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Implicit 2D surface flow models performance assessment: Shallow Water Equations vs. Zero-Inertia Model

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Abstract. Zero-Inertia (ZI) models are used in overland flow simulation due to their mathematical simplicity, compared to more complex formulations such as Shallow Water (SW) models. The main hypothesis in ZI models is that the flow is driven by water surface and friction gradients, neglecting local accelerations. On the other hand, SW models are a complete dynamical formulation that provide more information at the cost of a higher level of complexity. In realistic problems, the usually huge number of cells required to ensure accurate spatial representation implies a large amount of computing effort and time. This is particularly true in 2D models. Hence, there is an interest in developing efficient numerical methods. In general terms, numerical schemes used to solve time dependent problems can be classified in two groups, attending to the time evaluation of the unknowns: explicit and implicit methods. Explicit schemes offer the possibility to update the solution at every cell from the known values but are restricted by numerical stability reasons. This can lead to very slow simulations in case of using fine meshes. Implicit schemes avoid this restriction at the cost of generating a system of as many equations as computational cells multiplied by the number of variables to solve. In this work, an implicit finite volume numerical scheme has been used to solve the 2D equations in both ZI and SW models. The scheme is formulated so that both quadrilateral and triangular meshes can be used. A conservative linearization is done for the flux terms, leading to a non-structured matrix for unstructured meshes thus requiring iterative methods for solving the system. A comparison between 2D SW and 2D ZI is done in terms of performance, efficiency and mesh requirements, in which both models benefit of an implicit temporal discretization in steady and nearly-steady situations.

1 Introduction

The interest in the development of efficient hydraulic/hydrologic models has increased over the last decades. A wide range of natural phenomena can be studied by means of numerical simulation tools for predictive purposes. Flood events, rainfall-runoff-infiltration processes, river swelling, etc are some examples of topics of interest in which computer simulation can improve the management of natural hazards. It is traditionally accepted that the most accurate

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mathematical models for simulating surface flows are based on the Shallow Water equations (SW) [7, 12]. Nevertheless, the wide range of applicability of the SW models to any type of overland flow has a counterpoint in the usually high computational cost. In order to deal with this issue, simplified models have been developed (Kinematic Wave, Zero Inertia (a.k.a. Diffusion Wave), Gravity Wave). The Zero Inertia model (ZI) is usual choice for modeling overland flow under certain conditions [3, 9]. This model neglects all the intertia terms of the SW model momentum equations.

In general, numerical methods used to solve time dependent equations or system of equations can be classified in two groups, depending on the time evaluation of the unknowns: explicit and implicit methods. The first group updates the solution at every cell of the computational domain from the known values at the current time. On the other hand, implicit schemes generate a system of equations to solve the whole mesh at the same time. Implicit schemes are unconditionally stable, hence, these methods avoid stability issues when using large time steps. This constitutes one of the main reasons for using them in steady state computations. In the particular case of the ZI model, its explicit discretization has been reported as inefficient [2, 3, 6]. The performance and efficiency of the SW model is widely studied in [5].

In this work, a comparison between implicit SW and implicit ZI models is carried out in terms of performance and efficiency. Both models are applied to two realistic cases, the first one purely hydraulic and the second one with hydrological components. In both cases, the numerical results are compared and the computational cost is measured for both models in order to test the efficiency.

2 Mathematical models

2.1 2D Shallow Water Equations

The 2D Shallow Water equations (also known as Dynamic-Wave model) represent mass and momentum conservation averaged in the vertical direction [7, 12]

$$\frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = R \tag{1}$$

$$\frac{\partial q_x}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q_x^2}{h} + \frac{1}{2}gh^2\right) + \frac{\partial}{\partial y}\left(\frac{q_x q_y}{h}\right) = gh\left(S_{0x} - S_{fx}\right) \tag{2}$$

$$\frac{\partial q_y}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q_x q_y}{h}\right) + \frac{\partial}{\partial y}\left(\frac{q_y^2}{h} + \frac{1}{2}gh^2\right) = gh\left(S_{0y} - S_{fy}\right) \tag{3}$$

where the conserved variables are h representing the water depth (m) and $q_x = hu$ and $q_y = hv$ the unit discharges (m^2/s), with u and v (m/s) the depth averaged components of the velocity vector $\mathbf{u}$ along the x and y coordinates respectively. The acceleration due to gravity is represented with g (m/s^2). The source terms on the right hand side of the equations are written in terms of the net rainfall intensity R (m/s), the bed slopes of the bottom level z (m) in the x and y direction, S_{0x} and S_{0y}, respectively, given by:

$$S_{0x} = -\frac{\partial z}{\partial x}, \qquad S_{0y} = -\frac{\partial z}{\partial y} \tag{4}$$

