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Abstract

This paper aims to analyze the non-neutrality of monetary policy incorporating the Lucas (1988) type endogenous growth model in the standard New Keynesian macroeconomic model with nominal wage rigidities. It is shown that the monetary policy summarized in the level of trend inflation is non-neutral in the long-run economic growth in the presence of nominal wage rigidities. The growth-inflation nexus depends on the degree of nominal rigidities and the degree of differentiation of the labor services.

1. Introduction

Since the so-called New Keynesian model has been frequently used in analysis of the effects of a monetary policy shock, there has been a certain accumulation of investigations in this field. These analyses aim to address the short-run non-neutrality of monetary policy and to explain the impulse response functions detected in the empirical literature. Nevertheless, there is not yet a sufficient stock of investigations with respect to the long-run relationship between the trend inflation and the economic growth. As in Galí (2008), the “standard” New Keynesian models typically assume zero inflation at the steady state, although the majority of the central banks of the developed countries conduct the inflation targeting policy with a mild positive rate[1]. This paper aims to fill in this gap and to analyze a long-run non-neutrality of monetary policy.

The empirical literature provides some evidences on a non-linear relation between inflation and economic growth (among others, Khan and Senhadji, 2001; López-Villavicencio and Mignon, 2012). Khan and Senhadji (2001) detect the threshold effects of inflation on growth, based on a panel econometric model that incorporates threshold parameter, with data on 140 countries including both developed and developing ones. According to their estimation, there is a threshold inflation rate above which inflation significantly slows down the economic growth, which is estimated at 1–3 percent for industrial countries and 11–12 percent for developing countries. In line with Khan and Senhadji, López-Villavicencio and Mignon (2012) analyze the threshold effects of inflation taking advantage of the Panel Smooth Transition Regression (PSTR) model that allows the estimation of threshold effects with a smooth transition to one regime to another, or law-inflation regime to high-inflation regime. The estimated marginal effect of inflation on growth is, then, smoothed between law-inflation and high-inflation regimes. Their estimation results support the findings of Khan and Senhadji. That is, the threshold inflation rate differs across developed and developing countries, with 1.2 percent for the former and 10-20 percent for the latter.

From the theoretical point of view, there is only a few number of precedent investigations on this topic: among others, Amano et al. (2009; 2012) and Vaona (2012). Both back up the empirical evidences on the non-linear growth-inflation nexus. Amano et al. (2012) incorporate the endogenous growth model fueled by the expansion of varieties a la Romer (1990) in the New Keynesian model with Taylor-type price and wage contracts. Their results show a non-linear concave relationship between the trend inflation and the long-run real output growth. Under the basic calibration, shifting trend inflation from -5 to 5 percent provokes 50-point-basis variations in the long-run growth rate. The main channel of this

[1] This targeted inflation rate is typically set at 2 percent with a band of 0.1 percent or so around it. Although the ECB (European Central Bank) does not use the term of “inflation target”, it sets 2 percent of inflation as the “Definition of Price Stability”. The Bank of Japan was an only exception that had not explicitly declared formally a targeted inflation rate. However, under the newly selected governor, Haruhiko Kuroda, in January, 2013, the Bank has decided to introduce the “Price Stability Target” of 2 percent (http://www.boj.or.jp/en/mopo/outline/sgp.htm/).

effect is the labor supply effect, in which as the trend inflation increases, those who can re-optimize their wage try to front-end load it and thereby increase the economy's average wage markup, which in turn decreases availability of aggregate labor inputs. Moreover, their basic calibration indicates that the optimal trend inflation in a sense that maximizes the long-run growth rate is a substantial deflation of 3.15 percent.

On the other hand, Vaona (2012) incorporates the endogenous growth model based on knowledge externalities a la Romer (1986) with Taylor-type wage rigidities, where firms' aggregate knowledge is proportional to aggregate capital stock and considered as a public good that contributes to increasing production. Moreover, unlike the model developed by Amano et al. (2012), money is explicitly introduced in the model in a way that real money balances generate households' utility. Under the basic calibration, there is a threshold money growth rate around 2 percent, below which an impact of money growth on real output growth is slightly positive, while above which an impact falls to negative one.

The common features in these models are the followings; first, they put an emphasis on the importance of wage rigidities in the long-run growth-inflation nexus; and second, both are based on the model of uni-growth engine with physical capital accumulation. To the best of our knowledge, there is no literature that merges an endogenous growth model that incorporates human capital accumulation a la Lucas (1988) in the New Keynesian framework, in order to analyze the long-run non-neutrality of monetary policy. However, it would be of great importance to consider the human capital accumulation, partly because household's labor supply decision is closely related to its decision on human capital accumulation, and partly because the growth-inflation nexus in the presence of wage rigidities might be subject to qualitative change under the dual-growth engine model with both physical and human capital accumulation.

Based on the above mentioned motivations, this paper aims to analyze the long-run growth-inflation nexus, merging the endogenous growth model with human capital a la Lucas (1988) with the New Keynesian model with wage rigidities, and permitting non-zero trend inflation. The next section provides a brief explanation on the model structure. In the third section, the model properties at the steady state will be analyzed. The fourth section then provides analysis on the growth-inflation nexus, and the fifth section concludes.

2. The model

The main features of this model are the followings:

(i) Nominal wage rigidities in the form of Taylor (1980) type wage contracts

(ii) Endogenous growth model of dual-engine with human capital a la Lucas (1988)

There are four main agents in this economy: intermediate goods producer, final goods producing retail firms, household, and the Central Bank. Since the interest of this paper is in the long-run equilibrium, the monetary policy taken by the Central Bank is simply to set the trend inflation. Moreover, there is no money introduced in this model, following the “cashless economy” hypothesis (Woodford, 2003; Galí, 2008) typically taken in the New Keynesian macroeconomic models.

