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The Inheritance of Advantage

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Preliminary and Incomplete

Abstract

Some agents are better treated by the market than others. In our model this discrimination arises from statistical discrimination based on the observables on the background of the individual (her parents). Advantages thus created increase the intergenerational correlation of income. This has some strong implications. First, it implies that intergenerational mobility and income inequality should correlate negatively. Second, the amplification mechanism generated by advantages may produce a multiplicity of steady states. Third, the introduction of "meritocracy" (informative signals on talent) may actually decrease mobility due to general equilibrium effects: by increasing income dispersion, they also increase the value of background.

1 Introduction

" 'We know all men are not created equal in the sense some people would have us believe - Some people are smarter than others, some people have more opportunity because they're born with it, some men make more money than others, some ladies make better cakes than others - some men are born gifted beyond the normal scope of most men.' " ∼ Atticus Finch, To Kill a Mockingbird (Harper Lee, 1960)

Throughout the book *To Kill a Mockingbird*, Atticus Finch, the hero and father of the narrator Scout, is a keen observer of humanity. He highlights to his daughter how people's attitudes and morality set them apart from one another and provides these socio-economic determinants of the separation of some men from others: some men smarter; some have greater opportunities; and some make more money (we say little in this paper on cake making ability). Of particular interest to us are the opportunities with which we are born. We consider this: given that some people make more money than others, to what extent is the dispersion of income, and its correlation across generations, related to the inheritance of advantages. In so doing we attempt to go beyond a story of capital market imperfections and consider how firms use the information available to them.

The correlation between intergenerational income mobility and inequality was recently considered by Alan Krueger in a presentation he gave to the Center for American Progress (Krueger (2012)). He discussed the "Great Gatsby Curve," the literary namesake of which provides an example, of sorts, of upward mobility. Krueger's illustration drew on data provided by Miles Corak and Figure 1 reproduces Corak's version of the Great Gatsby Curve (Corak (2012)). Income inequality and the intergenerational earnings elasticity are on the x- and y-axes respectively. We can see that

Intergenerational Earnings Elasticity

.8

.6 · Peru
· China
· Brazil
Italy · Chile
United Kingdom · United States · Argentina
Pakistan · · Switzerland
· Singapore
France · ·
Spain
.4
Japan ·
Germany ·
· New Zealand
Sweden · · Australia

.2 Finland · Canada
Norway ·
Denmark ·

20   30   40   50   60
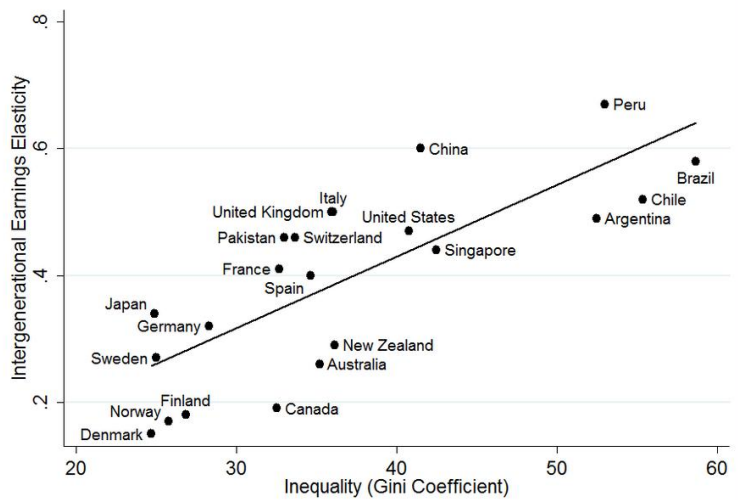Inequality (Gini Coefficient)

Figure 1: Corak's Great Gatsby Curve

more unequal countries have a greater correlation between the income of fathers and sons or, put another way, that there is a negative correlation between income inequality and mobility.

There exist several explanations for this negative correlation in the literature. One such explanation is given by the "distance effect" in Hassler et al. (2007). In their model, greater mobility leads to a long-run equilibrium in which there are fewer unskilled workers relative to skilled. This reduces the distance between their two incomes, which is their measure of income inequality, and increases the ability of unskilled parents to pay for the eduction of their children. This feeds back into increased mobility. Another explanation is provided in Solon (2004). In Solon's model, both intergenerational income elasticity and inequality are a function of the same factors, including the inheritance of income generating traits and more policy-related factors such as the progressivity of public human capital investment. This would again give rise to the upward sloping line illustrated in figure 1. This paper will offer an alternative explanation – the inheritance of advantage.

To be clear, we are not talking, as Solon did, about the greater opportunity of the children of the rich to accumulate talent. We assume that this is true, perhaps through some sort of capital market imperfection allowing the rich to invest more in their children's development. What we are talking about is the greater efficacy with which the children of the rich inherit the ability to *look* talented. This relates our paper to the literature on statistical discrimination (such as Coate & Loury (1993) and Moro & Norman (2004)). We differ from these papers in that our firms discriminate based on endogenously determined variables and do so within a dynamic model. This naturally produces the observed negative correlation between mobility and inequality. As income inequality increases, people differ more, and it becomes easier to identify talented individuals within society. As firms become more certain about who is talented and who is not, this feeds back into income dispersion. This feedback mechanism may lead to multiplicity of steady states. In addition, the better firms get at identifying talent, the lower is mobility, both because the talented tend to be from rich backgrounds and because one of the ways which firms identify talent is through information on family background.

These are the two key points which we wish to highlight in this paper. First, that there exists a multiplier effect whereby increased inequality improves the information available to firms when selecting workers, reducing mobility and encouraging further inequality in the incomes that they pay. When there is strong inheritability, in the sense that talent is closely related to family background, this can lead to multiplicity. Second, that it implies a perverse effect from meritocracy: if a meritocratic society is one in which firms can more readily identify the quality of workers and pay them accordingly, this could *decrease* mobility. As a result, societies with meritocratic

institutions (those in which it is easy to signal talent) should tend to have more inequality and less mobility than those without. This is what we observe in the likes of the US and UK, with a heirarchy of higher education institutions from the Ivy League and Oxbridge downwards, compared to the Scandinavian countries with their more egalitarian educational institutions. We will elaborate further on these points in the models which follow.

2 Labour market sorting where firms receive a signal on parental income

In this section we will consider how firms react to a signal on the parental income of a worker. This is going to be relevant to the firm because parental income and talent will be related in the following way,

$$\tau = \alpha \left(y_{-1} - \overline{y}_{-1} \right) + \epsilon_\tau \tag{1}$$

where τ is a worker's talent, y_{-1} is his parent's income, $\overline{y}_{-1}$ is the mean of parental income, and $\epsilon_\tau \sim N \left(0, \sigma^2_{\epsilon_\tau} \right)$. The talent process is a function of parental income and luck, where the extent of the role played by parental income is governed by the parameter $\alpha > 0$. Talent is the post-education, pre-labour force level of skill of a worker, and its development has a tournament component which will keep average talent centered on zero. We do not consider the factors underlying this relationship but you could imagine it being the result of capital market imperfections.

2.1 A firm's beliefs about talent

There are firms which pay wages according to their beliefs about the talent an individual has. Let θ_1 be the information available to a firm. θ_1 can be broken down into two elements: μ, information on the distribution of income in the parents' generation; and s_1, a signal on y_{-1} given by,

$$s_1 = y_{-1} + \epsilon_{s_1} \tag{2}$$

where $\epsilon_{s_1} \sim N \left(0, \sigma^2_{\epsilon s_1} \right)$. Firms do not, at this stage, receive a direct signal on talent. We assume that the firm knows the distributions of ϵ_{s_1} and ϵ_τ.

