$$i(m_e - m_\nu)b_1 = 2\tilde{b}_1(2im_e\tilde{b}_3 + 2im_\nu\tilde{b}_4)\cos\Theta = i(m_e - m_\nu)\varepsilon_1\tilde{b}_1$$

and the relation $\tilde{b}_1 = \varepsilon_1 b_1$ follows. □

Of course, this procedure cannot tell us whether $\varepsilon_1 = +1$ or $\varepsilon_1 = -1$. The second of these appears to be Nature's decision.

Equations (7.4) now dictate that $b_1^2 = \tilde{b}_1^2 = 1/8$. This determines b_1, up to a sign; we choose $b_1 = -1/2\sqrt{2}$.

Observe that the proof of chirality requires the presence of a higgs, at the level of tree graphs. (Indeed, were $\tilde{b}_3 = 0$ or $\tilde{b}_4 = 0$, it would follow that $b_1 = \tilde{b}_1 = 0$ too, and the whole term S_1^F would vanish. Thus, none of these coefficients are zero, and (7.2) is confirmed, with $c_0 \neq 0$ and $c_5 \neq 0$ as well.) There are several consistency cases for the scalar particle of the Standard Model. But it is hard to think of a simpler one. (We owe this remark to Alejandro Ibarra.)

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[18] This implies that all two-point obstructions of the first type (6.9a) also vanish, see (6.10a). Those of the second type (6.9b) can be freely set to zero, since they involve the up-to-now free parameters $b_{\alpha\beta\mu}$, see (6.10b).

7.3. Step 3: Electric Charge

The coefficient $e = gb_5$ of the coupling $A_\mu \bar{e}\gamma^\mu e$ in (5.7) is just the electric charge. An important tenet of electroweak theory [36] is that $e = g\sin\Theta$, with Θ being the Weinberg angle.

Lemma 7. *The relation* $gb_5 = g\sin\Theta$ *holds.*

Proof. Consider the term $-i\sin\Theta\, w_- A^\lambda F_{\mu\lambda}^+$ in (4.3), crossed with the term $b_1 W_{-\kappa}\bar{e}\gamma^\kappa\nu$ in (5.7), and the crossing of $b_1 w_-\bar{e}\gamma^\mu\nu$ with $b_5 A_\kappa\bar{e}\gamma^\kappa e$. These are the only terms yielding the field content $w_- A_\kappa\bar{e}\gamma^\kappa\nu$. The total obstruction is

$$(2b_1 b_5 - 2b_1\sin\Theta)w_-(x,l)A_\kappa(x,l)\bar{e}(x)\gamma^\kappa\nu(x)\,\delta(x-x').$$

This vanishes if and only if $b_5 = \sin\Theta$. □

The case could also have been made from the crossings with field content $w_+ A_\kappa\bar{\nu}\gamma^\kappa e$, *mutatis mutandis*.

7.4. Step 4: Mopping Up

We still have to determine the couplings b_3 and b_4 for the neutral current. For that, we seek first the contributions with content $w_- W_{+\kappa}\bar{e}\gamma^\kappa e$. The crossings are of four classes:

$$i\sin\Theta\, w_- W_+^\lambda F_{\mu\lambda} \quad \text{with} \quad b_5 A_\kappa\bar{e}\gamma^\kappa e,$$
$$i\cos\Theta\, w_- W_+^\lambda F_{\mu\lambda}^Z \quad \text{with} \quad b_3 Z_\kappa\bar{e}\gamma^\kappa e,$$
$$b_1 w_-\bar{e}\gamma^\mu\nu \quad \text{with} \quad b_1 W_{+\kappa}\bar{\nu}\gamma^\kappa e,$$
$$\tilde{b}_1 w_-\bar{e}\gamma^\mu\gamma^5\nu \quad \text{with} \quad \tilde{b}_1 W_{+\kappa}\bar{\nu}\gamma^\kappa\gamma^5 e.$$

The cancellation of the total obstruction now entails

$$b_3\cos\Theta + \sin^2\Theta = b_1^2 + \tilde{b}_1^2 = \frac{1}{4}\,, \text{that is, } b_3 = \frac{1}{4\cos\Theta} - \frac{\sin^2\Theta}{\cos\Theta}\,.$$

Similarly, from the crossing of $i\cos\Theta\, w_- W_+^\lambda F_{\mu\lambda}^Z$ with $b_4 Z_\kappa\bar{\nu}\gamma^\kappa\nu$, and the same fermionic terms as before, the contributions with content $w_- W_{+\kappa}\bar{\nu}\gamma^\kappa\nu$ cancel only if

$$b_4\cos\Theta = -b_1^2 - \tilde{b}_1^2 = -\frac{1}{4}\,, \text{ and thus } b_4 = -\frac{1}{4\cos\Theta}\,.$$

The expected result of the neutral current containing a right-handed component has been obtained.

