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CORRECTIONS ON REPEATING GROUND-TRACK ORBITS AND THEIR APPLICATIONS IN SATELLITE CONSTELLATION DESIGN

David Arnas[*], Daniel Casanova[†], and Eva Tresaco[‡]

The aim of the constellation design model shown in this paper is to generate constellations whose satellites share the same ground-track in a given time, making all the satellites pass over the same points of the Earth surface. The model takes into account a series of orbital perturbations such as the gravitational potential of the Earth, the atmospheric drag, the Sun and the Moon as disturbing third bodies or the solar radiation pressure. It also includes a new numerical method that improves the repeating ground-track property of any given satellite subjected to these perturbations. Moreover, the whole model allows to design constellations with multiple tracks that can be distributed in a minimum number of inertial orbits.

INTRODUCTION

Space has become a strategic resource that offers an unlimited number of possibilities. Scientific and military missions, telecommunications or Earth observation are some of its most important applications and have led the sector to a quick expansion with an increasing number of satellites orbiting the Earth.

Satellites lie in a very advantageous position that allows the observation of vast regions of the Earth in short periods of time, an objective which is difficult to achieve with human and technical means in ground. This advantage can be improved even further with the concept of satellite constellations. Satellite constellations are groups of satellites that, having the same mission, work cooperatively to achieve it. This concept increases the complexity of the celestial mechanics problem to solve, but opens new and interesting possibilities for future missions.

Satellite constellation design has been since its beginning a process that required a high number of iterations due to the lack of established models for the generation and study of constellations. This situation resulted in the necessity of specific studies for each particular mission, being unable of extrapolate the results from one mission to another. Fortunately, in the last decade, new theoretical models have been developed that include in their formulation all of the former configurations. One of these models is the Flower Constellations Theory.

The Flower Constellations were introduced for the first time by Mortari[1] in the year 2004. The most relevant feature of this model consists of the visualization and study of the constellations using a rotating frame of reference instead of an inertial frame of reference. That way, a relative orbit whose geometry reminds the shape of the petals of a flower is obtained.

[*]Centro Universitario de la Defensa, Zaragoza, Spain. IUMA - Universidad de Zaragoza, Spain.
[†]Centro Universitario de la Defensa, Zaragoza, Spain. IUMA - Universidad de Zaragoza, Spain.
[‡]Centro Universitario de la Defensa, Zaragoza, Spain. IUMA - Universidad de Zaragoza, Spain.

The initial Flower Constellations model was later reformulated in the 2-D Lattice[2] and 3-D Lattice[3] models which improved the parametrization of the problem. However, due to the strictly keplerian formulation of the model, the inclusion of orbital perturbations is required to enhance the precision. In Casanova[4] the perturbation created by the J_2 term of the gravitational potential of the Earth was introduced in the model. Nevertheless, other orbital perturbations are also significantly modifying the orbits, so it is important to include them in the design process of the constellation.

The goal of the model introduced in this work is to generate satellite constellations that include the effects of orbital perturbations such as the gravitational potential of the Earth, the atmospheric drag, the Sun and the Moon as disturbing third bodies or the solar radiation pressure. The proposed constellation design allows to generate a configuration in which a number of different ground-tracks is defined, each of these ground-tracks containing a number of satellites that present the same instantaneous ground-track over time. Moreover, in order to decrease the number of orbital launches to build the constellation, another constraint will be set: satellites from different ground-tracks have to share the same inertial orbit, allowing a decrease in the number of inertial orbits.

Furthermore, the repeating ground-track property is introduced in the constellation model, taking into account the orbital perturbations considered. In that respect, Wagner[5] developed a numerical method to achieve this by modifying the semi-major axis of the orbits. However, a new numerical method has been developed to improve the precision obtained no matter the orbital configuration used. The numerical orbit correction method proposed also modifies the semi-major axis as in Wagner's method, but in this case the methodology is different, focusing the study on the rotational frame of reference and considering all the orbital parameters non constant.

ANALYTICAL MODEL FOR CONSTELLATION DESIGN

Throughout this paper, the so called classical orbital elements will be used, namely: a the semi-major axis, e the eccentricity, i the inclination, ω the argument of perigee, Ω the right ascension of the ascending node and M the mean anomaly. Other common parameters used are: f the true anomaly, $\omega_\oplus$ the angular velocity of the Earth, μ the Earth gravitational constant, $R_\oplus$ the Equatorial Earth radius and J_2 the second order term of the gravitational potential of the Earth.

In an unperturbed dynamic model, the classical orbital parameters (a, e, i, ω, Ω) are constant whilst the true anomaly (f) varies through time. This property will be used to show in a clear way the analytical model behind the constellation design proposed in this paper.

During this section, three different constellation designs will be shown. First, a constellation design model in which satellites share the same relative trajectory will be presented. Second, this model will be expanded with the possibility of distribution of the satellites in several different relative trajectories. And finally, a constraint will be set in order to reduce the number of inertial orbits to a minimum.

Constellation design with a common relative trajectory

The objective of this design model is to generate a constellation whose satellites share the same relative trajectory over time. The first thing required to achieve this condition is to define that particular relative trajectory.

The position of a satellite along its trajectory in the perifocal frame of reference is:

$$[\mathbf{x}] = (r\cos f,\ r\sin f,\ 0)\,, \tag{1}$$

where r is the radium of the orbit in each instant of time:

$$r = \frac{a\left(1 - e^2\right)}{1 + e\cos f}. \tag{2}$$

These positions can be expressed in the inertial frame of reference (ECI: Earth Centered Inertial) using rotational matrices ($\mathcal{R}_3$ and $\mathcal{R}_1$):

$$[\mathbf{x}]_{ECI} = \mathcal{R}_3\left(\Omega\right)\mathcal{R}_1\left(i\right)\mathcal{R}_3\left(\omega\right)[\mathbf{x}], \tag{3}$$

and it can also be expressed in the ECEF (Earth Centered - Earth Fixed) frame of reference:

$$[\mathbf{x}]_{ECEF} = \mathcal{R}_3\left(-\psi_{G0} - \omega_{\oplus}t\right)[\mathbf{x}]_{ECI}, \tag{4}$$

where ψ_{G0} is the longitude of Greenwich at the time of reference t_0 and $\omega_{\oplus}$ is the angular velocity of rotation of the Earth.