It may seem unreasonable to the reader to assume that firms receive an observation (albeit a noisy one) on parental income. It may be easier to think of it as an observation on an individual's background: where an individual grew up, how they speak, how they dress, and so on. Using this information, the firm takes expectations of talent, conditional on the information available, in the following way,

$$E \left(\tau | \theta_1 \right) = \alpha E \left(y_{-1} - \overline{y}_{-1} | \theta_1 \right) + E \left(\epsilon_\tau | \theta_1 \right) \tag{3}$$

Since both μ and s_1 are independent of ϵ_τ, the firms expectation of ϵ_τ conditional on θ_1 will be equal to zero and the firm can concentrate on forming beliefs on parental income. To do this they initially form their prior, conditional on μ, and then update it with the information contained in an individual's signal using Bayesian inference. This leads to an expectation of parental income (as a deviation from its mean) given by,

$$E\left(y_{-1} - \overline{y}_{-1}|\theta_1\right) = \frac{\sigma_{y_{-1}}^2}{\sigma_{y_{-1}}^2 + \sigma_{\epsilon s_1}^2}\left(s_1 - \overline{s}_1\right)$$

Substituting this back into equation 3 gives a posterior belief on talent of,

$$E\left(\tau|\theta_1\right) = \frac{\alpha\sigma_{y_{-1}}^2}{\sigma_{y_{-1}}^2 + \sigma_{\epsilon s_1}^2}\left(s_1 - \overline{s}_1\right) \tag{4}$$

$\frac{\alpha\sigma_{y_{-1}}^2}{\sigma_{y_{-1}}^2 + \sigma_{\epsilon s_1}^2}$ is equal to $\frac{\sigma_{\tau, s_1}}{\sigma_{s_1}^2}$ which is the coefficient from an OLS regression of τ on s_1. Thus the firms adjusts away from their prior belief (zero) and towards the signal to the extent that a change in the signal implies a change in talent. When parental income variance is high, the prior gives little information as people are very different. The signal is more heavily used in determining the posterior and hence talent. When parental income variance is zero, the prior gives perfect information and the signal is disregarded. Our point, as we will see in the next section, is that income variance is endogenous.

2.2   Steady State Beliefs

Suppose that firms set income equal to their belief about an individual's talent having observed their signal. They have no further opportunity to learn about the talent of their worker. We will write this income as a linear function of the deviation of the signal from its mean value.

$$y = E\left(\tau|\theta_1\right) = \beta_1\left(s_1 - \overline{s}_1\right) \tag{5}$$

where β_1 can be read from equation 4. Since mean income is then zero in every generation, this implies that the mean signal is zero. We can then write equation 5 as

$$y = E\left(\tau|\theta_1\right) = \beta_1 s_1 \tag{6}$$

β_1 measures the optimal reaction of the firm to the signal. The more the firms react to the signals, the more income variance there is. But, as we saw above, the more income variance there is, the more value there is in the signal relative to the prior and so the more the firms react to the signal. β_1 both causes and is a reaction to income variance.

By substitution from equation 2 we can calculate the variance of income as a function of the variance in parental income and, by setting $\sigma_y^2 = \sigma_{y_{-1}}^2$, the steady state income variance. This is given by,

$$\sigma_y^2\left(\beta_1\right) = \begin{cases} \dfrac{\beta_1^2\sigma_{\epsilon s_1}^2}{1 - \beta_1^2} & \text{if } \beta_1 < 1 \\[2mm] \infty & \text{if } \beta_1 \geq 1 \end{cases}$$

We can see that, at least up to a point, σ_y^2 is a function of β_1. We also know from equation 4 that $\beta_1 = \frac{\alpha\sigma_y^2}{\sigma_y^2 + \sigma_{\epsilon s_1}^2}$ or, more generally, β_1 is a function of σ_y^2 (which we shall call $F\left(\cdot\right)$). Finding a steady state value of β_1 is thus a matter of finding a fixed point of the equation $\beta_1 = F\left(\sigma_y^2\left(\beta_1\right)\right)$ which is given by,
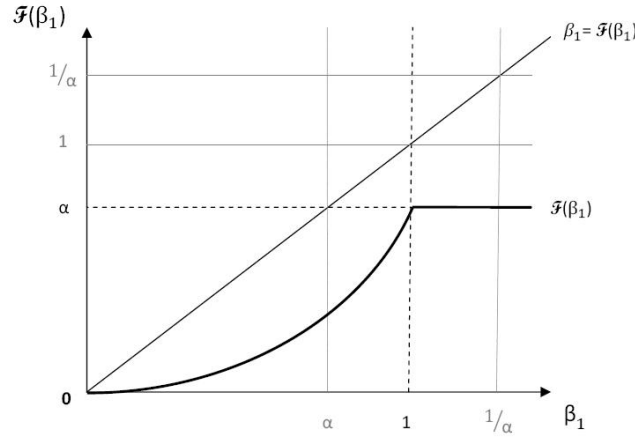
Figure 2: Steady State for $\alpha < 1$

$$\beta_1 = F\left(\sigma_y^2\left(\beta_1\right)\right) = \frac{\alpha\sigma_y^2\left(\beta_1\right)}{\sigma_y^2\left(\beta_1\right) + \sigma_{\epsilon s_1}^2} = \begin{cases} \alpha\beta_1^2 & \text{if } \beta_1 < 1 \\ \alpha & \text{if } \beta_1 \geq 1 \end{cases}$$

There are a maximum of three steady state values of β_1: $\beta_1 = 0$ will always be a steady state; $\beta_1 = 1/\alpha$ is a steady state when α is greater than one; and $\beta_1 = \alpha$ is a steady state when α is greater than one.

The simplest case is where α is less than one. In this instance there is only one steady state value of β_1 equal to zero, corresponding to a steady state income variance of zero. This is very intuitive. If the firm pays every worker the same income then by equation 1 their children's' talent will be distributed entirely by luck. By definition the firm can infer nothing about luck through the signal, and so they ignore it and pay all the children the same income. The case where α is less than one is shown in figure 2 where $F\left(\beta_1\right)$ is shorthand for $F\left(\sigma_y^2\left(\beta_1\right)\right)$. The economy will always tend to this steady state.

When α is greater than or equal to one there still exists a steady state with no income variance and no reaction by the firm to an individual's signal. This is stable in the sense that for small perturbations around zero the economy will return to this steady state. There also exist two other steady states: there is an unstable steady state where β_1 is equal to $\frac{1}{\alpha}$; and there is a stable steady state with β_1 equal to α. We are more interested in the latter. The situation where α is greater than one is shown in figure 3.

The stable steady state with β_1 equal to α has income variance tending towards infinity. As income variance in the economy becomes very high, incomes will be so different that it will be perfectly evident who is the child of whom. As a result the firm fully uses the signal to the extent that parental income tells them about talent (α).

2.3 The feedback mechanism

The fact that two stable steady states may emerge is one of the main findings that we will consider in the rest of this paper. When income variance is decreasing, firms care less about the signal because people are increasingly similar and the signal is known to be uninformative. They use it less and income variance continues to fall. When income variance is increasing, firms care more
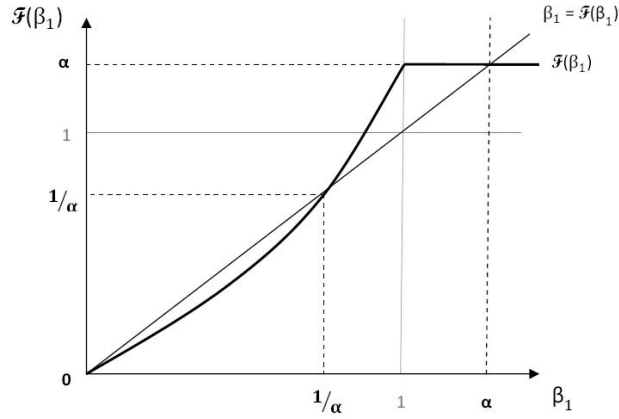
Figure 3: Steady States for $\alpha > 1$

about the signal because it is becoming more informative and people more different. Increasingly there is an extra meaningful dimension in which people differ. As firms use the signal more, this feeds back into greater income inequality. As a result of this latter process, the emergence of a portion of the population who are rich gives advantages to their children that go beyond their talents - the ability to display their privileged upbringing allows them to maintain it for the next generation.