The terms S_{fx}, S_{fy} represent the friction slopes in both directions, here written in terms of the Manning's roughness coefficient n ($sm^{-1/3}$):

$$S_{fx} = \frac{n^2 u\sqrt{u^2+v^2}}{h^{4/3}}, \qquad S_{fy} = \frac{n^2 v\sqrt{u^2+v^2}}{h^{4/3}} \tag{5}$$

2.2 2D Zero Inertia Model

One of the most commonly used strategy to simplify the SW system is the Zero-Inertia model [3, 9], which neglects acceleration terms in (2) and (3). The resulting expressions are listed below:

$$\frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = R \tag{6}$$

$$\frac{\partial h}{\partial x} = S_{0x} - S_{fx}, \qquad \frac{\partial h}{\partial y} = S_{0y} - S_{fy} \tag{7}$$

By defining the water surface slope vector as

$$\mathbf{S} = \left(S_x, S_y\right) = -\left(\frac{\partial(h+z)}{\partial x}, \frac{\partial(h+z)}{\partial y}\right) = -\nabla(h+z) \tag{8}$$

the system (6)-(7) can be compacted in a single non-linear scalar equation (see [3] for further details):

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = R, \qquad \mathbf{q} = \left(\frac{h^{5/3}}{n\sqrt{|\mathbf{S}|}} S_x\, , \; \frac{h^{5/3}}{n\sqrt{|\mathbf{S}|}} S_y\right) \tag{9}$$

3 Numerical models

An implicit first-order upwind finite volume numerical scheme is used for the discretization of both mathematical models (1)-(3) and (9). The numerical scheme has proven to be well-balanced, with a good tracking of wet/dry fronts. The use of a distributed surface flow model allows to calculate all the hydraulic and hydrologic variables, such as the water depth h or the flow velocities u, v in every cell of the computational mesh. The details of the numerical model used for the discretization of SW and ZI models can be found in [5] and [3], respectively. Both implicit discretizations generate a system matrix which needs to be solved by an iterative technique in case of using unstructured triangular meshes. In this work, the BiConjugate Gradient Stabilized (BiCGStab) is used as matrix solver in combination with a dual threshold incomplete LU factorization preconditioner [10, 11].

There is an important consideration that should be taken into account when performing a simulation with an implicit numerical scheme. As the implicit numerical scheme is unconditionally stable, a total control of the time step is reached and, hence, any value can be chosen. Nevertheless, as shown in [3, 5], a larger time step does not mean a faster simulation. Larger time steps require huge number of matrix solver iterations, which implies a high computational cost per time step. Usually, there is an optimal time step choice that minimizes the implicit simulation cost.

4 Numerical results

4.1 Test 1: Hydraulic case

The setup for this test is an adaptation of the case proposed in [8] to establish a comparison among different simulation models applied to valley flood simulation. Figure 1 (left) shows the valley bed elevation map. The domain is discretized by means of an unstructured triangular mesh (7592 cells). A uniform Manning's roughness coefficient of $n = 0.04 sm^{-1/3}$ is set all over the domain. All the boundaries remain closed except the inlet segment in which a