Thus, using Equation (1), (3) and (4), the position of a certain satellite is obtained in the ECEF frame of reference:

$$[\mathbf{x}]_{ECEF} = \mathcal{R}_3\left(-\psi_{G0} - \omega_{\oplus}t\right)\mathcal{R}_3\left(\Omega\right)\mathcal{R}_1\left(i\right)\mathcal{R}_3\left(\omega\right)\begin{bmatrix} r\cos f \\ r\sin f \\ 0 \end{bmatrix}. \tag{5}$$

Furthermore, combining the first two matrices, the following expression is obtained:

$$[\mathbf{x}]_{ECEF} = \mathcal{R}_3\left(\Omega - \psi_{G0} - \omega_{\oplus}t\right)\mathcal{R}_1\left(i\right)\mathcal{R}_3\left(\omega\right)\begin{bmatrix} r\cos f \\ r\sin f \\ 0 \end{bmatrix}. \tag{6}$$

The aim now is to create a constellation of satellites whose trajectories in the ECEF frame of reference are the same. Let a, e, i, ω, Ω_0 be the orbital parameters of the reference trajectory and let t_0 be the reference time of the constellation which also locates ψ_{G0} (a, e, i and ω are common for all the satellites of the constellation). This reference trajectory can be expressed in the relative frame of reference as:

$$[\mathbf{x_0}]_{ECEF} = \mathcal{R}_3\left(\Omega_0 - \psi_{G0} - \omega_{\oplus}t\right)\mathcal{R}_1\left(i\right)\mathcal{R}_3\left(\omega\right)\begin{bmatrix} r\cos f \\ r\sin f \\ 0 \end{bmatrix}, \tag{7}$$

where r and f are a function of ($t_0 + t$). This relative trajectory must be fulfilled by every satellite in the constellation, so it is fixed in the design of the constellation. If another point of this relative trajectory is considered, a satellite that shares the same relative trajectory can be obtained. If the value of t_0 is modified, this relative trajectory remains the same. Let t_1 be the changed value of t_0, then, the right ascension of the ascending node suffers a variation of $\Delta\Omega = -\omega_{\oplus}(t_1 - t_0)$ due to the fact that ψ_{G0} has its reference in t_0. Thus, the relative trajectory of the satellite when t_1 is considered is:

$$[\mathbf{x_1}]_{ECEF} = \mathcal{R}_3\left(\Omega_0 - \psi_{G0} - \omega_{\oplus}(t_1 - t_0 + t)\right)\mathcal{R}_1\left(i\right)\mathcal{R}_3\left(\omega\right)\begin{bmatrix} r\cos f \\ r\sin f \\ 0 \end{bmatrix}, \tag{8}$$

where r and f are now a function of $(t_1 + t)$. Varying t, the inertial orbit of this second satellite can be obtained through:

$$[\mathbf{x_1}]_{ECI} = \mathcal{R}_3\left(\psi_{G0} + \omega_{\oplus} t\right)[\mathbf{x_1}]_{ECEF}\,, \tag{9}$$

so the inertial orbit is:

$$[\mathbf{x_1}]_{ECI} = \mathcal{R}_3\left(\Omega_0 - \omega_{\oplus}(t_1 - t_0)\right)\mathcal{R}_1\left(i\right)\mathcal{R}_3\left(\omega\right)\begin{bmatrix} r\cos f \\ r\sin f \\ 0 \end{bmatrix}. \tag{10}$$

Note that the right ascension of the ascending node Ω_1 and the mean anomaly M_1 of the second satellite can be expressed as a function of the values of the first one:

$$\begin{aligned} \Omega_1 &= \Omega_0 - \omega_{\oplus}(t_1 - t_0); \\ M_1 &= M_0 + n(t_1 - t_0); \end{aligned} \tag{11}$$

where:

$$n = \sqrt{\frac{\mu}{a^3}}. \tag{12}$$

As it can be seen, combining both equations, there exists a function between M_1 and Ω_1:

$$M_1 = \left(M_0 + \frac{n}{\omega_{\oplus}}\Omega_0\right) - \frac{n}{\omega_{\oplus}}\Omega_1; \tag{13}$$

which represent a straight line as it can be seen in the $(\Omega,\ M)$-space[6] representation of the relative trajectory shown in Figure 1, where each vertical line represents the inertial orbit and the diagonal represents the relative trajectory of the satellite.

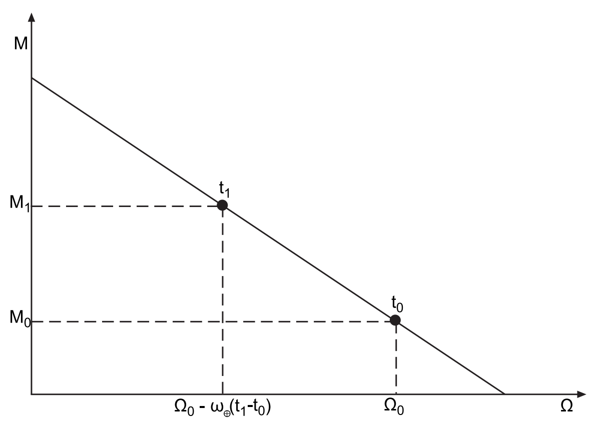

Figure 1. $(\Omega,\ M)$**-space representation of a relative trajectory.**

If instead of only one satellite, a certain number of them are taken, it is possible to generate a constellation whose satellites share the same relative trajectory. Let t_q be the temporal positions in the relative trajectory (in the same sense as t_1 worked) and let N_{st} be the number of satellites in the relative trajectory, where $q = 1, ..., N_{st}$ represent each particular satellite of the constellation. Then, for each q:

$$M_q = \left(M_0 + \frac{n}{\omega_{\oplus}}\Omega_0\right) - \frac{n}{\omega_{\oplus}}\Omega_q; \tag{14}$$

and the inertial orbits can be expressed as:

$$[\mathbf{x_q}]_{ECI} = \mathcal{R}_3\left(\Omega_0 - \omega_\oplus(t_q - t_0)\right)\mathcal{R}_1\left(i\right)\mathcal{R}_3\left(\omega\right)\begin{bmatrix} r\cos f \\ r\sin f \\ 0 \end{bmatrix}. \tag{15}$$

As Equation (15) shows, the first matrix corresponds to a rotation in a modified right ascension of the ascending node for each satellite. Let Ω_q be the right ascension of the ascending node of the satellite q, then:

$$\Omega_q = \Omega_0 - \omega_\oplus(t_q - t_0). \tag{16}$$

Note that $(t_q - t_0)$ represent a distribution over time with respect of the reference trajectory defined in the beginning, and as such, it does not depend on the time (t) used in the propagation. Moreover, Ω_q and $\omega_\oplus$ are also constant in time, so it can be concluded that Ω_q is fixed for each satellite of the constellation. On the other hand, the initial value of the true anomaly of each satellite of the constellation (f_q) only depends on t_q. Then, it is possible to generate the full constellation by the only use of the parameter of distribution t_q. Each inertial orbit of the constellation is obtained by:

$$[\mathbf{x_q}]_{ECI} = \mathcal{R}_3\left(\Omega_0 - \omega_\oplus(t_q - t_0)\right)\mathcal{R}_1\left(i\right)\mathcal{R}_3\left(\omega\right)\begin{bmatrix} \dfrac{a\left(1-e^2\right)}{1+e\cos f_q}\cos f_q \\ \dfrac{a\left(1-e^2\right)}{1+e\cos f_q}\sin f_q \\ 0 \end{bmatrix}. \tag{17}$$

Equation (17) allows to design a distribution of satellites in which all have the same relative trajectory (and thus, they share the same ground-track, its projection over the Earth surface). This distribution is done over time, with no constraints in the selection of the different values of t_q which is the parameter of distribution in the configuration.