2.1. Intermediate goods producer

It is assumed that there is a representative perfectly competitive intermediate goods producer with technology given by:

$$(1) \quad Y_t^m = AK_t^\alpha L_t^{1-\alpha}$$

where Y_t^m is the output of homogeneous intermediate goods, A total factor productivity, K_t stock of physical capital, and L_t a composite index of differentiated labor services measured by effective labor defined as follows:

$$(2) \quad L_t = \left[\int_0^1 L_{i,t}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$$

where $L_{i,t}$ represents differentiated labor service input in terms of effective labor provided by an individual i at time t, and θ the elasticity of substitution across labor services.

Since the market is perfectly competitive, the intermediate goods producer's profit maximization problem is:

$$(3) \qquad \max_{K_t, L_{i,t}} P_t^m A K_t^\alpha L_t^{1-\alpha} - \int_0^1 W_{i,t} L_{i,t} di - R_t P_t K_t$$

where P_t^m is the market price of intermediate goods, $W_{i,t}$ the nominal wage rate, and R_t the rental price of physical capital. The first order condition implies that the value of marginal productivity of physical capital equals to marginal cost:

$$(4) \qquad \alpha P_t^m A K_t^{\alpha-1} L_t^{1-\alpha} = R_t P_t$$

On the other hand, the demand for differentiated labor service i is obtained as follows:

$$(5) \qquad L_{i,t} = [(1-\alpha) A {K_t}^\alpha]^\theta \left(\frac{W_{i,t}}{P_t^m}\right)^{-\theta} {L_t}^{1-\theta\alpha}$$

Imposing the definition of a composite index of differentiated labor services, we get the aggregated demand for labor as follows:

$$(6) \qquad L_t = \left[\frac{(1-\alpha)A}{\widetilde{w_{a,t}}}\right]^{\frac{1}{\alpha}} K_t$$

where $\widetilde{w_{a,t}}$ represents the wage dispersion in terms of intermediate goods price given by:

$$(7) \qquad \widetilde{w_{a,t}} = \left[\int_0^1 \left(\frac{W_{i,t}}{P_t^m}\right)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$$

2.2. Final goods producing retail firms

There is an infinite number of retail firms over a continuum of [0,1], which repackage the homogeneous intermediate goods and sell them to the household. It is assumed that they have the same simplified production technology that converts one unit of homogeneous intermediate goods into one unit of differentiated final goods. The retail firms have a market power in the goods market so that they can set the own price facing the downward-sloping demand for each variety. Unlike the standard New Keynesian model, this model does not assume the price rigidities in the final goods market nor in the intermediate goods market. Then, the profit maximization problem of the retail firms is given by:

$$(8) \qquad \max_{P_{j,t}^*} \left(P_{j,t}^* - P_t^m\right) C_{j,t}\left(P_{j,t}^*\right)$$

where $C_{j,t}\left(P_{j,t}^*\right)$ represents the demand for each variety of final goods. As in the standard New Keynesian model, the representative household consumes a composite index of a continuum of differentiated final products over the range of $[0, 1]$, defined as:

$$(9) \qquad C_t = \left[\int_0^1 C_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where ε is the elasticity of substitution across the different varieties. Therefore, the demand for each variety is given by:

$$(10) \qquad C_{j,t}\left(P_{j,t}^*\right) = \left(\frac{P_{j,t}^*}{P_t}\right)^{-\varepsilon} C_t$$

The first order condition implies the standard pricing rule for monopolistically competitive market:

$$(11) \qquad P_{j,t}^* = \left(\frac{\varepsilon}{\varepsilon - 1}\right) P_t^m$$

Due to the symmetric equilibrium, the aggregate price will be determined by the intermediate good price times a mark-up, as follows:

$$(12) \quad P_t = \left[\int_0^1 {P_{j,t}^{*}}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = \left(\frac{\varepsilon}{\varepsilon-1}\right) P_t^m$$

Using this relation on the aggregate price, the economy's real wage dispersion is given by:

$$(13) \quad w_{a,t} = \left[\int_0^1 \left(\frac{W_{i,t}}{P_t}\right)^{1-\theta} di\right]^{\frac{1}{1-\theta}} = \left(\frac{\varepsilon-1}{\varepsilon}\right) \widetilde{w_{a,t}}$$

Then, the intermediate goods producer's optimal conditions can be rewritten as follows:

$$(14) \quad L_t = \left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{(1-\alpha)A}{w_{a,t}}\right]^{\frac{1}{\alpha}} K_t$$

$$(15) \quad R_t = \alpha \left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}} \left[\frac{1-\alpha}{w_{a,t}}\right]^{\frac{1-\alpha}{\alpha}}$$

2.3. Household

2.3.1. Basic settings

For simplicity, it is assumed that there is a multi-agent, infinitely lived representative household. The household consists of a continuum of members i $(i \in [0, 1])$ across which decisions on the effective labor supply and human capital accumulation can vary. However, as mentioned earlier, the representative household collectively consumes a composite index of differentiated final products, invests in physical capital and rent it to the intermediate goods producer.

In the labor market, each member supplies differentiated labor service to the intermediate goods producers. Each of them possesses market power to set its own wage rate facing downward-sloping labor demand, but it cannot affect the average wage rate of the economy. That is, the lobar market is monopolistically competitive. Moreover, as in the previous literature (Amano et al., 2012; Vaona, 2012), it is assumed that the labor market exhibits the Taylor (1980) type nominal rigidities. Each individual is supposed to make a contract which is valid for the next I periods. Obviously, I is a parameter for the nominal rigidities.