Finally, crossing the term $-\frac{1}{2}m_Z\sec\Theta w_Z\phi_Z\,\partial_\mu\phi_4$ in (4.4) with the terms $\tilde{c}_0\phi_4\bar{e}\gamma^5 e$ and $\tilde{c}_5\phi_4\bar{\nu}\gamma^5\nu$ of (5.7) gives rise to terms with content $w_Z\phi_Z\bar{e}\gamma^5 e$ and $w_Z\phi_Z\bar{\nu}\gamma^5\nu$, respectively.

The crossings of $b_3 w_Z\bar{e}\gamma^\mu e$ with $2im_e\tilde{b}_3\phi_Z\bar{e}\gamma^5 e$ and $b_4 w_Z\bar{\nu}\gamma^\mu\nu$ with $2im_\nu$ $\tilde{b}_4\phi_Z\bar{\nu}\gamma^5\nu$, respectively, vanish of their own accord: see table (B.1). Therefore, they cannot cancel the former crossings, and so $\tilde{c}_0 = \tilde{c}_5 = 0$ must hold. That is to say, the couplings of the higgs *are not chiral*.

In conclusion, we exhibit the leptonic couplings (for one family) of the SM, as derived from string independence. For definiteness, we take $\varepsilon_1 = -1$, which is the experimental fact. Here, then, is the chirality theorem in full.

The Chirality Theorem

Theorem 8. *The couplings of electroweak bosons to the fermion sector of the Standard Model are fully determined from string independence and the choice of sign $\varepsilon_1 = -1$. Given that choice, the absence of obstructions to string independence, at tree level up to second order, entails that:*

$$
\begin{aligned}
S_1^F = g\Bigg\{ & -\frac{1}{2\sqrt{2}}\, W_{-\mu}\bar{e}\gamma^\mu(1-\gamma^5)\nu - \frac{1}{2\sqrt{2}}\, W_{+\mu}\bar{\nu}\gamma^\mu(1-\gamma^5)e \\
& +\frac{1-4\sin^2\Theta}{4\cos\Theta}\, Z_\mu\bar{e}\gamma^\mu e - \frac{1}{4\cos\Theta}\, Z_\mu\bar{e}\gamma^\mu\gamma^5 e \\
& -\frac{1}{4\cos\Theta}\, Z_\mu\bar{\nu}\gamma^\mu(1-\gamma^5)\nu + \sin\Theta\, A_\mu\bar{e}\gamma^\mu e \\
& +i\frac{m_e - m_\nu}{2\sqrt{2}}\,(\phi_-\bar{e}\nu - \phi_+\bar{\nu}e) - i\frac{m_e + m_\nu}{2\sqrt{2}}\,(\phi_-\bar{e}\gamma^5\nu + \phi_+\bar{\nu}\gamma^5 e) \\
& -i\frac{m_e}{2\cos\Theta}\,\phi_Z\bar{e}\gamma^5 e + i\frac{m_\nu}{2\cos\Theta}\,\phi_Z\bar{\nu}\gamma^5\nu + \frac{m_e}{2m_W}\,\phi_4\bar{e}e + \frac{m_\nu}{2m_W}\,\phi_4\bar{\nu}\nu \Bigg\}.
\end{aligned}
\tag{7.5}
$$

Amazingly, this differs from what is known from the standard treatment by little more than a divergence.