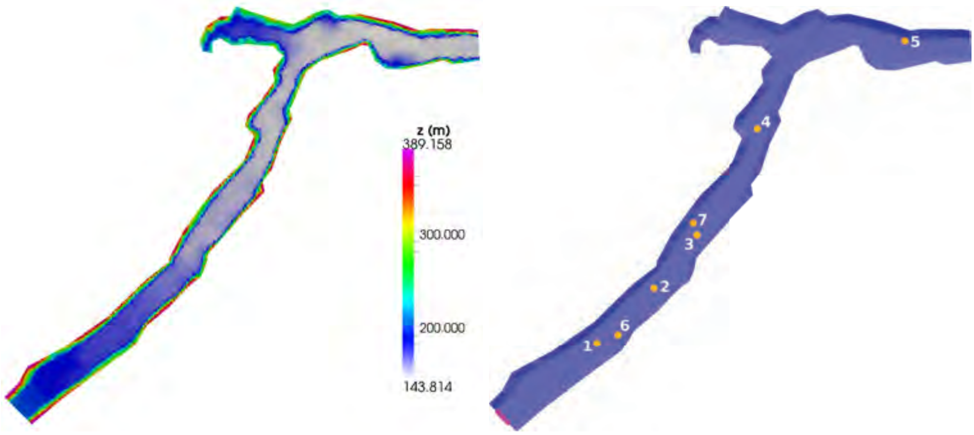

Figure 1. Bed elevations map (left) and probes location (right).

constant water depth of $0.5m$ is imposed (magenta line in Figure 1, right). The evolution of the water depth is registered in several gauges located as in Figure 1 (right).

Figure 2 shows the numerical results for the flood extension at two different times for SW model with $\Delta t = 16.2s$ (optimal for SW), ZI model with $\Delta t = 16.2s$ and ZI model with $\Delta t = 19s$ (optimal for ZI). Figure 3 shows the results for a selection of water depth probes for both SW and ZI implicit models. In general terms, both models produce very similar numerical solutions at all the gauges, observing minor differences at probes 2 and 5. Figure 3 (lower, right) shows the representation of the CPU time vs. Δt for both ZI and SW models. In this case, the implicit ZI model is slightly faster than the implicit SW model when choosing optimal time steps in both cases.

4.2 Test 2: Hydrologic case

This laboratory test case was originally presented in [2] in order to perform a comparison between SW and ZI explicit models. It was also used in [3] to compare implicit and explicit versions of the ZI model. A constant rainfall of $300mm/h$ during $20s$ is assumed over the impervious domain of a lab-scale catchment provided with several obstacles which are modeled as holes in the mesh (Figure 4). The Manning's roughness coefficient is set to $0.016sm^{-1/3}$.

Figure 5 shows the distributed water depth values at $t = 10s$ and $t = 15s$ for three different simulations: 1) ZI model with the optimal choice of the time step $\Delta t = 0.344s$ (left), 2) SW model with the same time step as in 1) ($\Delta t = 0.344s$) (middle) and 3) SW model with its optimal choice of the time step $\Delta t = 0.428s$ (right). Figure 6 shows the temporal evolution of the water level at the two gauges. Both models predict the same arriving time of the water level peak but ZI generates lower values than SW model. This is in concordance with the conclusions reached in [2], where the relevance of the inertia terms in some specific situations is pointed out. Figure 7 shows the representation of the CPU time vs. Δt for both models. In this particular case, the implicit ZI model is 2.5x faster than the implicit SW model when choosing optimal time steps in both cases.

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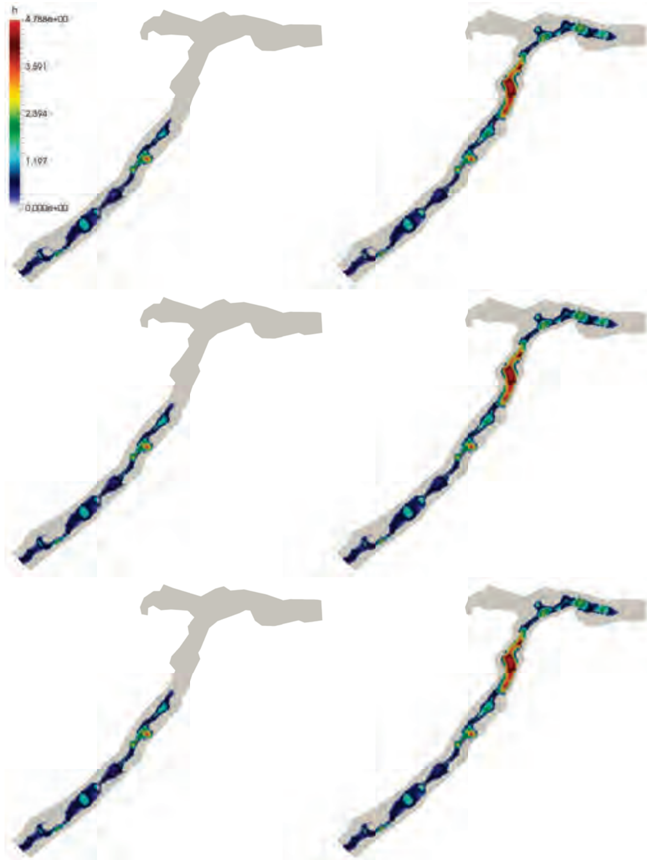