3 Labour market sorting where firms receive a signal on talent

In the model of section 2 the information set of the firm consisted of two parts: μ, prior information about the distribution of income in the parent's generation; and s_1, a signal on parental income. In this section we consider the actions of a firm when faced with an alternative information set θ_2. They receive the same prior information but an alternative signal, s_2, on an individual's talent given by,

$$s_2 = \tau + \epsilon_{s_2} \tag{7}$$

They do not receive signal s_1. Talent is still given by equation 1 and income by $E(\tau|\theta_2)$. We will model an improvement in meritocracy as a fall in $\sigma_{\epsilon s_2}^2$ since this improves the precision of the signal and allows talent to be better identified and rewarded.

We calculate the posterior beliefs of the firm, given this new information, which gives it beliefs about talent of,

$$E(\tau|\theta_2) = \frac{\alpha^2 \sigma_{y_{-1}}^2 + \sigma_{\epsilon\tau}^2}{\alpha^2 \sigma_{y_{-1}}^2 + \sigma_{\epsilon\tau}^2 + \sigma_{\epsilon s_2}^2} s_2 \tag{8}$$

The income that the firm pays a worker is a linear function of the new signal,

$$y = E(\tau|\theta_2) = \beta_2 s_2 \tag{9}$$

where the value of β_2 can be read from equation 8. From this we can quite easily calculate the steady state income variance as a function of β_2, remembering that $\sigma_y^2 = \sigma_{y_{-1}}^2$ in steady state,
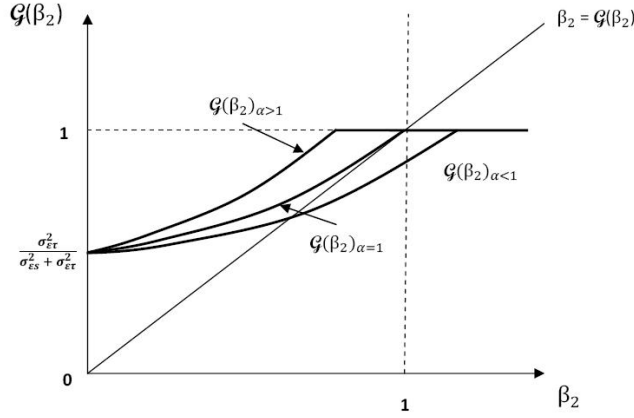
6

Figure 4: Only one steady state exists for a given α and $\sigma_{\epsilon\tau}^2 \geq \sigma_{\epsilon s_2}^2$

$$\sigma_y^2(\beta_2) = \begin{cases} \dfrac{\beta_2^2 \left(\sigma_{\epsilon s_2}^2 + \sigma_{\epsilon\tau}^2\right)}{1 - (\alpha\beta_2)^2} & \text{if } \beta_2 < \frac{1}{\alpha} \\ \infty & \text{if } \beta_2 \geq \frac{1}{\alpha} \end{cases}$$

We are now in a position to write the fixed point equation $\beta_2 = G\left(\sigma_y^2(\beta_2)\right)$ where $G\left(\cdot\right)$ is the coefficient of signal 2 from equation 8. This gives,

$$\beta_2 = G\left(\sigma_y^2(\beta_2)\right) = \frac{\alpha^2 \sigma_y^2(\beta_2) + \sigma_{\epsilon\tau}^2}{\alpha^2 \sigma_y^2(\beta_2) + \sigma_{\epsilon\tau}^2 + \sigma_{\epsilon s_2}^2} = \begin{cases} \dfrac{(\alpha\beta_2)^2 \sigma_{\epsilon s_2}^2 + \sigma_{\epsilon\tau}^2}{\sigma_{\epsilon s_2}^2 + \sigma_{\epsilon\tau}^2} & \text{if } \beta_2 < \frac{1}{\alpha} \\ 1 & \text{if } \beta_2 \geq \frac{1}{\alpha} \end{cases}$$

This is a similar result to what we found in section 2. There are up to two steady state values of β_2 less than $\frac{1}{\alpha}$ given by,

$$\beta_2 = \frac{\sigma_{\epsilon s_2}^2 + \sigma_{\epsilon\tau}^2 \pm \sqrt{\left(\sigma_{\epsilon s_2}^2 + \sigma_{\epsilon\tau}^2\right)^2 - 4\alpha^2 \sigma_{\epsilon s_2}^2 \sigma_{\epsilon\tau}^2}}{2\alpha^2 \sigma_{\epsilon s_2}^2}$$

We will call the lower of these $\underline{\beta}_2$ and the higher $\overline{\beta}_2$. There is a third possible steady state: $\beta_2 = 1$ for values of β_2 greater than $\frac{1}{\alpha}$. We will consider two possible cases: in the first, $\sigma_{\epsilon\tau}^2 \geq \sigma_{\epsilon s_2}^2$; in the second $\sigma_{\epsilon\tau}^2 < \sigma_{\epsilon s_2}^2$.

3.1 Steady state solutions where $\sigma_{\epsilon\tau}^2 \geq \sigma_{\epsilon s_2}^2$

The simplest case is where $\sigma_{\epsilon\tau}^2 \geq \sigma_{\epsilon s_2}^2$. When this condition holds there is only ever one steady state value of β_2.[1] When the variance of talent is largely exogenous ($\sigma_{\epsilon\tau}^2$ is high), and the signal is precise ($\sigma_{\epsilon s_2}^2$ is low), a firms decision over how much to use the signal is largely independent

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[1]Begin by considering $\alpha = 1$. The three possible steady states are then: $\beta_2 = \underline{\beta}_2 = 1$ for $\beta_2 < 1$; $\beta_2 = \overline{\beta}_2 = \sigma_{\epsilon\tau}^2/\sigma_{\epsilon s_2}^2 > 1$ for $\beta_2 < 1$; and $\beta_2 = 1$ for $\beta_2 \geq 1$. The first two of these are contradictions, leaving only one steady state with $\beta_2 = 1$ and $\sigma_y^2 \to \infty$. An increase in α causes the $G(\beta_2)$ curve to pivot upwards leaving the only steady
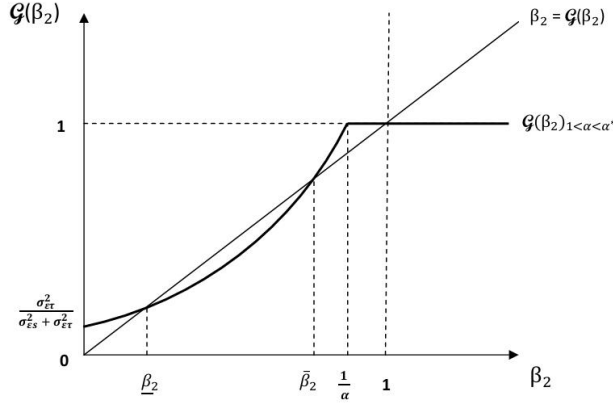
Figure 5: When $1 < \alpha < \alpha^*$ and $\sigma^2_{\epsilon\tau} < \sigma^2_{\epsilon s_2}$ there are 3 steady states, 2 stable

of what other firms are doing. Since $G(\beta_2)$ measures the firms response to the prevailing β_2 in society it has little slope in this case. The steady state solutions for α in various ranges are shown in figure 4.