Scholium 9. *One can write $S_1^F = S_1^{F,\mathrm{p}} + (\partial V)$, where $S_1^{F,\mathrm{p}}$ is almost pointlike,*

$$
\begin{aligned}
S_1^{F,\mathrm{p}} = g\Bigg\{ & -\frac{1}{2\sqrt{2}}\, W_{-\mu}^{\mathrm{p}}\bar{e}\gamma^\mu(1-\gamma^5)\nu - \frac{1}{2\sqrt{2}}\, W_{+\mu}^{\mathrm{p}}\bar{\nu}\gamma^\mu(1-\gamma^5)e \\
& +\frac{1-4\sin^2\Theta}{4\cos\Theta}\, Z_\mu^{\mathrm{p}}\bar{e}\gamma^\mu e - \frac{1}{4\cos\Theta}\, Z_\mu^{\mathrm{p}}\bar{e}\gamma^\mu\gamma^5 e \\
& -\frac{1}{4\cos\Theta}\, Z_\mu^{\mathrm{p}}\bar{\nu}\gamma^\mu(1-\gamma^5)\nu + \sin\Theta\, A_\mu\bar{e}\gamma^\mu e \\
& +\frac{m_e}{2m_W}\,\phi_4\bar{e}e + \frac{m_\nu}{2m_W}\,\phi_4\bar{\nu}\nu \Bigg\};
\end{aligned}
\tag{7.6}
$$

where V^μ is given by

$$
\begin{aligned}
V^\mu = g\Bigg\{ & -\frac{1}{2\sqrt{2}}\, \phi_-\bar{e}\gamma^\mu(1-\gamma^5)\nu - \frac{1}{2\sqrt{2}}\, \phi_+\bar{\nu}\gamma^\mu(1-\gamma^5)e \\
& +\frac{1-4\sin^2\Theta}{4\cos\Theta}\, \phi_Z\bar{e}\gamma^\mu e - \frac{1}{4\cos\Theta}\, \phi_Z\bar{e}\gamma^\mu\gamma^5 e \\
& -\frac{1}{4\cos\Theta}\, \phi_Z\bar{\nu}\gamma^\mu(1-\gamma^5)\nu \Bigg\}.
\end{aligned}
$$

That is to say, the divergence of the expression V sweeps away the escort fields.

We wrote "almost pointlike" because the fields in (7.6) are pointlike, except for the photon field A_μ, which remains stringlike—for the good reason that $W_\pm$ and Z can be lodged in a Hilbert space, whereas A cannot. Incidentally, this causes the interacting electron field to be string-localized, thus making direct contact with the early literature on stringlike fields [8, 9]. A key observation is that (∂V) is *not renormalizable* by power counting, whereas (∂Q) is.

We rest our case. The only way to disprove it would be to find an inconsistency coming from crossings not discussed so far. To verify that this does not happen is a routine, if utterly tedious, exercise.

A last remark is in order. In the stringlike version of electroweak theory, the eventual need of "renormalizing" the original time-ordered product T_0, as in (6.4a), arises. We only found that the skewsymmetric part of $c_{\lambda\mu\kappa}$ in that formula must vanish. Whether or not the theory requires a time-ordered product different from T_0 remains an open question.

8. Conclusion and Outlook

To repeat ourselves: interactions of quanta should spring from a simple underlying principle. Gauge field theory has played this unifying role so far. That flows from the embarrassing clash of the positivity axioms of Quantum Mechanics with the convenient description of electromagnetic and other forces in terms of potentials. Not unreasonably, the difficulty was elevated into a principle, and one that put geometry in the saddle. The resulting top-down approach, with the need of "quantizing" the Lagrangian description, has ridden us (without much mercy) for many a year. It should be recognized, however, that the gauge-plus-BRST-invariance framework is just a very useful theoretical *technology* to grapple with elementary particle physics problems. Other theoretical technologies can and sometimes are and should be used to address them. Stringlike field theory is but one of those. With the early dividends that the mentioned clash fades away, and unbounded-helicity particles take their due place among quantum fields [6].

To be sure, the extra variable complicates renormalized perturbation theory and the proof of renormalizability of physical models in general. Notwithstanding, the string independence principle becomes a powerful guide to interacting models. Internal symmetries are shown as consequences of quantum mechanics in the presence of Lorentz symmetry, and a bottom-up construction of the string-local equivalent for self-interaction of the Yang–Mills type ensues [37]. Fortunately, as with the chirality theorem itself, all that and more requires only construction of time-ordered products associated with tree graphs.[19]

All that being said, the model expounded here is of course anomalous, which manifests itself in S_3. The cure is the same as in the standard treatments. The computation of the chiral anomaly in our framework will be published elsewhere.