Figure 2. Test 1: Flood extension for $t = 9500s$ (left) and $t = 30000s$ (right) for SW model with $\Delta t = 16.2s$ (upper), ZI model with $\Delta t = 16.2s$ (middle) and ZI model with $\Delta t = 19s$ (lower).

5 Conclusions

The implementation of an implicit upwind scheme for both SW and ZI models has been presented in this work. The performance of both models has been compared through the application to two different hydraulic/hydrologic test cases. In both situations presented, the numerical challenges, as the dry/wet fronts tracking, are well resolved in both models without observing any stabilty issues. Test case 1 shows that SW and ZI models produce very similar numerical results, both in the flood extension and in the water level values at several selected gauges. Regarding the computational cost, implicit ZI model becomes slightly more efficient than implicit SW. The low slope values (in general) of this particular topography contribute to a good performance of the ZI model. The second test represents a small-scale catchment for rainfall/runoff simulation. Several town houses randomly placed and represented as holes in the mesh add more complexity to the case. The comparison of the numerical results generated in two different gauges shows that both models ZI model produce the same rising peak time but ZI predicts smaller values for the water level. Implicit ZI model show a better performance (2.5x faster) than the implicit SW for this particular case.

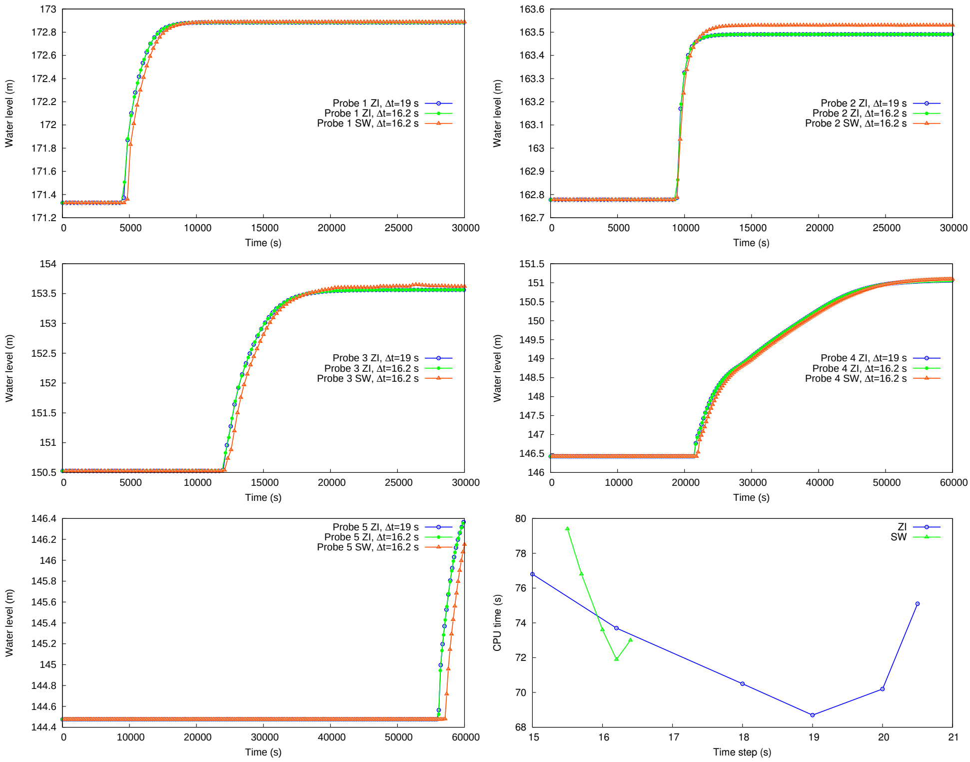

Figure 3. Test 1: Water level at probes 1 to 5 for for SW model with $\Delta t = 16.2s$, ZI model with $\Delta t = 16.2s$ and ZI model with $\Delta t = 19s$. CPU times as a function of the chosen time step for the simulation in both models (lower, right)