In order to supply the demanded amount of effective labor, individuals are supposed to make two decisions[2]. First, as in the Lucas model, each member of household chooses a fraction of time devoted to the production activity, $u_{i,t}$ ($u_{i,t} \in [0,1]$) and a fraction to human capital accumulation, $1-u_{i,t}$. Second, each individual also chooses the total time dedicated to non-leisure activities, that is, production activity plus accumulation of human capital, $N_{i,t}$[3]. Therefore, the effective labor is defined as follows:

$$(15) \qquad L_{i,t} = u_{i,t} N_{i,t} h_{i,t}$$

It is assumed that the human capital accumulation has a following technology:

$$(16) \qquad h_{i,t+1} = \left[1 + \xi\left(1 - u_{i,t}\right) N_{i,t}\right] h_{i,t}$$

where ξ is a productivity parameter of human capital accumulation. The law of motion for the economy's total human capital is then given by:

[2] As explained later, this assumption will be replaced by the Assumption (i) and (ii) in the Appendix I, due to the contradiction which exist in the first order condition for $u_{i,t+\tau}$.

[3] Given a certain level of wage rate, we can observe two types of trade-offs in the selection of $N_{i,t+\tau}$ and $u_{i,t+\tau}$. First, since the total time spent for non-leisure activities generate disutility, there is a trade-off between a decrease in disutility today and an increase in current or future income flows by devoting to production activity or accumulating human capital. This is the trade-off in the selection of $N_{i,t+\tau}$. Second, another trade-off lies between an increase in time dedicated to production which results in higher disposal income today and an increase in the income flows in the future through human capital accumulation today. This is the trade-off with respect to the selection of $u_{i,t+\tau}$.

$$(17) \quad h_{t+1} = \int_0^1 h_{i,t+1} di = \left\{ \int_0^1 \left[1 + \xi\left(1 - u_{i,t}\right) N_{i,t}\right] \frac{h_{i,t}}{h_t} \right\} h_t$$

Finally, as in Christiano et al.(2005), the representative household holds a stock of physical capital, rents it to the intermediate goods producers, and decides how much physical capital to accumulate. For simplicity, it is assumed that there are neither adjustment costs nor flow adjustment costs of investment. Then, the law of motion of physical capital is given as follows:

$$(18) \quad K_{t+1} = (1 - \delta) K_t + I_t$$

where δ represents a depreciation rate of physical capital.

2.3.2. Optimal decisions

Now, remember that the only rigidity lies in the nominal wage of differentiated labor services and the representative household can flexibly decide optimal trajectory of all the variables other than the wage rate offer. Then, the decision of household can be divided into two stages. First, each member of household decides the optimal wage rate, considering the trade-off that lies among (i) an increase in the unit income of efficient labor, (ii) a decrease in the demand for efficient labor, and (iii) a decrease in disutilities generated by non-leisure activities. The last factor enters in the trade-off by imposing the market-clearing for labor market. Second, given the trajectory of wage rate offer, and therefore that of demand for effective labor over time, the representative household chooses the consumption, the time dedicated to non-leisure activities, and its fraction for production activity. Then, from the law of motion of physical capital and that of human capital, we will get the trajectory for all the variables. As shown below, these trajectories are obtained by solving the optimal control problem.

2.3.2.1. Optimal wage setting rule

In the first stage, the representative household maximizes its expected present value of utility over the contract period of the individual i's contract made at time t, choosing its optimal wage offer:

$$(19)\qquad \max_{W_{i,t}^*} E_t \sum_{\tau=0}^{I-1} \beta^\tau \left[\log(C_{t+\tau}) - \frac{1}{1+\nu}\int_0^1 \left(N_{i,t+\tau}\right)^{1+\nu} di\right]$$

The restrictions are given by (5), (12) and the following equations:

$$(20)\qquad C_{t+\tau} + K_{t+\tau+1} = \mathrm{D}_{t+\tau} + \int_0^1 \left(\frac{W_{i,t}^*}{P_{t+\tau}}\right) L_{i,t+\tau}\left(\frac{W_{i,t}^*}{P_{t+\tau}^m}\right) di + (1 + R_{t+\tau} - \delta)K_{t+\tau}$$

$$(21)\qquad L_{i,t+\tau}\left(W_{i,t}^*\right) = u_{i,t+\tau} N_{i,t+\tau} h_{i,t+\tau}$$

where $\mathrm{D}_{t+\tau}$ indicates dividends. The first order condition implies the following wage rate setting rule:

$$(22)\qquad W_t^* = \left(\frac{\theta}{\theta-1}\right)\frac{E_t \sum_{\tau=0}^{I-1} \beta^\tau P_{t+\tau}^{\theta} K_{t+\tau}^{\theta\alpha} L_{t+\tau}^{1-\theta\alpha} N_{i,t+\tau}^{\nu}\left(u_{i,t+\tau} h_{i,t+\tau}\right)^{-1}}{E_t \sum_{\tau=0}^{I-1} \beta^\tau C_{t+\tau}^{-1} P_{t+\tau}^{\theta-1} K_{t+\tau}^{\theta\alpha} L_{t+\tau}^{1-\theta\alpha}}$$

Note that in case of no nominal rigidities in wage, the wage rule becomes a simple one that imposes an ordinary mark-up, $\theta/(\theta-1)$, on the competitive wage rate, which makes equalized the marginal utility through an increase in consumption and the marginal disutility generated by an increase in non-leisure activities.