Constellation design with multiple relative trajectories

The objective now is to distribute the satellites in several different relative trajectories instead of just one. The methodology is similar to the one seen before, but in this case, other degrees of freedom are added in the spacing of the relative trajectories in the ECEF frame of reference. Let N_t be the number of relative trajectories in which the constellation is distributed and let $k = 1, ..., N_t$ be the parameter that names each one of this trajectories. Therefore, the total number of satellites in the constellation N_s is:

$$N_s = N_{st}N_t, \tag{18}$$

where N_{st} is the number of satellites in each relative trajectory.

Furthermore, the satellites named with the sub-index $k0$ are the leading satellites of each k relative trajectory, that is, the reference satellites that define the trajectories in the ECEF frame of reference. Moreover, the leading satellite, named with the sub-index 00, represents the reference origin of the whole constellation. Thus, as seen before, the relative trajectories can be defined as:

$$[\mathbf{x_{kq}}]_{ECEF} = \mathcal{R}_3\left(\Omega_0 + \Delta\Omega_k - \psi_{G0} - \omega_\oplus(t_{kq} - t_0 + t)\right)\mathcal{R}_1\left(i\right)\mathcal{R}_3\left(\omega\right)\begin{bmatrix} r\cos f \\ r\sin f \\ 0 \end{bmatrix}, \tag{19}$$

where $\Delta\Omega_k$ is the space distribution of the relative trajectories in the ECEF frame of reference and t_{kq} represents the distribution parameter related to the initial time t_0 (required to locate the longitude of Greenwich). Note that r and f are now functions of t_{kq}. The parameter t_{kq} distributes the satellites in a k relative trajectory and the q position in that relative trajectory. As it can be seen, two degrees of freedom control the distribution of the constellation: $\Delta\Omega_k$ and t_{kq}.

Transforming those coordinates to the ECI frame of reference, and naming f_{kq} the true anomaly of the satellite q of the k relative trajectory at the initial time, the following inertial orbits for each satellite of the constellation are obtained:

$$[\mathbf{x_{kq}}]_{ECI} = \mathcal{R}_3\left(\Omega_0 + \Delta\Omega_k - \omega_\oplus(t_{kq} - t_0)\right)\mathcal{R}_1\left(i\right)\mathcal{R}_3\left(\omega\right)\begin{bmatrix} \dfrac{a\left(1-e^2\right)}{1+e\cos f_{kq}}\cos f_{kq} \\ \dfrac{a\left(1-e^2\right)}{1+e\cos f_{kq}}\sin f_{kq} \\ 0 \end{bmatrix}. \tag{20}$$

As it can be seen, the right ascension of the ascending node of each constellation satellite is:

$$\Omega_{kq} = \Omega_0 + \Delta\Omega_k - \omega_\oplus(t_{kq} - t_0), \tag{21}$$

which means that each satellite presents a different inertial orbit, a fact that increases the expenses of building the constellation in orbit. Therefore, in the next subsection the constraint of minimum number of inertial orbits will be set in order to correct this situation.

This distribution can also be represented in the $(\Omega,\ M)$-space. As done before:

$$\begin{aligned} \Omega_{kq} &= \Omega_0 + \Delta\Omega_k - \omega_\oplus(t_{kq} - t_0), \\ M_{kq} &= M_0 + n(t_{kq} - t_0), \end{aligned} \tag{22}$$

and the relation between Ω_{kq} and M_{kq} is:

$$M_{kq} = \left(M_0 + \frac{n}{\omega_\oplus}\Omega_0\right) + \frac{n}{\omega_\oplus}\Delta\Omega_k - \frac{n}{\omega_\oplus}\Omega_{kq}; \tag{23}$$

which is a distribution of points over a family of straight lines that have the same slope. Figure 2 shows a particular case of a satellite with respect to the reference trajectory (named 0). There, the satellite 11 ($k = 1, q = 1$) is located in the relative trajectory 1 which presents a rotation of $\Delta\Omega_1$ with respect to the reference trajectory.

Constellation design with minimum number of inertial orbits

Once a distribution over different relative trajectories is done, it is time to impose the restriction that the constellation has to be built in the least number of inertial orbits. As t_{kq} is a distribution, it can be separated in two different contributions, one depending on the inertial orbit (parameter q) and the other depending on the relative trajectory (parameter k). This separation is made in order to avoid the dependence in the right ascension of the ascending node with the number of relative trajectory. That way:

$$t_{kq} = t_k + t_q, \tag{24}$$

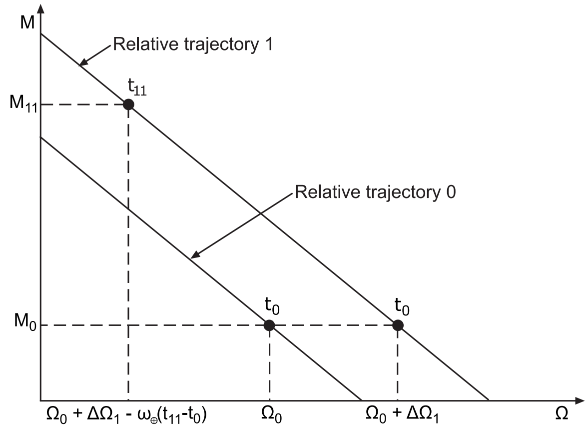

Figure 2. $(\Omega,\ M)$**-space representation of the configuration for multiple relative trajectories.**

where t_k represents the different distribution of each relative trajectory and t_q the distribution over the same relative trajectory. Thus, Equation (21) can now be written as:

$$\Omega_{kq} = \Omega_0 + \Delta\Omega_k - \omega_\oplus(t_k + t_q - t_0). \tag{25}$$

In Equation (25) is possible to eliminate the dependence on k imposing:

$$t_k = \frac{\Delta\Omega_k}{\omega_\oplus}, \tag{26}$$

and thus, introducing this value for t_k in Equation (20) the following equation is obtained:

$$[\mathbf{x_{kq}}]_{ECI} = \mathcal{R}_3\left(\Omega_0 - \omega_\oplus(t_q - t_0)\right)\mathcal{R}_1\left(i\right)\mathcal{R}_3\left(\omega\right)\begin{bmatrix} \dfrac{a\left(1-e^2\right)}{1+e\cos f_{kq}}\cos f_{kq} \\ \dfrac{a\left(1-e^2\right)}{1+e\cos f_{kq}}\sin f_{kq} \\ 0 \end{bmatrix}. \tag{27}$$

With this formulation it can be seen how the first matrix corresponds to the right ascension of the ascending node of each satellite, that is:

$$\Omega_{kq} = \Omega_0 - \omega_\oplus(t_q - t_0). \tag{28}$$

Note that now, Ω_{kq} does not depend on the terms in k, and as such, is the same for every satellite that shares the value of t_q, one for each relative trajectory. That leads to a distribution in which the satellites with the same q are distributed in the same inertial orbit whilst the satellites with the same k are distributed in the same relative trajectory (remember that f_{kq} is a function of $t_q + t_k$). Figure 3 shows how the distribution works in the ECEF and the ECI frames of reference for two generic relative trajectories.