3.2 Steady state solutions where $\sigma^2_{\epsilon\tau} < \sigma^2_{\epsilon s_2}$

Where $\sigma^2_{\epsilon\tau} < \sigma^2_{\epsilon s_2}$ there is a possibility that multiple steady state values of β_2 exist. Intuitively when $\sigma^2_{\epsilon\tau}$ is low and $\sigma^2_{\epsilon s_2}$ is high, the variance in talent and quality of the signal are largely driven by income variance. Since this is endogenously determined by the prevailing β_2 in society, any firm's response to changes in the prevailing β_2 is greater than in the previous section. This is why $G(\beta_2)$ has a steeper slope and why we may reach more than one steady state value of β_2. This is illustrated in figure 5. Qualitatively the situation is very similar to what we saw for β_1 is section 2. There are two stable steady states: one at $\beta_2 = \underline{\beta}_2$ with finite income variance; and one at $\beta_2 = 1$ with income variance tending to infinity. We will refer to these as the "low" and "high" (stable) steady states.

In this case there is an upper and lower bound on α, equal to one and $\alpha^* = \frac{\sigma^2_{\epsilon s_2} + \sigma^2_{\epsilon\tau}}{2\sqrt{\sigma^2_{\epsilon s_2}\sigma^2_{\epsilon\tau}}}$ respectively, for which this multiplicity exists. For values of α less than one there is only one steady state value of β_2 less than $\sigma^2_{\epsilon\tau}/\sigma^2_{\epsilon s_2}$. There is insufficient inheritance (parental income is not an important enough driver of talent) for heavy use of the signal to translate into high enough quality that this heavy use be sustained. For values of α greater than α^* there is only one steady state value of β_2 equal to one. Here the opposite is true. The quality of the talent signal generated by even a small amount of income variance would be sufficient for the talent signal to be used more and more.

The feedback mechanism works in a similar way to what we saw in section 2.3. More income inequality leads to greater dispersion of talent. This in turn leads to greater dispersion of the signals and more value being given to the signals by firms, the combined effect of which is greater income inequality.

state as $\beta_2 = 1$. A decrease in α will cause $\underline{\beta}_2$ to fall below one, confirming it as a valid steady state value. $\overline{\beta}_2$ is ruled out once we consider that any steady state solution must lie on the $G(\beta_2)$ curve which increases monotonically to a maximum value of one at $\beta_2 = 1/\alpha$. Therefore any solution to $\beta_2 = G(\beta_2)$ must have a solution with a value less than or equal to one. A decrease in α below one would cause $\overline{\beta}_2$ to increase, and since it was already above one, this rules it out as a steady state. $\beta_2 = 1$ when $\beta_2 \geq 1/\alpha > 1$ is also ruled out by contradiction. There is therefore never multiplicity when $\sigma^2_{\epsilon\tau} \geq \sigma^2_{\epsilon s_2}$ for any value of α.

8

3.3 The curse of meritocracy I

We model an increase in meritocracy as a fall in $\sigma_{\epsilon s_2}^2$. This improves the quality of the information which firms have about talent and makes them better able to pay workers according to their talent. From figure 5 we can see that a fall in $\sigma_{\epsilon s_2}^2$ causes the intercept of $G(\beta_2)$ to increase and, since it must reach the same point at $\beta_2 = 1/\alpha$, the slope to fall.[2] $\underline{\beta}_2$ increases. The steady state intergenerational correlation of incomes is given by,

$$\rho_{y,y_{-1}} = \begin{cases} \alpha\beta_2 & \text{if } \beta_2 < 1/\alpha \\ 1 & \text{if } \beta_2 \geq 1/\alpha \end{cases}$$

and so increases in the low steady state as $\sigma_{\epsilon s_2}^2$ falls. Improvements in meritocracy *reduce* mobility.

If we exogenously improve the quality of the signal, firms use it more, increasing the variance of income and talent and further improving the quality of the signal (the feedback mechanism). As this happens, firms get better at identifying talented individuals and paying them accordingly but these individuals tend to be from rich backgrounds, reducing mobility. We produce the observed negative correlation between mobility and inequality in the low steady state. The high steady state is unaffected.

4 Labour market sorting where firms receive two signals

In the previous sections we have considered the actions of a firm trying to work out the talent of individual workers with the aid of two alternative information sets, θ_1 and θ_2. We are now going to combine these so that the firm's information set, which we will now call θ', has three parts; the prior, μ, based on information on the distribution of income in the parents' generation; signal s_1 on parental income given by equation 2; and signal s_2 on talent given by equation 7. By reintroducing the signal on parental income, we will show that this makes the curse of meritocracy worse. In addition to firms being better able to pick out the talented, who happen to be predominantly from rich backgrounds, they will also use the signal on parental income more.

We assume that talent is still given by equation 1. As before, the distributions of the errors ϵ_{s_1}, ϵ_{s_2} and ϵ_τ are known to the firm and all are independently distributed.

Using Bayesian inference we can calculate the posterior beliefs of a firm conditional on the new information set θ'. This gives a steady state posterior belief of,[3]

$$E(\tau|\theta') = \frac{\alpha\sigma_{\epsilon s_2}^2\sigma_y^2}{\alpha^2\sigma_y^2\sigma_{\epsilon s_1}^2 + \left(\sigma_{\epsilon\tau}^2 + \sigma_{\epsilon s_2}^2\right)\left(\sigma_y^2 + \sigma_{\epsilon s_1}^2\right)}s_1 + \frac{\alpha^2\sigma_y^2\sigma_{\epsilon s_1}^2 + \sigma_{\epsilon\tau}^2\left(\sigma_y^2 + \sigma_{\epsilon s_1}^2\right)}{\alpha^2\sigma_y^2\sigma_{\epsilon s_1}^2 + \left(\sigma_{\epsilon\tau}^2 + \sigma_{\epsilon s_2}^2\right)\left(\sigma_y^2 + \sigma_{\epsilon s_1}^2\right)}s_2 \quad (10)$$

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[2]Formally, the slope of the $G(\beta_2)$ function is increasing in $\sigma_{\epsilon s_2}^2$ at any given value of $\beta_2 < \frac{1}{\alpha}$,

$$\frac{\partial}{\partial\sigma_{\epsilon s_2}^2}\left[\frac{\partial G(\beta_2)}{\partial\beta_2}\right]|_{\beta_2 = \widehat{\beta}_2 < \frac{1}{\alpha}} = \frac{2\alpha^2\widehat{\beta}_2\sigma_{\epsilon\tau}^2}{\left(\sigma_{\epsilon s_2}^2 + \sigma_{\epsilon\tau}^2\right)} > 0$$

[3] The firm's belief, given in equation 10 is a weighted average of three things: a prior on talent, $E(\tau|\mu)$, equal to zero; a belief about talent based on signal 1, $E(\tau|s1)$, equal to αs_1; and a belief about talent based on signal 2, $E(\tau|s_2)$, equal to s_2. As σ_y^2 goes to infinity the prior is useless and the weights given to the other parts sum to one. Similarly, if $\sigma_{\epsilon s_1}^2$ equals zero then the prior is useless since the first signal perfectly informs the firm about parental income. Again it is thrown away. The weights given to other parts then reflect the extent to which parental income and the second signal inform about talent and sum to one. When $\sigma_{\epsilon s_2}^2$ is zero only the second signal is used since it perfectly informs about talent so tis weight is equal to one.