2.3.2.2. Optimal control problem

Now, given the optimal trajectory of nominal wage of each individual, we can solve the representative household's optimal control problem, with the following objective function:

$$(23)\qquad E_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\log(C_{t+\tau}) - \frac{1}{1+\nu}\int_0^1 \left(N_{i,t+\tau}\right)^{1+\nu} di\right]$$

subject to (5), (12), (14), (17), (18), (20), (21) and (22). The first order conditions imply the following relations (refer to the Appendix 1 for details):

$$(24) \qquad \frac{\beta}{C_{t+\tau+1}/C_{t+\tau}} = (1+\delta) - \left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{w_{a,t}}\right)^{\frac{1-\alpha}{\alpha}}$$

$$(25) \qquad \xi N_{i,t+\tau}\left[\frac{\left(W_{i,t+\tau}^{*}/P_{t+\tau}\right)}{\overline{w}_{t+\tau}}u_{i,t+\tau} + \left(1-u_{i,t+\tau}\right)\right] = 1 - \left(\frac{\beta}{C_{t+\tau+1}/C_{t+\tau}}\right)\left(\frac{\overline{w}_{t+\tau+1}}{\overline{w}_{t+\tau}}\right)$$

where $\overline{w}_{t+\tau}$ represents economy's average real wage given by:

$$\overline{w}_{t+\tau} = \int_0^1 \frac{W_{i,t+\tau}^{*}}{P_{t+\tau}} di$$

As explained in the Appendix 1, these relations can be obtained by taking the assumption that the representative household aims to adjust $u_{i,t+\tau}$ in a way that (A3) will be satisfied for the average real wage of the economy.

3. Steady State

3.1. BGP growth

In this section, several properties of the steady state will be analyzed. First of all, from the intermediate production function, the intermediate output, the physical capital and the effective labor grow at the same rate at the steady state. Since the final product market has the symmetric equilibrium, it is deduced that the intermediate output and a final output composite index coincide, $Y = Y^m$. Letting $g(.)$ be the growth rate of a variable at the steady state, the steady state of this economy implies the following BGP (Balanced Growth Path) relations:

$$(26) \qquad g(Y) = g(Y^m) = g(K) = g(L)$$

It also implies that the steady state output to physical capital ratio is constant. From the homogeneous of degree one Cobb-Douglas production function of intermediate goods, we get the following output to physical capital ratio:

$$(27) \qquad \frac{Y}{K} = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\frac{R_t}{\alpha} = A^{\frac{1}{\alpha}}\left[\left(\frac{\varepsilon - 1}{\varepsilon}\right)\frac{1 - \alpha}{w_{a,t}}\right]^{\frac{1-\alpha}{\alpha}}$$

On the other hand, from the market clearing condition of the final goods market, $Y_t = C_t + I_t$, the steady state consumption to physical capital ratio, C/K, is determined as follows:

$$(28) \qquad \frac{C}{K} = \frac{Y}{K} - g(K) - \delta$$

Since the right-hand side is constant over time, consumption and capital grow at the same rate, and therefore:

$$(29) \qquad \frac{C}{K} = A^{\frac{1}{\alpha}}\left[\left(\frac{\varepsilon - 1}{\varepsilon}\right)\frac{1 - \alpha}{w_{a,t}}\right]^{\frac{1-\alpha}{\alpha}} - g(C) - \delta$$

$$(30) \qquad g(Y) = g(Y^m) = g(K) = g(L) = g(C)$$

3.2. The wage rule and non-leisure activities

Unlike the model developed by Amano et al. (2012), the re-optimized real wage should be constant at the steady state since the nominal wage is expressed in terms of effective labor. Therefore, the steady state average real wage, $\bar{w}_t$, and the real wage dispersion, $w_{a,t}$, are also constant and can be expressed in terms of the re-optimized real wage. Letting the re-optimized real wage at the steady state be written with a notation of (W_t^{**}/P_t) in order to distinguish it from the optimal trajectory of the real wage for each individual, and assuming that members of the representative household are uniformly distributed across I cohorts, the steady state average real wage and the real wage dispersion can be expressed as follows:

$$(31) \qquad \bar{w} = \frac{W_t^{**}}{P_t}\left[\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{1}{\Pi}\right)^{\tau}\right]$$

$$(32) \quad w_a = \frac{W_t^{**}}{P_t}\left[\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{1}{\Pi}\right)^{(1-\theta)\tau}\right]^{\frac{1}{1-\theta}}$$

where Π is the gross trend inflation decided by the Central Bank (i.e. one plus trend inflation rate).

From (31) and the Assumption (ii) in the Appendix 1, (A9) can be rewritten as follows, from which it is deduced that the time dedicated to non-leisure activities, $N_{i,t+\tau}$, is constant across the time and individuals:

$$(33) \quad \lambda_{2,t+\tau+1} - \lambda_{2,t+\tau} = -\lambda_{2,t+\tau}\,\xi N_{i,t+\tau} = -\lambda_{2,t+\tau}\,\xi N_{ss}$$

Then, (A10) can be simplified as follows:

$$(34) \quad \xi N_{ss} = 1 - \frac{\beta}{1+g(C)}$$

On the other hand, from (22), we obtain the following expression of re-optimized wage rule at the steady state (refer to the Appendix 2 for details):

$$(35) \quad \left(\frac{W_t^{**}}{P_t}\right)^{1-\theta} = \left(\frac{\theta}{\theta-1}\right)\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{{w_a}^{1-\alpha\theta}}{(1-\alpha)A}\right]^{\frac{1}{\alpha}}\left(\frac{C}{K}\right)\left[\frac{E_t\sum_{\tau=0}^{I-1}\beta^{\tau}}{E_t\sum_{\tau=0}^{I-1}\beta^{\tau}\Pi^{(\theta-1)\tau}}\right]N_{ss}^{1+\upsilon}$$

Note that the nominal re-optimized wage will grow at the same rate as the trend inflation. It implies that, at the individual level, nominal wage is fixed during the contract period and then jumps at the rate of Π^I at the next re-optimizing opportunity.