Moreover, using the two time distributions t_q and t_k, it is possible to achieve the configuration desired with no limitations on any parameter due to the fact that the distribution parameters are real numbers and it can be freely chosen.

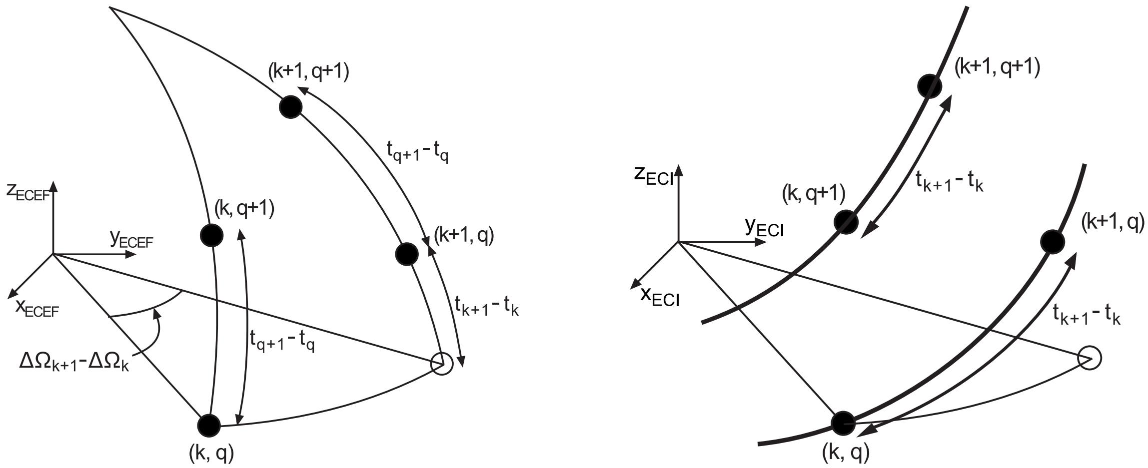

Figure 3. Constellation distribution in the ECEF (left) and ECI (right) frames of reference.

The $(\Omega,\ M)$-space representation can be defined as before:

$$\begin{aligned} \Omega_{kq} &= \Omega_0 - \omega_\oplus(t_q - t_0), \\ M_{kq} &= M_0 + n(\frac{\Delta\Omega_k}{\omega_\oplus} + t_q - t_0), \end{aligned} \tag{29}$$

obtaining the same expression as in Equation (23):

$$M_{kq} = \left(M_0 + \frac{n}{\omega_\oplus}\Omega_0\right) + \frac{n}{\omega_\oplus}\Delta\Omega_k - \frac{n}{\omega_\oplus}\Omega_{kq}. \tag{30}$$

The difference now is that the right ascension of the ascending node is shared by one satellite of each relative trajectory as seen in Figure 4. In fact it is a particular case of the one presented before.

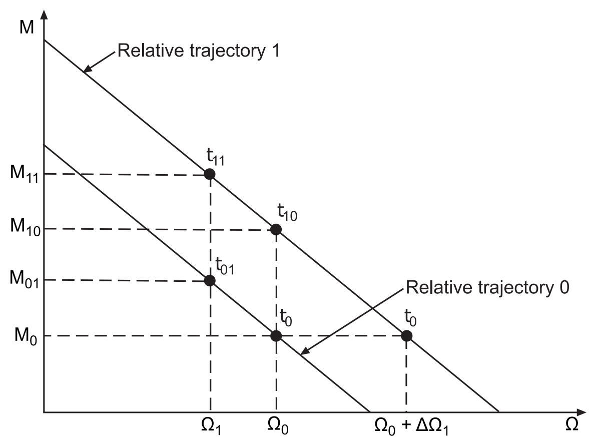

Figure 4. $(\Omega,\ M)$**-space representation of the configuration for minimum number of inertial orbits.**

SEMI-ANALYTICAL MODEL FOR CONSTELLATION DESIGN

It has previously been seen how to generate the constellation design in a keplerian model. The objective now is to apply that theory to the case of orbital perturbations. Orbital perturbations such as the gravitational potential of the Earth, the solar radiation pressure, the Sun and Moon as disturbing third bodies or the atmospheric drag, will destroy the analytical configuration proposed in a short period of time, so other complementary model has to be developed to solve this problem. The semi-analytical model proposed in this paper achieves the sharing of the projection of the relative trajectory over the Earth surface, that is, the ground-track, despite of being the satellites subjected to certain known orbital perturbations.

As done in the latter section, three different constellation designs will be presented, corresponding to the ones studied previously in the analytical model. However, due to the nature of the perturbations considered, it is more practical to design the constellation around the concept of the ground-track instead of the relative trajectory. This is the case due to the circumstance that perturbations such as the atmospheric drag will decrease the altitude of the orbit.

Constellation design with a common ground-track

The objective is to generate a constellation whose satellites share the same ground-track despite of being subjected to several known orbital perturbations. Note that sharing the same ground-track does not mean that their ground-track has to be closed, this is a property that will be presented afterwards in this paper.

The idea behind the semi-analytical model is to propagate first a reference satellite $[\mathbf{x_0}]$, which will be called the leading satellite, taking into account all the perturbations of the dynamical model chosen, and keeping the propagation results of times, positions and velocities of its relative trajectory for the instants in which the times of propagation correspond to:

$$t = t_q - t_0, \tag{31}$$

where as before, t_0 is the reference time useful to locate Greenwich, and t_q represents the parameter of distribution of each particular satellite compared to the leading satellite t_0. Moreover, using the nomenclature used in the analytical model, $q = 1, ..., N_{st}$.

Then, a transformation of this positions and velocities will be made in order to define the initial positions and velocities of the satellites of the constellation. Therefore, two transformations will be required: the first one to obtain the relative trajectory, and the second one to obtain the inertial orbits of the constellation. A schematic diagram of this two transformations can be seen in Figure 5, where it is clear that $[\mathbf{x_q}]_{ECI}$ and $[\tilde{\mathbf{x}}_\mathbf{q}]_{ECI}$ are not the same due to the different rotation performed.