A natural question is: to what extent, on the basis of string independence of the couplings, chirality of the interaction with fermions is a generic trait of physics models. We do not have a comprehensive answer to this. From our

[19] There is nothing much new in this: in the seventies it was generally understood that unitarity and renormalizability requirements impose internal symmetries and at least the presence of one scalar field, under appropriate circumstances [39,40]. For heavy vector boson interactions, the Higgs-mechanism shortcut replaced this wisdom in the textbooks. Similar bottom-up arguments surface nowadays in [41, Prob. 9.3 and Sect. 27.5].

treatment here one gathers that models with only massless bosons like QCD are purely vectorial, on the one hand. Limits of the SM, like the Georgi–Glashow model and the Higgs–Kibble model, on the other hand, must exhibit chirality.

Acknowledgements

Appendix A. Proof of Eq. (6.8)

We prove here the identities

$$\sum_{\varphi}\left(d_l\frac{\partial S_1}{\partial\varphi}\langle\!\langle \mathrm{T}\,\varphi\chi'\rangle\!\rangle + \frac{\partial S_1}{\partial\varphi}\langle\!\langle \mathrm{T}(d_l\varphi)\chi'\rangle\!\rangle\right) = [\mathrm{T}(d_lS_1)\chi']_{\mathrm{tree}}\,, \qquad (\mathrm{A.1})$$

$$\sum_{\psi}\left(\partial_\mu\frac{\partial Q^\mu}{\partial\psi}\langle\!\langle \mathrm{T}\,\psi\chi'\rangle\!\rangle + \frac{\partial Q^\mu}{\partial\psi}\langle\!\langle \mathrm{T}(\partial_\mu\psi)\chi'\rangle\!\rangle\right) = [\mathrm{T}(\partial_\mu Q^\mu)\chi']_{\mathrm{tree}}\,. \qquad (\mathrm{A.2})$$

Using the identity

$$d_lS_1 = \sum_{\varphi} :\frac{\partial S_1}{\partial\varphi}\, d_l\varphi:\,,$$

the right-hand side of Eq. (A.1) is

$$\sum_{\varphi}\left[\mathrm{T}:\frac{\partial S_1}{\partial\varphi}\,d_l\varphi:\chi'\right]_{\mathrm{tree}} = \sum_{\psi}\left\{\sum_{\varphi}:\frac{\partial^2 S_1}{\partial\varphi\partial\psi}\,d_l\varphi:\right\}\langle\!\langle \mathrm{T}\,\psi\chi'\rangle\!\rangle$$
$$+ \sum_{\varphi}\frac{\partial S_1}{\partial\varphi}\,\langle\!\langle \mathrm{T}(d_l\varphi)\chi'\rangle\!\rangle.$$

But the term in braces is just $d_l\frac{\partial S_1}{\partial\psi}$. Hence the right-hand side of the above equation coincides with the left-hand side of Eq. (A.1).

Similarly, using $\partial_\mu Q^\mu = \sum_\varphi :(\partial Q^\mu/\partial\varphi)\,\partial_\mu\varphi:$, the right-hand side of Eq. (A.2) becomes

$$\sum_\psi \left\{ \sum_\varphi :\frac{\partial^2 Q^\mu}{\partial\varphi\partial\psi}\,\partial_\mu\varphi: \right\} \langle\!\langle \mathrm{T}\,\psi\chi'\rangle\!\rangle + \sum_\varphi \frac{\partial Q^\mu}{\partial\varphi}\,\langle\!\langle \mathrm{T}(\partial_\mu\varphi)\chi'\rangle\!\rangle,$$

which equals the left-hand side of Eq. (A.2).

Appendix B. Fermionic Crossings

The crossings of fermionic type in Sect. 7 are computed as follows. When crossing $\bar{e}\gamma^\mu\nu$ with $\bar{\nu}'\gamma^k\gamma^5 e'$, say, one meets two obstructions of type (6.16): contracting the neutrinos gives a factor $\mathcal{O}(\gamma\nu,\bar{\nu}') = -\delta(x-x')$, whereas contraction of the electrons gives $\mathcal{O}(e',\bar{e}\gamma) = +\delta(x-x')$. Thus, the overall crossing yields a sum of two terms

$$-\bar{e}(x)\gamma^\kappa\gamma^5 e(x)\,\delta(x-x') + \bar{\nu}(x)\gamma^\kappa\gamma^5\nu(x)\,\delta(x-x').$$

On the other hand, the crossing of $\bar{e}\gamma^\mu\gamma^5 e$ with $\bar{e}'\gamma^5 e'$, say, involving both $\mathcal{O}(\gamma e,\bar{e}')$ and $\mathcal{O}(e',\bar{e}\gamma)$, gives two equal contributions of $\bar{e}(x)\,e(x)\,\delta(x-x')$ to the total obstruction.