3.3. Steady state system of equations

Finally, the steady state system of equations, which is characterized with 5 unknowns: W_t^{**}/P_t, w_a, C/K, $g(C)$ and N_{ss}, is given as follows:

$$\text{(S1)}\quad \left(\frac{W_t^{**}}{P_t}\right)^{1-\theta} = \left(\frac{\theta}{\theta-1}\right)\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{{w_a}^{1-\alpha\theta}}{(1-\alpha)A}\right]^{\frac{1}{\alpha}}\left(\frac{C}{K}\right)\left[\frac{E_t\sum_{\tau=0}^{I-1}\beta^{\tau}}{E_t\sum_{\tau=0}^{I-1}\beta^{\tau}\Pi^{(\theta-1)\tau}}\right]N_{ss}^{1+\upsilon}$$

$$\text{(S2)}\quad w_a = \frac{W_t^{**}}{P_t}\left[\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{1}{\Pi}\right)^{(1-\theta)\tau}\right]^{\frac{1}{1-\theta}}$$

$$\text{(S3)}\quad \frac{C}{K} = A^{\frac{1}{\alpha}}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1-\alpha}{w_a}\right]^{\frac{1-\alpha}{\alpha}} - g(C) - \delta$$

$$\text{(S4)}\quad 1+g(C) = \frac{\beta}{(1+\delta)-\left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{w_a}\right)^{\frac{1-\alpha}{\alpha}}}$$

$$\text{(S5)}\quad \xi N_{ss} = 1-\frac{\beta}{1+g(C)}$$

4. Analysis of growth-inflation nexus

4.1. Growth-inflation nexus in the presence of nominal rigidities

For simplicity, suppose that there is no depreciation of the physical capital, that is, $\delta = 0$. Operating the steady state system of equations, we can obtain the following two representative expressions:

$$\text{(36)}\quad (1-\alpha)\Lambda = \left(\frac{\theta}{\theta-1}\right)\Theta^{1-\theta}\mathrm{B}\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)\Lambda + 1 - \frac{\beta}{1-\Lambda}\right]\left(\frac{\Lambda}{\xi}\right)^{1+\upsilon}$$

$$\text{(37)}\quad 1+g(C) = \frac{\beta}{1-\Lambda}$$

where

$$\Lambda \equiv \left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{w_a}\right)^{\frac{1-\alpha}{\alpha}}$$

$$\Theta = \left[\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{1}{\Pi}\right)^{(1-\theta)\tau}\right]^{\frac{1}{1-\theta}}$$

$$\mathrm{B} = \left[\frac{E_t \sum_{\tau=0}^{I-1} \beta^\tau}{E_t \sum_{\tau=0}^{I-1} \beta^\tau \Pi^{(\theta-1)\tau}}\right]$$

The first derivative of (37) with respect to Λ implies that an increase in Λ causes a higher steady state growth rate as follows:

$$(38) \qquad \frac{dg(C)}{d\Lambda} = \frac{\beta}{(1-\Lambda)^2} > 0$$

Now, taking the first derivative of (36) with respect to the gross trend inflation, Π, we can obtain the following relation:

$$(39) \qquad \frac{d\Lambda}{d\Pi} = \frac{\left(\frac{\theta-1}{\Theta}\right)\frac{d\Theta}{d\Pi} - \frac{1}{\mathrm{B}}\frac{d\mathrm{B}}{d\Pi}}{\left[\left(\frac{\varepsilon}{\varepsilon-1} - \frac{\beta}{(1-\Lambda)^2}\right)\frac{K}{C} + \frac{v}{\Lambda}\right]}$$

where

$$\frac{d\Theta}{d\Pi} = -\left(\frac{\Theta^\theta}{I}\right)\left[\sum_{\tau=0}^{I-1} \tau \Pi^{(\theta-1)\tau-1}\right] < 0$$

$$\frac{d\mathrm{B}}{d\Pi} = -\left(\frac{\mathrm{B}(\theta-1)}{\sum_{\tau=0}^{I-1} \beta^\tau \Pi^{(\theta-1)\tau}}\right)\left[\sum_{\tau=0}^{I-1} \beta^\tau \Pi^{(\theta-1)\tau-1}\right] < 0$$

For a combination of the parameters that generates rational steady state growth rate, the denominator is supposedly positive. Then, the sign of $d\Lambda/d\Pi$ depends on the sign of numerator. Note that the parameters that affect the numerator are I, θ, and β. Figure 1 shows the variations of the numerator of (39) with respect to different trend inflation rates, assuming that $\beta = 0.99$ and $I = 8$. The main observations are the followings: first, for higher values of θ, there is a threshold below and above which the sign of numerator changes; second, this threshold lies in deflation area but gets closer to the zero inflation as the parameter of elasticity of substitution across differentiated labor services gets larger.