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Since firm's pay wages according to expected talent we can now write the equation for income as,

$$y = E\left(\tau | \theta'\right) = \beta_1 s_1 + \beta_2 s_2 \tag{11}$$

where β_1 and β_2 can be read from equation 10. This gives steady state income variance of,[4]

$$\sigma_y^2\left(\beta_1, \beta_2\right) = \begin{cases} \dfrac{\beta_1^2 \sigma_{\epsilon s_1}^2 + \beta_2^2 \left(\sigma_{\epsilon \tau}^2 + \sigma_{\epsilon s_2}^2\right)}{1 - \left(\beta_1 + \alpha \beta_2\right)^2} & \text{if } \beta_1 + \alpha \beta_2 < 1 \\ \infty & \text{if } \beta_1 + \alpha \beta_2 \geq 1 \end{cases}$$

This is a function of β_1 and β_2 which will be written $\sigma_y^2\left(\beta_1, \beta_2\right)$.

We now have three equations in three unknowns: one each for steady state values of β_1 and β_2 as function of the income variance which can be read from equation 10; and one for the steady state income variance as a function of β_1 and β_2. Unfortunately finding analytical solutions to these three unknowns for finite income variance is unmanageable. As such, we will proceed in a slightly different manner to before.

4.1 The firm's choice of β_1 when the value of β_2 is fixed

Since it is very difficult to find an analytical solution for β_1 when β_2 and σ_y^2 are both being endogenously determined, what we do instead is to exogenously impose a value of β_2 and ask the question: If this were the value of β_2 which firms faced, what value(s) of β_1 would they tend towards? This method allows us to draw a reaction correspondence for β_1 as a function of different exogenously imposed values of β_2.

To carry out this method, first we need to know the firms' choice of β_1 when β_2 is exogenous. In section 2 we noted that the posterior belief in equation 4 gave a value of β_1 equal to the coefficient from a regression of τ on s. This worked because the OLS regressor was the value of β_1 which minimised the variance of expected talent around its true value, which also happens to be how a rational agent using Bayes rule behaves. We use this result again here. The value of β_1 which minimises the variance of $E\left(\tau | \theta'\right)$ around τ is found by solving the minimisation,

$$\min_{\beta_1} E\left(\tau - \beta_1 s_1 - \beta_2 s_2\right)^2 \tag{12}$$

which gives,

$$\beta_1 = \alpha\left(1 - \beta_2\right) \frac{\sigma_y^2}{\sigma_y^2 + \sigma_{\epsilon s_1}^2} \tag{13}$$

Fixing the value of β_2 at zero shuts off the talent signal and returns us the model of section 2. Also, if $\beta_1 + \alpha \beta_2 \geq 1$, we can see from above that income variance will tend to infinity and

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[4]This is derived from,

$$\sigma_y^2 = \beta_1^2 \left(\sigma_{y_{-1}}^2 + \sigma_{\epsilon s_1}^2\right) + \beta_2^2 \left(\alpha^2 \sigma_{y_{-1}}^2 + \sigma_{\epsilon \tau}^2 + \sigma_{\epsilon s_2}^2\right) + 2\beta_1 \beta_2 \left(\alpha \sigma_{y_{-1}}^2\right)$$
$$= \left(\beta_1 + \alpha \beta_2\right)^2 \sigma_{y_{-1}}^2 + \beta_1^2 \sigma_{\epsilon s_1}^2 + \beta_2^2 \left(\sigma_{\epsilon \tau}^2 + \sigma_{\epsilon s_2}^2\right)$$

Solving for $\sigma_y^2 = \sigma_{y_{-1}}^2$ gives steady state income variance.

$\beta_1 = \alpha(1 - \beta_2)$. Since rearranging this gives $\beta_1 + \alpha\beta_2 = \alpha$ the requirement for the existence of this infinite variance steady state is again $\alpha \geq 1$.

By substituting the steady state income variance and solving for β_1, equation 13 gives the $\widehat{\beta_1}$ reaction correspondence, where the "hat" indicates that this is the value of β_1 that firms choose *given an exogenously imposed value of β_2*.

4.2   The firm's choice of β_2 when the value of β_1 is fixed

We proceed in exactly the same manner as in the previous section. First, we want to solve the minimisation in equation 12 for β_2 to give us the firm's choice of β_2 *given β_1*. Solving the minimisation gives,

$$\beta_2 = \frac{\alpha(\alpha - \beta_1)\sigma_y^2 + \sigma_{\epsilon\tau}^2}{\alpha^2\sigma_y^2 + \sigma_{\epsilon\tau}^2 + \sigma_{\epsilon s_2}^2} \tag{14}$$

Substituting in the steady state income variance and solving for β_2 gives us a correspondence $\widehat{\beta_2}$ which describes the firm's choice of β_2 in reaction to a particular exogenously imposed value of β_1. Setting β_1 equal to zero cuts off the signal on parental income and returns us to the model of section 3 where the first signal plays no role. Also, when $\beta_1 + \alpha\beta_2 \geq 1$ we again find that there is a steady state at which income variance tends towards infinity and $\beta_1 + \alpha\beta_2 = \alpha$.

4.3   A numerical example

The response correspondences in sections 4.1 and 4.2 do not easily lend themselves to analytical solutions so a numerical analysis was carried out. The correspondences were drawn for the parameter values: $\alpha = 1.1$; $\sigma_{\epsilon s_1}^2 = 5$; and $\sigma_{\epsilon s_2}^2 = 4$; $\sigma_{\epsilon\tau}^2 = 1$. These were chosen based on what had been learned from the one signal models – that to have multiplicity we require: α to be larger than one but below some upper bound α^*; and the variance in the signal errors ($\sigma_{\epsilon s_1}^2$ and $\sigma_{\epsilon s_2}^2$) to be large compared to the variance in the error on talent ($\sigma_{\epsilon\tau}^2$). Figure 6 shows how the reaction correspondences look for these parameter values. We have focussed only on non-negative, non-complex values of β_1 and β_2.

Along the x- and y-axis, β_1 and β_2 are respectively fixed at zero. Thus the solutions along the axes are as in sections 3 and 2 respectively. The highest of these solutions are connected by the line $\beta_1 + \alpha\beta_2 = \alpha$. From sections 4.2 and 4.3 we can see that the infinite variance solutions to both $\widehat{\beta_1}$ and $\widehat{\beta_2}$ lie on this line.

Where the two correspondences cross we find a finite variance steady state solution for β_1 and β_2 with the firm choosing β_1 as a best response to β_2 and β_2 as a best response to β_1. We find that there are two of these, one with relatively low values of β_1 and β_2 and one with relatively higher ones. The lower one is stable while the higher one is unstable.

The dynamics of the economy work as follows. Suppose β_1 and β_2 are fixed at certain values. This will imply a certain income variance. Then imagine that at some point in time firms are allowed to freely choose β_1 and β_2. They will do so taking the current income variance as given (since it is our state variable). As a result they act as if they were on the saddle path with the same variance as was created by the initial (β_1,β_2) combination. This is given by the point at which the "isovariance" curve through this (β_1,β_2) cuts the saddle path. They will jump to a new point on the saddle path, after which they continue to choose values of β_1 and β_2 which follow the saddle path towards steady state. If the initial (β_1,β_2) was inside the higher of the two isovariance curves
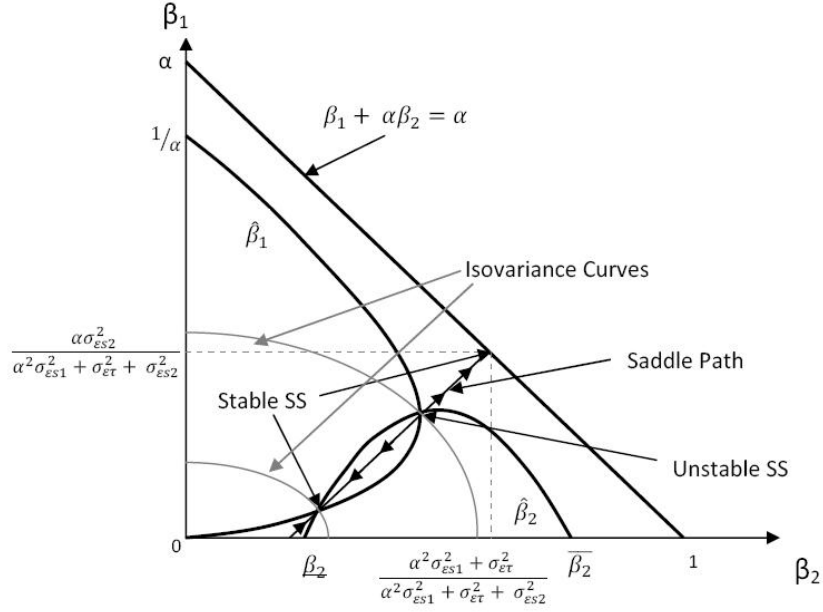
Figure 6: An example of multiplicity in the two signal model