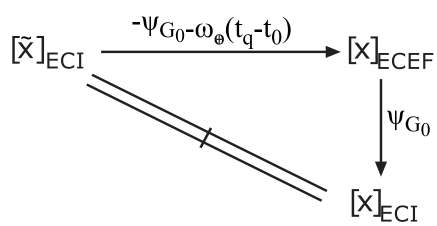

Figure 5. Schematic diagram of the two different frame transformations.

Note that each satellite requires a different rotation in the first transformation due to the fact that the position of each satellite is defined in a different time: the parameter of t_q is different for each satellite in Equation (31).

Let $[\tilde{\mathbf{x}}_{\mathbf{q}}]_{ECI}$ and $[\tilde{\mathbf{v}}_{\mathbf{q}}]_{ECI}$ be the positions and velocities of the leading satellite in the inertial frame of reference. The relative positions ($[\mathbf{x}_{\mathbf{q}}]_{ECEF}$) and velocities ($[\mathbf{v}_{\mathbf{q}}]_{ECEF}$) are obtained from the inertial ones by using the following expressions:

$$[\mathbf{x}_{\mathbf{q}}]_{ECEF} = \mathcal{R}_3\left(-\psi_{G0} - \omega_\oplus(t_q - t_0)\right)[\tilde{\mathbf{x}}_{\mathbf{q}}]_{ECI}\,, \tag{32}$$

$$[\mathbf{v}_{\mathbf{q}}]_{ECEF} = \mathcal{R}_3\left(-\psi_{G0} - \omega_\oplus(t_q - t_0)\right)[\tilde{\mathbf{v}}_{\mathbf{q}}]_{ECI} - \boldsymbol{\omega}_\oplus \times [\mathbf{x}_{\mathbf{q}}]_{ECEF}\,. \tag{33}$$

However, the initial inertial positions $[\mathbf{x}_{\mathbf{q}}]_{ECI}$ and velocities $[\mathbf{v}_{\mathbf{q}}]_{ECI}$ are required in order to define the constellation, thus, the second transformation of frames of reference is needed:

$$[\mathbf{x}_{\mathbf{q}}]_{ECI} = \mathcal{R}_3\left(\psi_{G0}\right)[\mathbf{x}_{\mathbf{q}}]_{ECEF}\,, \tag{34}$$

$$[\mathbf{v}_{\mathbf{q}}]_{ECI} = \mathcal{R}_3\left(\psi_{G0}\right)[\mathbf{v}_{\mathbf{q}}]_{ECEF} + \boldsymbol{\omega}_\oplus \times [\mathbf{x}_{\mathbf{q}}]_{ECI}\,. \tag{35}$$

One important thing to notice is that, having included the perturbations in the orbital propagation, the constellation designed presents the same ground-track for all its satellites for the perturbations considered in the constellation design. Thus, the more realistic the orbital perturbation model is, the better the constellation will perform in the reality.

Constellation design with multiple ground-tracks

The next step in complexity in the design of a constellation is to include multiple ground-tracks in the configuration. The process is similar as before, but now, several leading satellites are required in order to define the different ground-tracks, one leading satellite for each ground-track. Furthermore, the distribution of the satellites is done using two parameters: the time distribution over the different ground-tracks t_{kq} and the angular distribution of the relative trajectories in the ECEF frame of reference $\Delta\Omega_k$.

As it has been said, each ground-track requires a leading satellite. Those satellites have the same values of a_0, e_0, i_0 and w_0, whilst the right ascension of the ascending node follows:

$$\Omega_{k0} = \Omega_0 + \Delta\Omega_k, \tag{36}$$

where Ω_{k0} are the right ascension of the ascending nodes of the leading satellites and each ground-track is named as $k = 1, ..., N_t$.

Once the leading satellites are defined, each one of them is propagated numerically or analytically. This generates a number of relative trajectories equal to N_t, the number of different ground-tracks of the constellation. As before, the values of the positions and velocities of each ground-track are kept and two transformations are required:

$$\begin{aligned}
[\mathbf{x}_{\mathbf{kq}}]_{ECEF} &= \mathcal{R}_3\left(-\psi_{G0} - \omega_\oplus(t_{kq} - t_0)\right)[\tilde{\mathbf{x}}_{\mathbf{kq}}]_{ECI}\,, \\
[\mathbf{v}_{\mathbf{kq}}]_{ECEF} &= \mathcal{R}_3\left(-\psi_{G0} - \omega_\oplus(t_{kq} - t_0)\right)[\tilde{\mathbf{v}}_{\mathbf{kq}}]_{ECI} - \boldsymbol{\omega}_\oplus \times [\mathbf{x}_{\mathbf{kq}}]_{ECEF}\,; \\
[\mathbf{x}_{\mathbf{kq}}]_{ECI} &= \mathcal{R}_3\left(\psi_{G0}\right)[\mathbf{x}_{\mathbf{kq}}]_{ECEF}\,, \\
[\mathbf{v}_{\mathbf{kq}}]_{ECI} &= \mathcal{R}_3\left(\psi_{G0}\right)[\mathbf{v}_{\mathbf{kq}}]_{ECEF} + \boldsymbol{\omega}_\oplus \times [\mathbf{x}_{\mathbf{kq}}]_{ECI}\,.
\end{aligned} \tag{37}$$

The values of the inertial positions $[\mathbf{x_{kq}}]_{ECI}$ and velocities $[\mathbf{v_{kq}}]_{ECI}$ of each satellite determine the initial configuration of the constellation. This configuration distributes the constellation in N_t different ground-tracks and $N_s = N_{st}N_t$ (the number of satellites) number of inertial orbits (in general), that is, each satellite has a different inertial orbit. That is why it is required to include the constraint of minimum number of inertial orbits to decrease the expenses of the mission.

Constellation design with minimum number of inertial orbits

The latter configuration distributes the constellation in N_s different inertial orbits, which is a design decision that carries a lot of expenses to build the constellation in orbit. In order to solve that, as done in the analytical model, the distribution parameter can be separated in two different distribution parameters t_k and t_q where $t_{kq} = t_k + t_q$. Then, a relation can be established between t_k and Ω_k using Equation (26). However, due to the orbital perturbations, it is concluded that the configuration obtained in that way does not share the inertial orbits. That is why a correction over the analytical solution is required.

The orbital perturbations make the right ascension of the ascending node to shift, which means that the orbit sees the rotation of the Earth with a different angular velocity than the inertial one. Thus, if the movement of the right ascension of the ascending node is included in the formulation, the following expression is obtained:

$$t_k = \frac{\Delta\Omega_k}{\omega_\oplus - \dot{\Omega}_{kq}}, \tag{38}$$

where $\dot{\Omega}_{kq}$ is the derivative of the right ascension of the ascending node for each satellite, which can be obtained using the secular value of the perturbation. The value of t_k is introduced in Equation (37) leading to a constellation based on N_s satellites distributed in N_t ground-tracks and N_{st} inertial orbits. All this design includes the orbital perturbations considered in the propagations that were made.