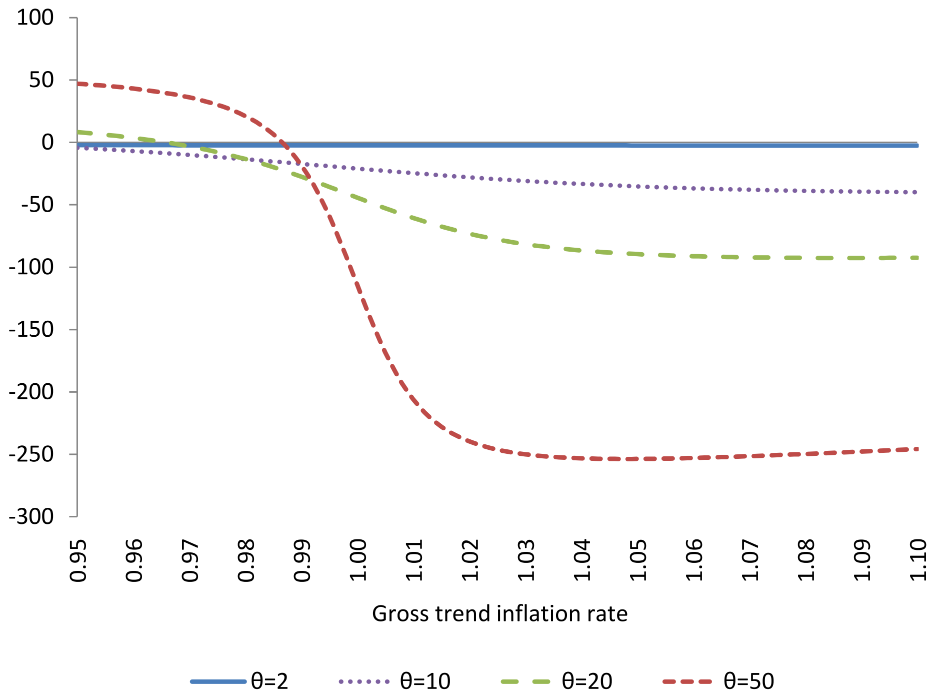

Figure 1. Variations of the numerator and the elasticity of substitution of differentiated labor services ($\beta = 0.99$ and $I = 8$)

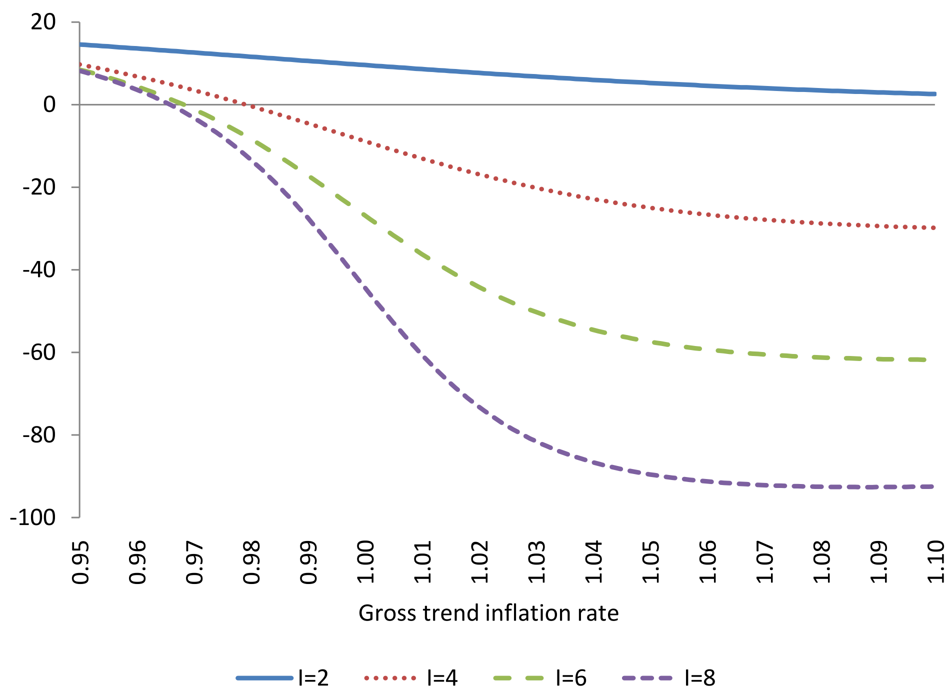

Figure 2. Variations of the numerator and the rigidities parameter ($\beta = 0.99$ and $\theta = 20$)

On the other hand, Figure 2 shows the variations of the numerator of (39) with respect to different trend inflation rates, assuming that $\beta = 0.99$ and $\theta = 20$. The main observations here are the followings: first, there is a threshold below and above which the sign of numerator changes for higher nominal rigidities; second, this threshold again lies in deflation area but gets closer to the zero inflation as the nominal rigidities get smaller.

4.2. The main channel of growth-inflation nexus

Finally, for the sake of comparative analysis, let us see the base line case when there are no nominal wage rigidities, that is, $I = 1$. Since each individual can re-optimize its wage every period, there would be no real wage dispersion, and therefore the steady state equations of (S1) and (S2) will be modified as follows:

$$\left(\frac{W_t^{**}}{P_t}\right)^{\frac{\alpha-1}{\alpha}} = \left(\frac{\theta}{\theta-1}\right)\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{1}{(1-\alpha)A}\right]^{\frac{1}{\alpha}}\left(\frac{C}{K}\right)N_{ss}^{1+\upsilon}$$

$$w_a = \frac{W_t^{**}}{P_t}$$

Since all the variations in Λ in (39) are attributed to the variations in the real wage dispersion, under the flexible wage condition, the trend inflation will not affect the steady state economic growth. Therefore, the long-run non-neutrality of the monetary policy appears only in the presence of nominal rigidities in wages.

5. Conclusion

In this paper, the Lucas type endogenous growth model is incorporated in the New Keynesian model with nominal wage rigidities. In line with the previous studies by Vaona (2012) and Amano et al. (2012), it is confirmed that, even in the model of dual-growth engine with the accumulation of human and physical capital, the monetary policy summarized in the trend inflation rate set by the

Central Bank is non-neutral in the long-run economic growth due to the presence of nominal wage rigidities. In other words, the trend inflation rate will affect the steady state economic growth through the variations in the real wage dispersion across individuals. In case of high nominal rigidities and highly differentiated labor market, there seems to be a threshold inflation rate, below and above which the sign of the effect of trend inflation on growth changes, in such a way that the marginal effect of increasing trend inflation is slightly positive below the threshold, while it becomes significantly negative above that. This threshold typically lies in the deflation area, which is consistent with Amano et al. (2012).