illustrated, the economy will converge towards the lower stable steady state (with finite variance). If it is outside this isovariance curve the economy will converge towards the high stable steady state and infinite income variance. The equation of the saddle path is given by,

$$\beta_1 = -\frac{\sigma_{\epsilon\tau}^2}{\alpha\sigma_{\epsilon s_1}^2} + \frac{\sigma_{\epsilon s_2}^2 + \sigma_{\epsilon\tau}^2}{\alpha\sigma_{\epsilon s_1}^2}\beta_2 \tag{15}$$

It is worth noting that, where two stable steady states exist, it is always the case that one has a lower level of both β_1 and β_2 and so there never exists two steady states which substitute one signal for the other.

4.4 The curse of meritocracy II

In the two signal model, the intergenerational income correlation is given by,

$$\rho_{y,y_{-1}} = \begin{cases} \beta_1 + \alpha\beta_2 & \text{if } \beta_1 + \alpha\beta_2 < 1 \\ 1 & \text{if } \beta_1 + \alpha\beta_2 \geq 1 \end{cases}$$

Income variance in the finite variance steady state is given by,

$$\sigma_y^2 = \frac{\beta_1^2\sigma_{\epsilon s_1}^2 + \beta_2^2\left(\sigma_{\epsilon s_2}^2 + \sigma_{\epsilon\tau}^2\right)}{1 - (\beta_1 + \alpha\beta_2)^2}$$

Since the analytical solutions for β_1 and β_2 are unmanageable, and intergenerational mobility and

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Parameter	Values the parameter may take
α	$0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5$
$\sigma^2_{\epsilon s_1}$	$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$
$\sigma^2_{\epsilon s_2}$	$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$
$\sigma^2_{\epsilon\tau}$	$0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6$

Table 1: Parameter values for the numerical exercise investigating the curse of meritocracy

income inequality are functions of β_1 and β_2, it is also very difficult to solve analytically the effects of a change in our exogenous parameters on them. We proceeded with a numerical analysis. First we will examine the effects of a fall in $\sigma^2_{\epsilon s_2}$. This is again how we now define an improvement in meritocracy. There are two forces at work as $\sigma^2_{\epsilon s_2}$ falls: *given β_1 and β_2* a fall in the noise will decrease income variance which will lead to a reduction in β_1 and β_2 as firms respond; but *for a given income variance* a fall in $\sigma^2_{\epsilon s_2}$ necessarily increases the use of the talent signal, which in turn will increase income variance and the use of both signals.

We first define our parameter sets to be any combination of parameters from table 1. α may take any one of fifteen values, $\sigma^2_{\epsilon s_1}$ any one of twelve, $\sigma^2_{\epsilon s_2}$ one of twelve, and $\sigma^2_{\epsilon\tau}$ one of twelve. This gives 25,920 possible combinations of parameters. However for some of those with α greater than one, the finite variance steady state will not exist. We eliminated these cases and were left with 17,918 parameter combinations which produce a finite variance steady state.

For each of these steady states we decreased $\sigma^2_{\epsilon s_2}$ by 0.001. We interpret this as an exogenous marginal increase in meritocracy. In all 17,918 cases for which a finite variance steady state existed, income variance and the intergenerational correlation of incomes both increased. We graph the effect of increasing meritocracy on inequality and mobility for a small number of different parameter sets in figure 7. Meritocracy was measured as $\frac{1}{\sigma^2_{\epsilon s_2}}$. The graph illustrates the increase in inequality and fall in mobility which accompanies an increase in meritocracy. There are two possible reasons for the fall in mobility: in all cases, the quality of the second signal has increased and so β_2 increases. This was the effect described in section 3.3 where firms get better at picking out the talented but they also tend to be from rich backgrounds; additionally, in some cases (all cases for which $\alpha > 1$) the first signal is also used more, reducing mobility since it directly relates to family background.

We also investigated whether the correlation of an individual's talent and income increased with a fall in $\sigma^2_{\epsilon s_2}$. If it did not, a fall in $\sigma^2_{\epsilon s_2}$ would not be a useful way of exogenously shocking meritocracy. It occurred in all cases.

In addition to investigating the effects of meritocracy we consider the effect of inherited advantages. An exogenous increase in inherited advantages was defined as a fall in $\sigma^2_{\epsilon s_1}$. With a fall in $\sigma^2_{\epsilon s_1}$, it is easier for firms to identify parental income and, since parental income is correlated with talent, to pay higher incomes to those from richer backgrounds. Thus the advantages of a rich background (and disadvantages of a poor background) are greater. We decreased $\sigma^2_{\epsilon s_1}$ by 0.001 and examined the effects for the parameter sets given in table 1. In all 17,981 cases for which a finite variance steady state existed, income variance, the intergenerational correlation of incomes, and the correlation of talent and income increased. Figure 8 illustrates how increases in advantages related to background (defined as $\frac{1}{\sigma^2_{\epsilon s_1}}$) lead to greater income inequality and lower intergenerational mobility.

In summary, an increase in the precision of either signal (an increase in meritocracy or advantages related to background) will lead to greater income inequality and further increases in both meritocracy and advantages related to background. This is because the endogenous response of firms to the more precise information is to use one or both of the signals more. We might have expected that when firms are better informed about an individual's talent, they would care less about his background. We show that more often than not (and in all cases where there is sufficient
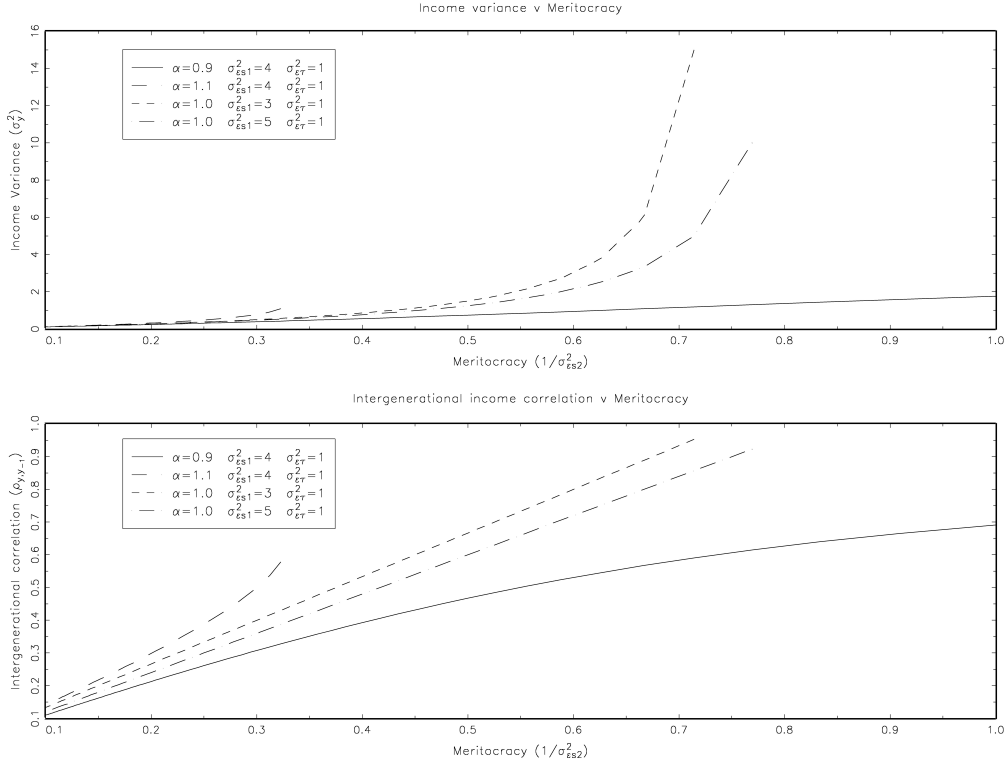
Figure 7: The effect of increasing meritocracy on steady state income variance and mobility

inheritance) this is not true and that even when it is true, mobility still falls due to increased use of the talent signal.