EXAMPLE OF APPLICATION

As an example of application, a constellation whose satellites are equally spaced in time will be introduced. Moreover, the satellites will present the repeating ground-track property and will be distributed in the least number of inertial orbits using the semi-analytical model.

During this example the following perturbations have been taken into account: the gravitational potential of the Earth[7] to 4th order terms (including tesserals), the Sun and Moon as disturbing third bodies,[8] the solar radiation pressure[9] and the atmospheric drag (Harris-Priester[10,11] model).

It will be supposed that the parameters N_p, N_d, N_{st} and N_t are know as part of the mission requirements. Furthermore, the eccentricity and the inclination will be free for the orbit designer to be chosen, so they will be considered already known.

Moreover, as a mission requirement, the pass of the constellation over a certain point of the Earth surface with coordinates in longitude and latitude $(\psi_r,\ \phi_r)$ will be imposed. This local coordinates are related to the inertial ones $(\psi_i,\ \phi_i)$ by:

$$\begin{aligned} \psi_i &= \psi_G + \psi_r, \\ \phi_i &= \phi_r, \end{aligned} \tag{39}$$

where ψ_G is the position of the Greenwich meridian in the ECI frame of reference. Furthermore, a relation between the coordinates of Equation (39) and the orbital parameters exist through the spherical trigonometry as shown in Figure 6 and Equation (40) (Bessel's equations):

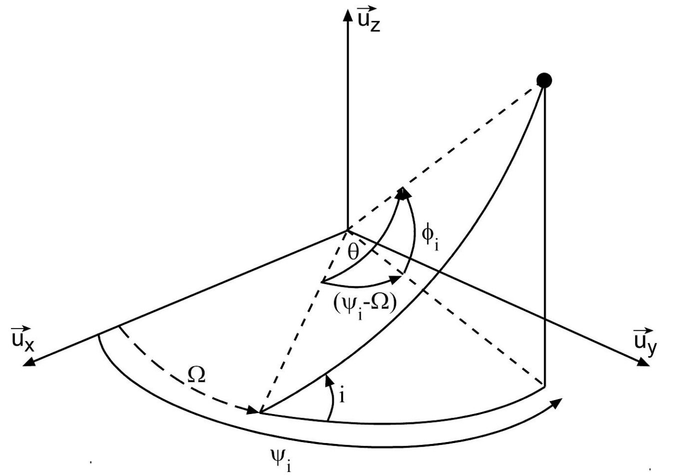

Figure 6. Spherical triangle visualization

$$\begin{bmatrix} \cos(\phi_i) \\ \sin(\phi_i) \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\psi_i - \Omega) & 0 & \sin(\psi_i - \Omega) \\ 0 & 1 & 0 \\ \sin(\psi_i - \Omega) & 0 & -\cos(\psi_i - \Omega) \end{bmatrix} \begin{bmatrix} \cos(\theta) \\ \sin(\theta)\sin(i) \\ \sin(\theta)\cos(i) \end{bmatrix}. \tag{40}$$

Equation (40) allows to obtain the right ascension of the ascending node (Ω) and the argument of latitude ($\theta = \omega + f$). These values are calculated as a function of the inclination (i) and the coordinates of the point in the Earth surface ($\psi_i,\ \phi_i$):

$$\sin\theta = \frac{sin\phi_i}{\sin i}, \tag{41}$$

$$\tan(\psi_i - \Omega) = \tan\theta \sin i, \tag{42}$$

whilst ω and f can be chosen in order to orientate the initial orbit in the orbital plane. That way, if the point that has been defined in the Earth's surface $(\psi_i,\ \phi_i)$ is required to have a maximum time of covering, the point above that region can be chosen as the apogee of the orbit by just imposing $f = \pi$. Thus, the argument of perigee can be calculated using the relation:

$$\theta = \omega + f. \tag{43}$$

On the other hand, a proper semi-major axis is required to achieve the repeating ground-track property and obtain a closed ground-track. In the example proposed and due to the fact that several orbital perturbations are considered, a numerical method for achieving the ground-track property will be used.

Let a cycle be the time that a satellite requires to repeat its ground-track, and let T_c be the period of this cycle. In order to achieve the repeating ground-track property, the orbital parameters have to fulfil a relation with the rotation of the Earth, given by:

$$T_c = N_p T_\Omega = N_d T_{\Omega G}, \tag{44}$$

where N_p is the number of orbital revolutions to cycle repetition, N_d is the number of sidereal days to cycle repetition, T_Ω is the nodal period of the orbit and $T_{\Omega G}$ is the nodal period of Greenwich.

The basis of the numerical method is to correct the value of the semi-major axis by adjusting the orbit of the satellite in the ECEF frame of reference. This correction is achieved by using a basic property in celestial mechanics: if the semi-major axis of an orbit increases, its period also increases and vice versa. Therefore, the goal of the correction is to find the value of the semi-major axis that allows the closing of the ground-track in a period of time equal to a cycle: $T_c = TN_p$ (being T the orbital period and N_p the number of orbital periods to complete a cycle). This is done by a series of iterations in which the secant method and the intermediate value theorem are used to find the value of the semi-major axis that allows the closing of the ground-track for the orbital perturbations considered.

Once the orbital parameters are established for one satellite, it is time to generate the initial configuration of a constellation with minimum number of inertial orbits. In order to do so, a constellation distribution must be chosen. For the sake of simplicity, the value of t_0 is fixed as $t_0 = 0$. Let N_{st} be the number of satellites in each different ground-track, and let $q = 1, ..., N_{st}$ be the integer that names each satellite of each ground-track of the constellation. The configuration is made using a time distribution, assigning a certain $t_q < T_c$ to each constellation satellite. Due to the equally spaced time distribution, the values of t_q are given by Equation (45):

$$t_q = (q-1)\frac{T_c}{N_{st}}. \tag{45}$$

Furthermore, let N_t be the number of different ground-tracks, and let $k = 1, ..., N_t$ be the integer that names each different ground-track of the constellation. The right ascension of the ascending nodes of the leading satellites of each relative trajectory are expressed as:

$$\Omega_k = \Omega_0 + (k-1)\frac{2\pi}{N_t}, \tag{46}$$

where:

$$\Delta\Omega_k = (k-1)\frac{2\pi}{N_t}. \tag{47}$$

Note that the right ascension of the ascending node of the leading satellites in not shared with the rest of the satellites situated in the same ground-track (see Equation (16)).

Using Equation (38), the distribution of t_k is obtained:

$$t_k = \frac{(k-1)\dfrac{2\pi}{N_t}}{\omega_\oplus - \dot{\Omega}_{kq}}, \tag{48}$$

thus, the distribution of each satellite ($t_{kq} = t_k + t_q$) for an equally spaced in time configuration is:

$$t_{kq} = (q-1)\frac{T_c}{N_{st}} + (k-1)\frac{2\pi}{N_t\left(\omega_\oplus - \dot{\Omega}_{kq}\right)}. \tag{49}$$

One thing to notice is that due to the possible symmetries of the configuration, it has to be assured two conditions by the designer. The first one is that the parameters N_d and N_p are relatively primes.