However, it should be noted that the above-mentioned form of growth-inflation nexus highly depends on the Assumption (i) and (ii) described in the Appendix 1. It might be the case that the selection of the total time dedicated to non-leisure activities and its fraction of production activity at the individual level might be different from this assumption. More sophisticated mechanism of determination of these variables is subject to future investigation.

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Appendix 1. Optimal control problem of the household

The Hamiltonian for this problem is:

$$H_{t+\tau} = \beta^{\tau}\left[\log(C_{t+\tau}) - \frac{1}{1+\nu}\int_0^1 \left(N_{i,t+\tau}\right)^{1+\nu} di\right]$$
$$+\lambda_{1,t+\tau}\left[\mathrm{D}_{t+\tau} + \int_0^1 \left(\frac{W^*_{i,t+\tau}}{P_{t+\tau}}\right) L_{i,t+\tau}\left(\frac{W^*_{i,t+\tau}}{P^m_{t+\tau}}\right) di + (r_{t+\tau}-\delta)K_{t+\tau} - C_{t+\tau}\right]$$
$$+\lambda_{2,t+\tau}\left\{\int_0^1 \xi\left(1-u_{i,t+\tau}\right)N_{i,t+\tau}h_{i,t+\tau}\right\}$$

subject to (5), (12), (14), (17), (18), (20), (21) and (22). The first order conditions are given as follows:

(A1) $\dfrac{\beta^{\tau}}{C_{t+\tau}} = \lambda_{1,t+\tau}$

(A2) $\beta^{\tau} {N_{i,t+\tau}}^{\upsilon} = \lambda_{1,t+\tau}\left(\dfrac{W^*_{i,t+\tau}}{P_{t+\tau}}\right) u_{i,t+\tau}h_{i,t+\tau} + \lambda_{2,t+\tau}\xi\left(1-u_{i,t+\tau}\right)h_{i,t+\tau} \quad \forall i \in [0,1]$

(A3) $\lambda_{2,t+\tau} = \dfrac{\lambda_{1,t+\tau}}{\xi}\dfrac{W^*_{i,t+\tau}}{P_{t+\tau}} \quad \forall i \in [0,1]$

(A4) $\lambda_{1,t+\tau+1} - \lambda_{1,t+\tau} = -\lambda_{1,t+\tau}(r_{t+\tau}-\delta) - \lambda_{1,t+\tau}\displaystyle\int_0^1 \left(\frac{W^*_{i,t+\tau}}{P_{t+\tau}}\right)\left(\frac{W^*_{i,t+\tau}}{P^m_{t+\tau}}\right)^{-\theta}\left[\frac{(1-\alpha)A}{\widetilde{W}^{1-\theta\alpha}_{a,t+\tau}}\right]^{\frac{1}{\alpha}} di$

(A5) $\lambda_{2,t+\tau+1} - \lambda_{2,t+\tau} = -\lambda_{1,t+\tau}\left(\dfrac{W^*_{i,t+\tau}}{P_{t+\tau}}\right) u_{i,t+\tau}N_{i,t+\tau} - \lambda_{2,t+\tau}\,\xi\left(1-u_{i,t+\tau}\right)N_{i,t+\tau} \quad \forall i \in [0,1]$

(A6) $K_{t+\tau+1} = \mathrm{D}_{t+\tau} + \displaystyle\int_0^1 \left(\frac{W^*_{i,t+\tau}}{P_{t+\tau}}\right) L_{i,t+\tau}\left(W^*_{i,t+\tau}\right) di + (1 + r_{t+\tau} - \delta)K_{t+\tau} - C_{t+\tau}$

(A7) $h_{t+\tau+1} = \left\{\displaystyle\int_0^1 \left[1+\xi\left(1-u_{i,t+\tau}\right)N_{i,t+\tau}\right]\frac{h_{i,t+\tau}}{h_{t+\tau}} di\right\} h_{t+\tau}$

Note that the first order condition for $u_{i,t+\tau}$ in (A3) implies that the real wage at time $t+\tau$ has to be the same across all individuals. However, since the nominal wage is expressed in terms of effective labor, the re-optimized real wage should be constant at the steady state, and therefore the nominal re-optimized wage grows at the same rate as the aggregate price. It implies that when the trend inflation is different from zero, there will be variations in the real wage across individuals. Obviously, it contradicts (A3).

In order to solve this problem, the following assumption is taken:

> <u>Assumption (i)</u>: In the presence of nominal wage rigidites, the condition (A3) can be interpreted as optimality reference in a way that closer to (A3) the trade-off for each individual by marginally increasing $u_{i,t+\tau}$ is, the better off the representative household will be in terms of utility. The representative household then aims to adjust $u_{i,t+\tau}$ for each individual in order to minimize the squared sum of each individual's distance from the optimal reference (A3)[4]. It is equivalent to say that (A3) is satisfied for the economy's average real wage, which is defined as a simple integral of each individual's real wage. Therefore, (A3) should be modified to the following condition:

$$(\text{A3}') \qquad \lambda_{2,t+\tau} = \frac{\lambda_{1,t+\tau}}{\xi}\overline{w}_{t+\tau} = \frac{\lambda_{1,t+\tau}}{\xi}\int_0^1 \frac{W^*_{i,t+\tau}}{P_{t+\tau}}di$$

> <u>Assumption (ii)</u>: The distribution of the total time dedicated to production activity, $u_{i,t+\tau}N_{i,t+\tau}$, across individuals is proportionate to the distribution of real wage for each individual, $W^*_{i,t+\tau}/P_{t+\tau}$.