5   A discrete model with finite income variance

Until now this paper has used a model where income and talent were continuous and normally distributed. It has been shown, using three variations on the normal model, how multiplicity may exist, producing two stable steady state outcomes. In the low steady state there was finite income variance, while the high steady state tended towards infinite income variance. Although the analysis concentrated on the low steady state, the existence of the infinite variance high steady state seems less realistic. We now turn to a model with discrete levels of income, which will provide a simple example of some of the features of the normal model, without the high variance steady state exhibiting infinite steady state income variance.

As in section 4, firm's are still unable to observe talent and receive two signals about each worker: the first, which we will call s_r, is a signal on parental income; the second, s_t, is a signal on talent. In a departure from the previous models, these signals can only take values of one or zero and their quality will be determined exogenously. Since there are two signals, each of which can take one of two values, there are four "types" of individual. Y_{rt} gives the income of each type, for example Y_{11} is the income of a type 11 worker (one for whom $s_r = 1$ and $s_t = 1$).

Talent, τ, is discrete and only able to take values one or zero. If an individual's parent is sufficiently rich, it is more likely to take a value of one. The probability of being talented is given by,
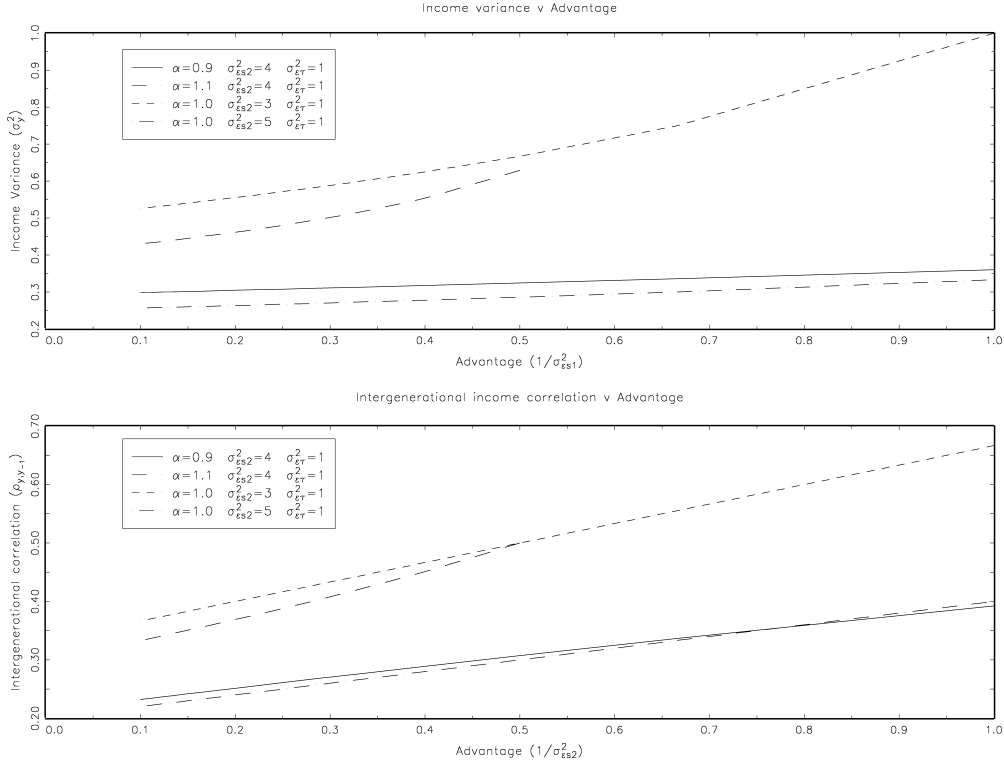
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Figure 8: The effect of increasing advantages on steady state income variance and mobility

$$P\left(\tau=1\right)=\begin{cases}\alpha_s+\mu_s\left(1-\alpha_s\right) & \text{if } \tilde{Y}_{rt} > \tilde{K} \\ \mu_s & \text{if } \tilde{Y}_{rt} = \tilde{K} \\ \mu_s\left(1-\alpha_s\right) & \text{if } \tilde{Y}_{rt} < \tilde{K}\end{cases} \qquad 0 \leq \alpha_s \leq 1$$

$$S_{rt} = P\left(\tau=1 | \tilde{Y}_{rt}\right)$$

$\tilde{K}$ is average talent in the parent's generation which, as we will see below, is equal in steady state to average income in the parent's generation. Those whose parent's have above average income are more likely to be talented than those whose parent's have below average income. α_s is a parameter controlling inheritance. When α_s is zero, everyone has the same probability of being talented, μ_s, irrespective of who their parent is. When α_s is one, for everyone other than those on the threshold, talent is entirely determined by parental "type". S_{rt} is defined to be the probability of being talented conditional on *parental* income $\tilde{Y}_{rt}$.

As before, the first signal is on parental income. We will call an individual who looks like she is from a rich background ($s_r=1$) "advantaged", while one who looks like she is from a poor background ($s_r=0$) will be termed "disadvantaged". An individual's likelihood of being advantaged is increased if her parent is sufficiently rich. The probability of being advantaged is given by,

$$P\left(s_r=1\right)=\begin{cases}\alpha_r+\mu_r\left(1-\alpha_r\right) & \text{if } \tilde{Y}_{rt} > (1+\lambda)\tilde{K} \\ \mu_r & \text{if } \tilde{Y}_{rt} = (1+\lambda)\tilde{K} \\ \mu_r\left(1-\alpha_r\right) & \text{if } \tilde{Y}_{rt} < (1+\lambda)\tilde{K}\end{cases} \qquad 0 \leq \alpha_r \leq 1, \, 0 \leq \lambda$$

$$R_{rt} = P\left(s_r=1 | \tilde{Y}_{rt}\right)$$

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$\tilde{Y}_{rt}$ represents parental income so the first line states that an individual's chances of being advantaged are larger if her parent has an income above $(1 + \lambda) \tilde{K}$. λ is a parameter controlling how rich your parent needs to be, relative to the average, in order to increase your chances of being advantaged. α_r is a parameter which controls the quality of the signal. If α_r is zero then the signal is randomly distributed in the population and hence not correlated with, or informative about, parental income. If α_r is one, the signal tells exactly whose parents have income above and below the threshold $(1 + \lambda) \tilde{K}$. R_{rt} is defined to be the probability that an individual is advantaged given her parent's income is Y_{rt}.

Finally, the second signal, s_t, is a signal on talent. We will refer to an individual who looks talented ($s_t = 1$) as "promising", since he is an attractive prospect to a firm, while one which looks untalented ($s_t = 0$) will be referred to to as "unpromising". The probability that an individual is promising is given by,

$$P(s_t = 1) = \begin{cases} \alpha_t + \mu_t (1 - \alpha_t) & \text{if } \tau = 1 \\ \mu_t (1 - \alpha_t) & \text{if } \tau = 0 \end{cases} \qquad 0 \le \alpha_t \le 1$$

This states that an individual is more likely be promising if he is actually talented and unpromising if he is not. α_t measures the quality of the signal, where zero is completely uninformative and one completely informative. It is going to be our measure of meritocracy, which will increase as α_t goes from zero to one.