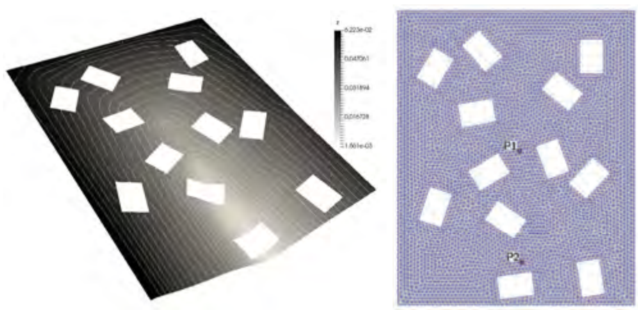

Figure 4. 3D representation of the bed elevations map (left, vertical scale is exaggerated 5x) and computational mesh with probe locations (right).

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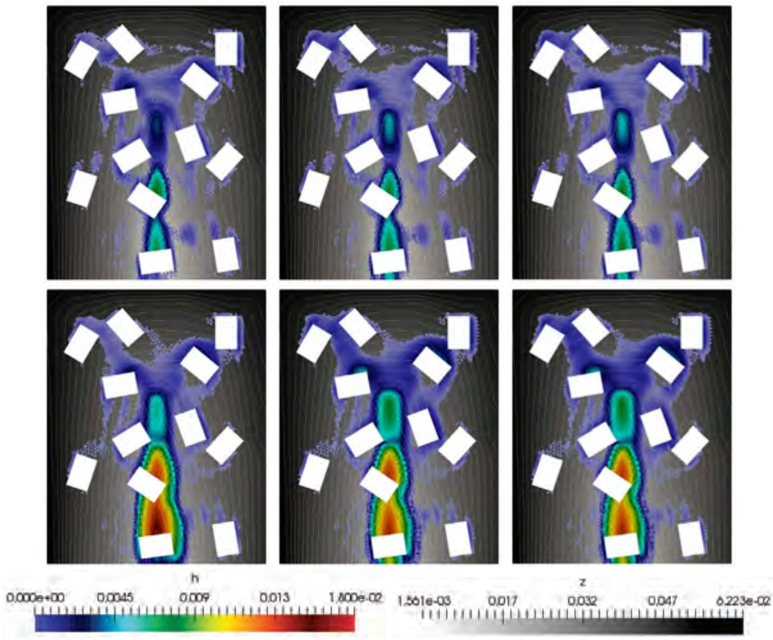

Figure 5. Test 2: Water depth values at $t = 10s$ (upper) and $t = 15s$ (lower) for ZI model with $dt = 0.344s$ (left), SW model with $dt = 0.344s$ (middle) and SW model with $dt = 0.428s$ (right).

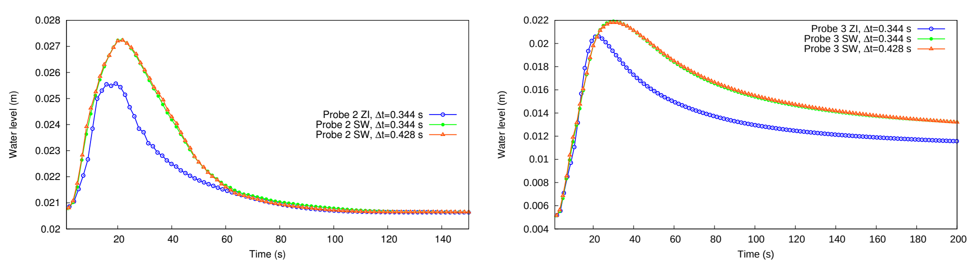

Figure 6. Test 2: Water level at two different locations.

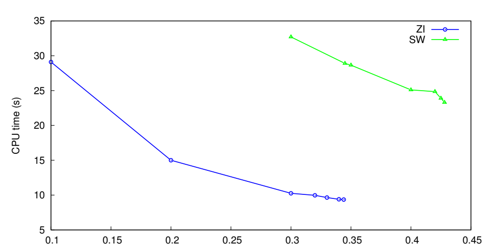

Figure 7. Test 2: CPU times as a function of the chosen time step for the simulation in both models.

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