Now, substituting (A1) in (A4), we obtain:

$$1-\left(\frac{\beta}{C_{t+\tau+1}/C_{t+\tau}}\right) = \left[\frac{(1-\alpha)A}{\widetilde{w_{a,t}}^{1-\theta\alpha}}\right]^{\frac{1}{\alpha}}\int_0^1\left(\frac{W^*_{i,t+\tau}}{P_{t+\tau}}\right)\left(\frac{W^*_{i,t+\tau}}{P^m_{t+\tau}}\right)^{-\theta}di + (r_{t+\tau}-\delta)$$

Then, substituting the real rental price and the relations on the aggregate price and on the average real wage ((12) and (15)), we will obtain:

$$(\text{A8}) \qquad \frac{\beta}{C_{t+\tau+1}/C_{t+\tau}} = (1+\delta) - \left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{w_{a,t}}\right)^{\frac{1-\alpha}{\alpha}}$$

---

4 It would be possible to define that each one's distance from optimal reference as $\left|\lambda_{2,t+\tau} - \frac{\lambda_{1,t+\tau}}{\xi}\frac{W^*_{i,t+\tau}}{P_{t+\tau}}\right|$.

From (A3') and (A5):

$$\text{(A9)} \qquad \lambda_{2,t+\tau+1} - \lambda_{2,t+\tau} = -\lambda_{2,t+\tau}\, \xi N_{i,t+\tau} \left[\frac{\left(W^{*}_{i,t+\tau}/P_{t+\tau}\right)}{\overline{w}_{t+\tau}} u_{i,t+\tau} + \left(1 - u_{i,t+\tau}\right) \right] \quad \forall i \in [0,1]$$

On the other hand, from (A1) and (A3'), we obtain:

$$\text{(A10)} \qquad \frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} - 1 = \frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} \frac{\overline{w}_{t+\tau+1}}{\overline{w}_{t+\tau}} - 1 = \frac{\beta}{C_{t+\tau+1}/C_{t+\tau}} \left(\frac{\overline{w}_{t+\tau+1}}{\overline{w}_{t+\tau}} \right) - 1$$

Then, (A9) and (A10) imply the following relation:

$$\text{(A11)} \qquad \xi N_{i,t+\tau} \left[\frac{\left(W^{*}_{i,t+\tau}/P_{t+\tau}\right)}{\overline{w}_{t+\tau}} u_{i,t+\tau} + \left(1 - u_{i,t+\tau}\right) \right] = 1 - \left(\frac{\beta}{C_{t+\tau+1}/C_{t+\tau}} \right) \left(\frac{\overline{w}_{t+\tau+1}}{\overline{w}_{t+\tau}} \right)$$

Appendix 2. Steady state wage rule

First of all, combining the definition of efficient labor together with the demand for labor services (15), we will obtain the following equation:

$$\text{(A12)} \quad \left(u_{i,t+\tau}h_{i,t+\tau}\right)^{-1} = N_{i,t+\tau}[(1-a)AK_{t+\tau}{}^{\alpha}]^{-\theta}\left(\frac{W_{i,t}^{*}}{P_{t+\tau}^{m}}\right)^{\theta} L_{t+\tau}{}^{\theta\alpha-1}$$

Substituting (A12) in the optimal wage rule given by (22), we get:

$$W_t^{*1-\theta} = \left(\frac{\theta}{\theta-1}\right)\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{(1-\alpha)A}\right]^{\theta}\frac{E_t\sum_{\tau=0}^{I-1}\beta^{\tau}N_{i,t+\tau}^{1+\upsilon}}{E_t\sum_{\tau=0}^{I-1}\beta^{\tau}C_{t+\tau}^{-1}P_{t+\tau}^{\theta-1}K_{t+\tau}^{\theta\alpha}L_{t+\tau}^{1-\theta\alpha}}$$

Substituting the aggregate labor demand (14),

$$W_t^{*1-\theta} = \left(\frac{\theta}{\theta-1}\right)\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{(1-\alpha)A}\right]^{\frac{1}{\alpha}}\frac{E_t\sum_{\tau=0}^{I-1}\beta^{\tau}N_{i,t+\tau}^{1+\upsilon}}{E_t\sum_{\tau=0}^{I-1}\beta^{\tau}C_{t+\tau}^{-1}P_{t+\tau}^{\theta-1}K_{t+\tau}w_{a,t+\tau}^{-\left(\frac{1-\alpha\theta}{\alpha}\right)}}$$

At the steady state, the re-optimized real wage is constant over time, and so is the real wage dispersion. Moreover, the capital to consumption ratio is also constant over time. Therefore, letting $(W_t{}^{**}/P_t)$ be the steady state re-optimizing real wage, the wage rule implies the following constant steady state real wage rule:

$$\text{(A13)} \quad \left(\frac{W_t^{**}}{P_t}\right)^{1-\theta} = \left(\frac{\theta}{\theta-1}\right)\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{w_a{}^{1-\alpha\theta}}{(1-\alpha)A}\right]^{\frac{1}{\alpha}}\left(\frac{C}{K}\right)\left[\frac{E_t\sum_{\tau=0}^{I-1}\beta^{\tau}}{E_t\sum_{\tau=0}^{I-1}\beta^{\tau}\Pi^{(\theta-1)\tau}}\right]N_{ss}^{1+\upsilon}$$