This imposes that the definition of the cycle is the minimal relation between both numbers N_d and N_p, other way, the configuration is multi-generated by several pairs of parameters.

The second condition is that the parameters N_p and N_t are relatively primes. This aims to avoid the overlapping of satellites in the configuration. This happens if the configuration is very symmetrical in time and space, and thus, reduces the number of practical satellites of the constellation due to the overlapping. There is also another way to solve this condition by slightly modifying the distribution formulation. Let N_f be the maximum common divisor between N_p and N_t. Then, the distribution over space is:

$$\Omega_k = \Omega_0 + (k-1)\frac{2\pi}{N_t N_f}, \tag{50}$$

and therefore, the distribution over time is:

$$t_{kq} = (q-1)\frac{T_c}{N_{st}} + (k-1)\frac{2\pi}{N_t N_f\left(\omega_\oplus - \dot{\Omega}_{kq}\right)}. \tag{51}$$

Equations (50) and (51) substitute Equations (46) and (49) in order to avoid the overlapping of satellites due to the high symmetry of the distribution. Note that the overlapping could have been solved by just taking into account that depending on the distribution, two different leading satellites can define the same relative trajectory, so it is of no use defining the same relative trajectory twice.

Once it has been defined the equally spaced in time configuration, it is time to apply this concept to a particular case and show the results. A constellation consisting of 24 satellites has been taken as an example. The constellation repeats its ground-track each two orbital revolutions ($N_p = 2$) and each day ($N_d = 1$). Furthermore, all satellites have an inclination of $i = 63.435^o$ and an eccentricity of $e = 0.5$. A high eccentricity orbit has been selected in order to show the possibilities of the constellation design model. The constellation is distributed in 6 different ground-tracks ($N_t = 6$) and 4 inertial orbits ($N_{st} = 4$), thus $N_s = N_t N_{st} = 24$.

Note that $N_t = 6$ and $N_p = 2$ have a maximum common divisor of $N_f = 2$, so, Equations (50) and (51) must be used to perform the distribution. As a further requirement, it has been imposed that one ground-track of the constellation passes over the city of Zaragoza (Spain) with coordinates ($\phi_r = 41.698169^o$ and $\psi_r = -0.874295^o$).

With these conditions, the constellation is designed following the semi-analytical model proposed in this work obtaining the initial positions and velocities shown in Table 1. These results are given in the inertial frame of reference and generate a constellation whose satellites are distributed in 4 different inertial orbits and 6 ground-tracks. The satellites are subjected to the orbital perturbations named at the beginning of this section: the gravitational potential of the Earth, the Sun and Moon as disturbing third bodies, the solar radiation pressure and the atmospheric drag.

This configuration can be seen in Figure 7, where the ground-track of the whole constellation is presented. There, it can be observed that the constellation is distributed in 6 different ground-tracks, being them completely closed and shared by 4 satellites each.

Figure 8 shows the inertial (left) and relative (right) trajectories of all the satellites in the constellation. There, it can be seen how the constellation is distributed in only 4 different inertial orbits, and how the relative trajectory is shared by groups of satellites (4 for each relative trajectory).

Table 1. Initial positions and velocities of the constellation.

Sat. (k,q)	x [km]	y [km]	z [km]	v_x [km/s]	v_y [km/s]	v_z [km/s]
1,1	29742.291461	-453.882562	26500.795390	-1.170867	1.357686	1.337337
1,2	154.757740	9918.577275	-8829.517691	-4.072424	-3.511337	-4.013089
1,3	-29744.258831	452.556797	26498.609462	1.170722	-1.357541	1.337610
1,4	-160.230634	-9924.332897	-8822.957115	4.071962	3.509064	-4.015544
2,1	16921.729936	8809.410393	31186.380496	-2.343357	1.134970	-0.134496
2,2	-14103.295941	-21475.851654	-5338.235302	-0.094244	-2.9690372	2.558953
2,3	-16924.509567	-8809.572429	31186.599923	2.343266	-1.134808	-0.134081
2,4	14100.582686	21472.679186	-5342.417411	0.094398	2.969823	2.558760
3,1	-2593.920310	13813.973852	22164.558045	-2.883874	-0.004426	-2.654988
3,2	-9567.104875	-32767.409591	13084.071119	1.092862	-0.422175	2.329501
3,3	2590.853241	-13813.672025	22168.896630	2.883967	0.004272	-2.654190
3,4	9564.986068	32767.859269	13080.263462	-1.092785	0.422482	2.329676
4,1	-9918.577275	154.757740	-8829.517691	3.511337	-4.072424	-4.013089
4,2	-452.556797	-29744.258831	26498.609462	1.357541	1.170722	1.337610
4,3	9924.332897	-160.230634	-8822.957115	-3.509064	4.071962	-4.015544
4,4	451.230563	29746.225806	26496.423089	-1.357397	-1.170578	1.337883
5,1	21475.851603	-14103.295946	-5338.235358	2.969037	-0.094244	2.558953
5,2	8809.572392	-16924.509661	31186.599943	1.134808	2.343266	-0.134081
5,3	-21472.679042	14100.582688	-5342.417547	-2.969823	0.094398	2.558760
5,4	-8809.734988	16927.289198	31186.818695	-1.134646	-2.343176	-0.133665
6,1	32767.409572	-9567.104748	13084.071327	0.422175	1.092862	2.329501
6,2	13813.671999	2590.853697	22168.896165	-0.004272	2.883967	-2.654190
6,3	-32767.859301	9564.985811	13080.263949	-0.422482	-1.092785	2.329676
6,4	-13813.370286	-2587.786786	22173.233127	0.004118	-2.884060	-2.653391