There are sixteen kinds of crossings in all, taking account of the order of the contractions, and the presence or absence of γ^κ and/or γ^5 factors. Let f denote a fermion (ν or e, as the case may be). When computing the crossings, we label the contracted terms with stars: either $\gamma^\mu f\,\bar{f}'$ is replaced by $\mathcal{O}(\gamma f,\bar{f}') = -\delta$, or $f'\,\bar{f}\gamma^\mu$ is replaced by $\mathcal{O}(f',\bar{f}\gamma) = +\delta$. In the table which follows, σ and τ denote uncontracted fermions:

$$\bar{\sigma}\gamma^\mu \overset{\star}{f}\, \overset{\star}{\bar{f}'}\gamma^\kappa\tau' \rightsquigarrow -\bar{\sigma}\gamma^\kappa\tau\cdot\delta, \qquad \overset{\star}{\bar{f}}\gamma^\mu\tau\,\bar{\sigma}'\gamma^\kappa \overset{\star}{f'} \rightsquigarrow +\bar{\sigma}\gamma^\kappa\tau\cdot\delta,$$

$$\bar{\sigma}\gamma^\mu\gamma^5 \overset{\star}{f}\, \overset{\star}{\bar{f}'}\gamma^\kappa\gamma^5\tau' \rightsquigarrow -\bar{\sigma}\gamma^\kappa\tau\cdot\delta, \qquad \overset{\star}{\bar{f}}\gamma^\mu\gamma^5\tau\,\bar{\sigma}'\gamma^\kappa\gamma^5 \overset{\star}{f'} \rightsquigarrow +\bar{\sigma}\gamma^\kappa\tau\cdot\delta,$$

$$\bar{\sigma}\gamma^\mu\gamma^5 \overset{\star}{f}\, \overset{\star}{\bar{f}'}\gamma^\kappa\tau' \rightsquigarrow -\bar{\sigma}\gamma^\kappa\gamma^5\tau\cdot\delta, \qquad \overset{\star}{\bar{f}}\gamma^\mu\gamma^5\tau\,\bar{\sigma}'\gamma^\kappa \overset{\star}{f'} \rightsquigarrow +\bar{\sigma}\gamma^\kappa\gamma^5\tau\cdot\delta.$$

$$\bar{\sigma}\gamma^\mu \overset{\star}{f}\, \overset{\star}{\bar{f}'}\gamma^\kappa\gamma^5\tau' \rightsquigarrow -\bar{\sigma}\gamma^\kappa\gamma^5\tau\cdot\delta, \qquad \overset{\star}{\bar{f}}\gamma^\mu\tau\,\bar{\sigma}'\gamma^\kappa\gamma^5 \overset{\star}{f'} \rightsquigarrow +\bar{\sigma}\gamma^\kappa\gamma^5\tau\cdot\delta,$$

$$\bar{\sigma}\gamma^\mu \overset{\star}{f}\, \overset{\star}{\bar{f}'}\tau' \rightsquigarrow -\bar{\sigma}\tau\cdot\delta, \qquad \overset{\star}{\bar{f}}\gamma^\mu\tau\,\bar{\sigma}' \overset{\star}{f'} \rightsquigarrow +\bar{\sigma}\tau\cdot\delta,$$

$$\bar{\sigma}\gamma^\mu\gamma^5 \overset{\star}{f}\, \overset{\star}{\bar{f}'}\gamma^5\tau' \rightsquigarrow +\bar{\sigma}\tau\cdot\delta, \qquad \overset{\star}{\bar{f}}\gamma^\mu\gamma^5\tau\,\bar{\sigma}'\gamma^5 \overset{\star}{f'} \rightsquigarrow +\bar{\sigma}\tau\cdot\delta,$$

$$\bar{\sigma}\gamma^\mu\gamma^5 \overset{\star}{f}\, \overset{\star}{\bar{f}'}\tau' \rightsquigarrow +\bar{\sigma}\gamma^5\tau\cdot\delta, \qquad \overset{\star}{\bar{f}}\gamma^\mu\gamma^5\tau\,\bar{\sigma}' \overset{\star}{f'} \rightsquigarrow +\bar{\sigma}\gamma^5\tau\cdot\delta,$$

$$\bar{\sigma}\gamma^\mu \overset{\star}{f}\, \overset{\star}{\bar{f}'}\gamma^5\tau' \rightsquigarrow -\bar{\sigma}\gamma^5\tau\cdot\delta, \qquad \overset{\star}{\bar{f}}\gamma^\mu\tau\,\bar{\sigma}'\gamma^5 \overset{\star}{f'} \rightsquigarrow +\bar{\sigma}\gamma^5\tau\cdot\delta. \qquad \text{(B.1)}$$

Appendix C. Proof of Locality of the Stringy Fields

We prove here locality in the sense that $A_\mu(x,l)$ and $A_\alpha(x',l')$ commute if the strings $\{x+tl\}$ and $\{x'+tl'\}$ are causally disjoint and not parallel. We begin

with some geometric considerations about wedge regions. These are Poincaré transforms of the wedge

$$W_1 := \{\, x \in \mathbb{R}^4 : x^1 > |x^0| \,\}.$$

Associated with W_1 are the one-parameter group $\Lambda_1(\cdot)$ of Lorentz boosts which leave W_1 invariant, and the reflection j_1 across the edge of the wedge. More specifically, $\Lambda_1(t)$ acts as

$$\begin{pmatrix} \cosh t & \sinh t \\ \sinh t & \cosh t \end{pmatrix}$$

and j_1 acts as the reflection on the coordinates x^0 and x^1, leaving the other coordinates unchanged. For a general wedge $W = LW_1 = a + \Lambda W_1$ with $L = (a, \Lambda)$, one defines the corresponding boosts $\Lambda_W(\cdot)$ and reflection j_W by

$$\Lambda_W(t) := L\,\Lambda_1(t)\,L^{-1}, \quad j_W := L\,j_1\,L^{-1}.$$

The reflection j_W results from analytic extension of the (entire analytic) matrix-valued function $\Lambda_W(z)$ at $z = i\pi$.

Note that in the definition of covariance in Sect. 2 the string direction transforms only under the homogeneous part of the Poincaré transformations. This leads us to consider the mapping $(a, \Lambda) : l \mapsto \Lambda l$ as the natural action of the Poincaré group on the manifold of string directions. In particular, if $W = a + \Lambda W_1$ then

$$\Lambda_W(t)l = \Lambda\Lambda_1(t)\Lambda^{-1}l. \tag{C.1}$$

Lemma 10. (i) *A string $\{x + tl\}$ is contained in the closure of a wedge $W = a + \Lambda W_1$ if and only if x and l are contained in the closures of W and ΛW_1, respectively.*

(ii) *Suppose that the strings $\{x + tl\}$ and $\{x' + tl'\}$ are causally disjoint and not parallel. Then there is a wedge W whose closure contains $\{x+tl\}$ and whose causal complement contains $\{x' + tl'\}$. The corresponding boosts, respectively, act as*

$$\Lambda_W(t)l = e^t l \qquad and \qquad \Lambda_W(t)l' = e^{-t}l'. \tag{C.2}$$

Proof. Item (i) is the same as in Lemma A.1. of [7], whose proof is valid for any direction $l \in \mathbb{R}^4$.

For item (ii), take $W := \frac{1}{2}(x + x') + W_{l,l'}$, where $W_{l,l'}$ is the wedge $\{\, y : (yl) < 0 < (yl') \,\}$. The causal complement of this W is the closure of $\frac{1}{2}(x+x') + W_{l',l}$, see [42]. Furthermore, l is—up to a factor—the only lightlike vector contained in the upper boundary of $W_{l,l'}$ (which is a part of the lightlike hyperplane $l^{\perp}$).

Using the elementary fact that $\{x+tl\}$ and $\{x'+tl'\}$ are causally disjoint if and only if $(x - x')^2 < 0$ and $\big((x' - x)l\big) \geq 0 \geq \big((x' - x)l'\big)$, one readily verifies [26] that these strings are contained in the respective wedges $\overline{W}$ and W', as claimed.