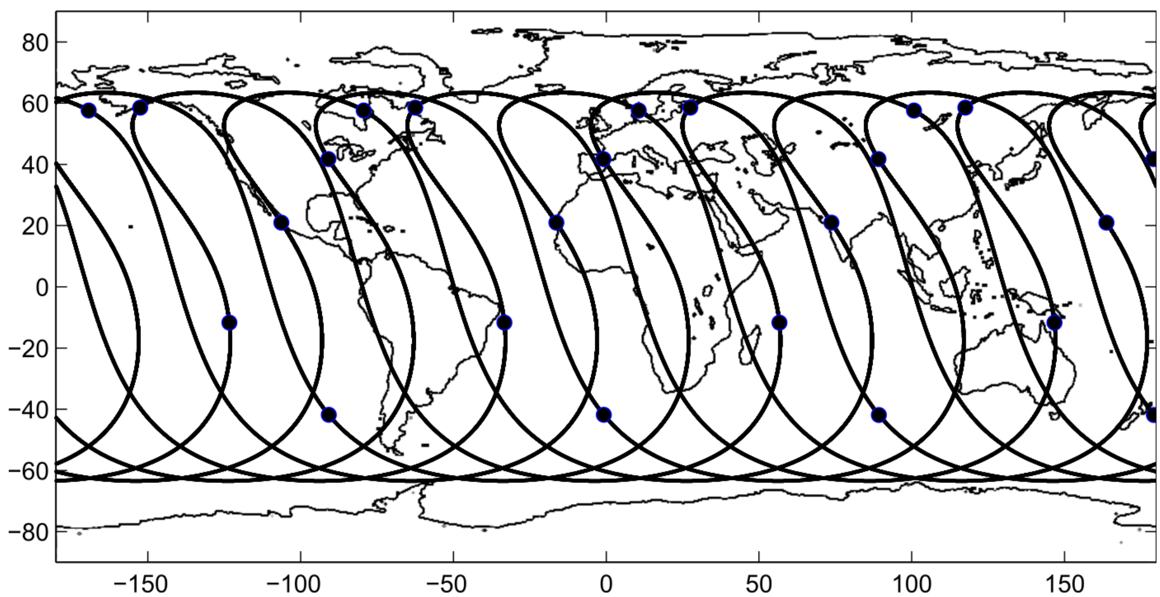

Figure 7. Ground-track of the constellation.

Finally, in Figure 9 the three-dimensional structure of the constellation in the ECEF frame of reference can be observed. It can be concluded that the satellites are able to share their relative trajectories (4 satellites in each trajectory) despite of being subjected to orbital perturbations. The figure allows to see the possibilities that the definition of the constellation in the relative frame of reference brings, generalizing the orbits from a conic shape in the inertial frame of reference into a more diverse group of configurations in the relative frame of reference.

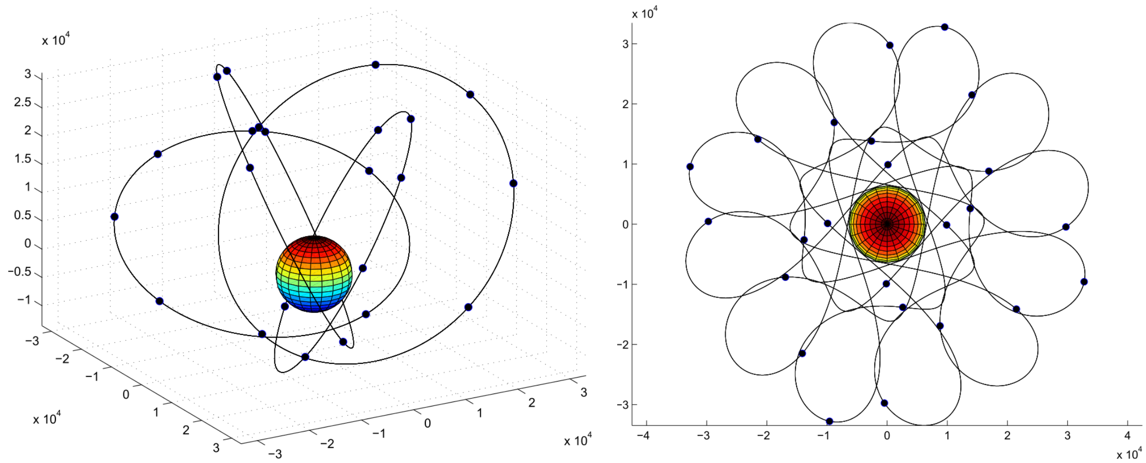

Figure 8. Inertial (left) and relative (right) trajectories of the constellation.

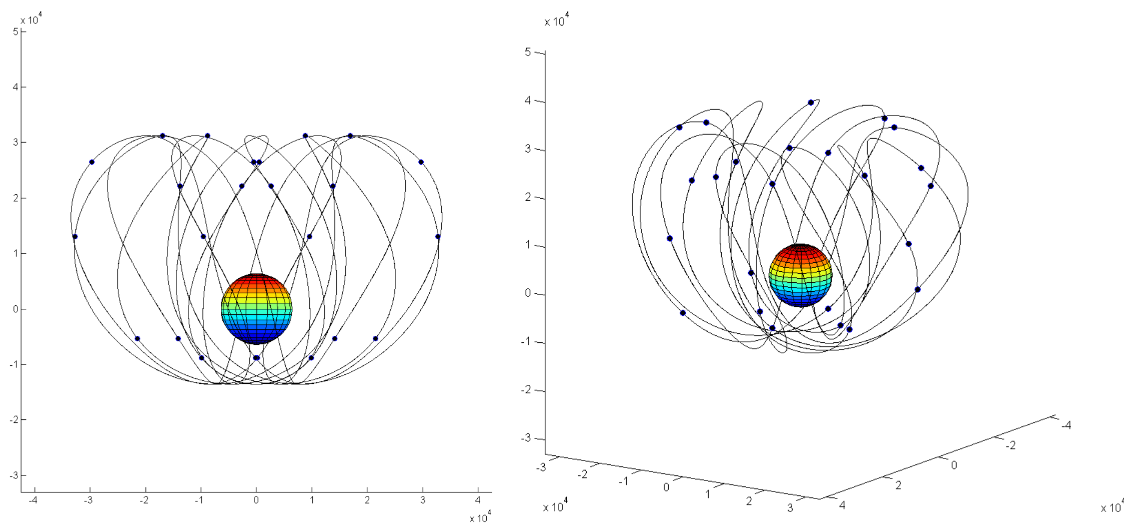

Figure 9. Three-dimensional structure of the constellation in the ECEF frame of reference.

CONCLUSIONS

This paper has shown a new design model to create constellations whose satellites share the same ground-track using time as parameter of distribution in the configuration. This design allows to distribute satellites in several relative trajectories without no restrictions at all in their distribution, a property that can be used to configure missions in which the satellites have to pass consecutively over a certain point of the Earth's surface.

This design model opens a wide variety of possibilities in the configuration of satellite constellations, and it is able to handle any combination of orbital parameters, being the model applicable even with constellations based on high eccentricity orbits.

Furthermore, two different approaches have been presented for this design model, an analytical model in which no orbital perturbation was considered, and a semi-analytical model that can handle orbital perturbations. These two methodologies represent the same idea, but each one has its own peculiarities and uses. Specifically, the semi-analytical model allows to include the orbital perturbations inside the design process, improving the results obtained.

Moreover, this constellation design model allows to include orbital properties to the basic design. In that respect, a semi-major axis correction has been applied to the example presented in the paper in order to achieve the repeating ground-track property in the constellation despite of being the satellites subjected to certain known orbital perturbations. The ability to include other properties such as the sun-synchrony will be studied in future work.

Finally the decrease on the number of inertial orbits to a minimum, represents a big design advantage, due to the fact that the reduction of inertial orbits allows to group satellites in their launches, therefore reducing the costs of the mission.

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