In terms of the lightlike vectors $l_{(\pm)} := (1, \pm 1, 0, 0)$, the standard wedge W_1 is just $W_{l_{(+)}, l_{(-)}}$. Since $l_{(+)}$ is, up to a factor, the only lightlike vector contained in the upper boundary of W_1, the Lorentz transformation Λ maps

the span of $l_{(+)}$ onto the span of l. Thus, $\Lambda_W(t)l \equiv \Lambda\Lambda_1(t)\Lambda^{-1}l$ is a multiple of $\Lambda\Lambda_1(t)l_{(+)}$. But one readily verifies that $\Lambda_1(t)l_{(+)} = e^t l_{(+)}$. This proves the first equation in (C.2). The second is shown analogously, using that Λ maps the span of $l_{(-)}$ onto that of l'. $\qquad\square$

We now prove locality of the two-point function, recalling first that the on-shell two-point function for not necessarily coinciding directions is given, instead of (2.7a), by

$$M_{\mu\nu}^{AA}(p, l, l') = -g_{\mu\nu} + \frac{p_\mu l_\nu}{(pl)} + \frac{p_\nu l'_\mu}{(pl')} - \frac{p_\mu p_\nu\,(l\,l')}{(pl)(pl')}, \qquad (C.3)$$

see [28]. Given the two causally disjoint and non-parallel strings, let W be a wedge whose closure contains $\{x + tl\}$ and whose causal complement contains $\{x' + tl'\}$ (as in the lemma), and let j_W and $\Lambda_W(t)$ be the reflection and the boosts, respectively, corresponding to W. Denote by g_t the proper non-orthochronous Poincaré transformation $\Lambda_W(-t)j_W$. By translation invariance of the two-point function, we may assume that the edge of W contains the origin. Then x and l are in the closure of W, while x' and l' lie in the causal complement of W. This implies that for t in the strip $\mathbb{R} + i(0, \pi)$ the imaginary parts of both $g_t x$ and $g_{-t}x'$ lie in the closed forward light cone—see, for example, Eq. (A.7) in [7].

Now consider the relation

$$\int \mathrm{d}\mu(p)\, e^{-i(p(x'-g_t x))}\, M_{\alpha\mu}^{AA}(p, l', g_t l)$$
$$= \int \mathrm{d}\mu(p)\, e^{-i(p(x-g_{-t}x'))}\, M_{\alpha\mu}^{AA}(-g_t p, l', g_t l), \qquad (C.4)$$

which is verified by applying the transformation $p \mapsto -g_t p$ on the mass shell. (We use $-g_t$ instead of g_t, since the former is an orthochronous Poincaré transformation, while the latter is not orthochronous and maps the positive onto the negative mass shell.) We may write $g_t^{-1} = g_{-t}$, since j_W and $\Lambda_W(t)$ commute. We wish to extend the function $F(t)$ defined by (C.4) analytically into the strip $\mathbb{R} + i(0, \pi)$. To this end, note that the Minkowski products of $g_t x$ and $g_{-t}x'$ with a covector p in the mass shell both have positive imaginary parts due to the remark before Eq. (C.4). This implies that the functions $|\exp i(pg_t x)|$ and $|\exp i(pg_{-t}x')|$ are uniformly bounded by 1 over the strip. Furthermore, $M_{\alpha\mu}^{AA}(p, l', g_t l) = M_{\alpha\mu}^{AA}(p, l', l)$ since $g_t l = e^t l$ by Eq. (C.2), and the factor e^t cancels as can be seen from Eq. (C.3). By the same token plus covariance, one obtains

$$M_{\alpha\mu}^{AA}(-g_t p, l', g_t l) \equiv (-g_t)_\alpha{}^\beta M_{\beta\nu}^{AA}(p, -g_{-t}l', -l)(-g_{-t})_\mu{}^\nu$$
$$= (g_t)_\alpha{}^\beta M_{\beta\nu}^{AA}(p, l', l)(g_{-t})_\mu{}^\nu.$$

These facts imply that $F(t)$ has an analytic extension into the strip, and Eq. (C.4) holds, by the Schwarz reflection principle, also at $t = i\pi$. But $g_{\pm i\pi} = 1$, and thus at $t = i\pi$ the left-hand side of Eq. (C.4) reduces, up to a factor $(2\pi)^3$, to the vacuum expectation value $\langle\!\langle A_\alpha(x', l')A_\mu(x, l)\rangle\!\rangle$. On the right-hand side, one verifies that $M_{\alpha\mu}^{AA}(p, l', l) = M_{\mu\alpha}^{AA}(p, l, l')$. Thus, at $t = i\pi$ the

right-hand side of (C.4) reduces, up to a factor $(2\pi)^3$, to $\langle\!\langle A_\mu(x,l)A_\alpha(x',l')\rangle\!\rangle$. In short, Eq. (C.4) at $i\pi$ is just the locality of the two-point functions. This implies locality of the fields by a standard argument in the proof of the Jost–Schroer theorem [43].

Appendix D. A Model of Leptons

Engineering the GWS model from our formalism is not overly desirable. But we do it here, as promised in the introduction. Let us reconsider the three first lines of expression (7.5). We begin by introducing the notation

$$\Psi_L := \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} := \begin{pmatrix} \frac{1}{2}(1-\gamma^5)\nu \\ \frac{1}{2}(1-\gamma^5)e \end{pmatrix}.$$

First,

$$-\frac{1}{\sqrt{2}}W_{-\mu}\bar{e}\gamma^\mu\frac{(1-\gamma^5)}{2}\nu = -\frac{1}{\sqrt{2}}\,\overline{\Psi}_L\gamma^\mu\begin{pmatrix} 0 & 0 \\ W_{-\mu} & 0 \end{pmatrix}\Psi_L = -\tfrac{1}{2}\overline{\Psi}_L\gamma^\mu W_{-\mu}\tau_-\Psi_L$$

where $\tau_\pm = (\tau_1 \pm i\tau_2)/\sqrt{2}$, with τ_i denoting here the Pauli matrices. Similarly,

$$-\frac{1}{\sqrt{2}}W_{+\mu}\bar{\nu}\gamma^\mu\frac{(1-\gamma^5)}{2}e = -\tfrac{1}{2}\overline{\Psi}_L\gamma^\mu W_{+\mu}\tau_+\Psi_L.$$

The first two terms in (7.5) are therefore of the form

$$-\tfrac{1}{2}g\overline{\Psi}_L\gamma^\mu(W_{+\mu}\tau_+ + W_{-\mu}\tau_-)\Psi_L = -\tfrac{1}{2}g\overline{\Psi}_L\gamma^\mu(W_{1\mu}\tau_1 + W_{2\mu}\tau_2)\Psi_L. \quad \text{(D.1)}$$

Knowing, as we know, that the interaction is governed by a $U(2)$ symmetry, it is tempting to regard ν and e as isospin components valued $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively. The "right-handed leptons" $e_R := \frac{1}{2}(1+\gamma^5)e$ and $\nu_R := \frac{1}{2}(1+\gamma^5)\nu$ are isospin singlets.

Denote by Q the electric charge, so that $Q(e) = -1$ and $Q(\nu) = 0$, and isospin by I_3. Observe that, putting $\Psi = \Psi_L + \Psi_R$, the next four terms of (7.5) are rendered into:

$$-g\sin\Theta\,\overline{\Psi}\gamma^\mu(A_\mu - Z_\mu\tan\Theta)Q\Psi - \frac{g}{\cos\Theta}\,\overline{\Psi}_L\gamma^\mu Z_\mu I_3\Psi_L. \quad \text{(D.2)}$$

In order to translate this into the received framework, with its "covariant gauge transformation" technology, we now introduce the unobservable fields

$$\begin{aligned} W_{3\mu} &:= \cos\Theta\,Z_\mu + \sin\Theta\,A_\mu \\ B_\mu &:= -\sin\Theta\,Z_\mu + \cos\Theta\,A_\mu \end{aligned} \quad \text{inverted by} \quad \begin{aligned} A_\mu &= \cos\Theta\,B_\mu + \sin\Theta\,W_{3\mu} \\ Z_\mu &= -\sin\Theta\,B_\mu + \cos\Theta\,W_{3\mu}. \end{aligned}$$

Then, with $g_B := g\tan\Theta$, we can rewrite (D.2) as

$$-g_B\overline{\Psi}\gamma^\mu B_\mu Q\Psi + g_B\overline{\Psi}_L\gamma^\mu B_\mu I_3\Psi_L - \tfrac{1}{2}g\overline{\Psi}_L\gamma^\mu W_{3\mu}\tau_3\Psi_L. \quad \text{(D.3)}$$

One can now bring in the convention

$$Y = 2(Q - I_3), \quad \text{that is:} \quad Y(e_L) = Y(\nu_L) = -1; \ Y(e_R) = -2, \ Y(\nu_R) = 0.$$

Then the first two summands in (D.3) are rewritten as $-\frac{1}{2}g_B\overline{\Psi}\gamma^\mu B_\mu Y\Psi$, while the last one plus the right side of (D.1) yields $-\frac{1}{2}g\overline{\Psi}_L(\gamma^\mu\mathbb{W}_\mu\cdot\tau)\Psi_L$.

In fine, we have manufactured the interaction parts of the GWS Lagrangian.

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