In what follows we are going to make the simplifying assumptions that $\mu_s = \mu_r = \mu_t = \mu$.

5.1 Steady state conditions

Let π_{rt} represent the fraction of the population which are of that type. In steady state π_{rt} is constant across generations. We will begin with π_{11}, the proportion of individuals who are both advantaged and promising. The probability that an individual is of type 11 depends on what type their parent is and what their probability of being 11 is conditional on their parents type. This gives,[5]

$$\begin{aligned} P(s_r = 1, s_t = 1) &= \pi_{11} \\ &= \mu (1 - \alpha_t) [R_{11} \tilde{\pi_{11}} + R_{10} \tilde{\pi_{10}} + R_{01} \tilde{\pi_{01}} + R_{00} \tilde{\pi_{00}}] \\ &+ \alpha_t [R_{11} S_{11} \tilde{\pi}_{11} + R_{10} S_{10} \tilde{\pi}_{10} + R_{01} S_{01} \tilde{\pi}_{01} + R_{00} S_{00} \tilde{\pi}_{00}] \end{aligned}$$

We define a steady state to be one in which the proportion of each type is constant over time (i.e.

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[5]The proportion of the population which is advantaged and promising is given by,

$P(s_r = 1, s_t = 1) = \pi_{11}$

$$\begin{aligned} &= P\left(s_r = 1 | \tilde{Y}_{11}\right) P\left(s_t = 1 | \tilde{Y}_{11}\right) P\left(\tilde{Y}_{11}\right) + P\left(s_r = 1 | \tilde{Y}_{10}\right) P\left(s_t = 1 | \tilde{Y}_{10}\right) P\left(\tilde{Y}_{10}\right) \\ &+ P\left(s_r = 1 | \tilde{Y}_{01}\right) P\left(s_t = 1 | \tilde{Y}_{01}\right) P\left(\tilde{Y}_{01}\right) + P\left(s_r = 1 | \tilde{Y}_{00}\right) P\left(s_t = 1 | \tilde{Y}_{00}\right) P\left(\tilde{Y}_{00}\right) \\ &= R_{11} [\alpha_t S_{11} + \mu (1 - \alpha_t)] \tilde{\pi_{11}} + R_{10} [\alpha_t S_{10} + \mu (1 - \alpha_t)] \tilde{\pi_{10}} \\ &+ R_{01} [\alpha_t S_{01} + \mu (1 - \alpha_t)] \tilde{\pi_{01}} + R_{00} [\alpha_t S_{00} + \mu (1 - \alpha_t)] \tilde{\pi_{00}} \\ &= \mu (1 - \alpha_t) [R_{11} \tilde{\pi_{11}} + R_{10} \tilde{\pi_{10}} + R_{01} \tilde{\pi_{01}} + R_{00} \tilde{\pi_{00}}] \\ &+ \alpha_t [R_{11} S_{11} \tilde{\pi}_{11} + R_{10} S_{10} \tilde{\pi}_{10} + R_{01} S_{01} \tilde{\pi}_{01} + R_{00} S_{00} \tilde{\pi}_{00}] \end{aligned}$$

The third line uses the fact that $P\left(s_t = 1 | \tilde{Y}_{rt}\right) = P(s_t = 1 | \tau = 1) P\left(\tau = 1 | \tilde{Y}_{rt}\right) + P(s_t = 1 | \tau = 0) P\left(\tau = 0 | \tilde{Y}_{rt}\right)$

and that $P\left(\tilde{Y}_{rt}\right) = \tilde{\pi}_{rt}$ since all parents of type rt are paid an income Y_{rt}.

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$\pi_{rt} = \tilde{\pi}_{rt}$). We will also define a variable A, which is the proportion of the population which is both advantaged and talented, and is given by,

$$A = R_{11}S_{11}\pi_{11} + R_{10}S_{10}\pi_{10} + R_{01}S_{01}\pi_{01} + R_{00}S_{00}\pi_{00}$$

Then our first steady state condition is,

$$\pi_{11} = \alpha_t A + \mu(1-\alpha_t)[R_{11}\pi_{11} + R_{10}\pi_{10} + R_{01}\pi_{01} + R_{00}\pi_{00}]$$

Next we want to find the steady state solution for π_{10}, the proportion of the population that is advantaged but unpromising. To do so we will actually find the proportion that are advantaged which equals the unconditional probability that $s_r = 1$. This gives,[6]

$$\begin{aligned} P(s_r = 1) &= \pi_{11} + \pi_{10} \\ &= R_{11}\tilde{\pi_{11}} + R_{10}\tilde{\pi_{10}} + R_{01}\tilde{\pi_{01}} + R_{00}\tilde{\pi_{00}} \end{aligned}$$

By letting $\pi_{rt} = \tilde{\pi}_{rt}$ and substituting our second steady state equation into our first, we are left with the following two steady state conditions:

$$\pi_{11} = \alpha_t A + \mu(1-\alpha_t)[\pi_{11} + \pi_{10}] \tag{16}$$

$$\pi_{11} + \pi_{10} = R_{11}\pi_{11} + R_{10}\pi_{10} + R_{01}\pi_{01} + R_{00}\pi_{00} \tag{17}$$

We derive our next steady state equation by focussing on the proportion of the population which is promising. K is the proportion of the population which is talented and so is given by,

$$K = P(\tau = 1) = S_{11}\tilde{\pi}_{11} + S_{10}\tilde{\pi}_{10} + S_{01}\tilde{\pi}_{01} + S_{00}\tilde{\pi}_{00}$$

or in steady state,

$$K = P(\tau = 1) = S_{11}\pi_{11} + S_{10}\pi_{10} + S_{01}\pi_{01} + S_{00}\pi_{00}$$

Using K, we find the following equation for the steady state proportion of promising individuals,[7]

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[6]The proportion of the population which is advantaged is given by,

$$\begin{aligned} P(s_r = 1) &= \pi_{11} + \pi_{10} \\ &= P\left(s_r = 1|\tilde{Y}_{11}\right)P\left(\tilde{Y}_{11}\right) + P\left(s_r = 1|\tilde{Y}_{10}\right)P\left(\tilde{Y}_{10}\right) + P\left(s_r = 1|\tilde{Y}_{01}\right)P\left(\tilde{Y}_{01}\right) + \\ &\quad + P\left(s_r = 1|\tilde{Y}_{00}\right)P\left(\tilde{Y}_{00}\right) \\ &= R_{11}\tilde{\pi_{11}} + R_{10}\tilde{\pi_{10}} + R_{01}\tilde{\pi_{01}} + R_{00}\tilde{\pi_{00}} \end{aligned}$$

[7]The proportion of the population which is promising is given by,

$$\begin{aligned} P(s_t = 1) &= \pi_{11} + \pi_{01} \\ &= P(s_t = 1|\tau = 1)P(\tau = 1) + P(s_t = 1|\tau = 0)P(\tau = 0) \\ &= [\alpha_t + \mu(1-\alpha_t)]K + \mu(1-\alpha_t)(1-K) \\ &= \alpha_t K + \mu(1-\alpha_t) \end{aligned}$$

$$\pi_{11} + \pi_{01} = \alpha_t K + \mu (1 - \alpha_t) \tag{18}$$

Our last steady state equation draws on the fact that the π_{rt} terms for each generation must sum to one and so,

$$\pi_{11} + \pi_{10} + \pi_{01} + \pi_{00} = 1 \tag{19}$$

Equations 16 through 19 can be solved for the four unknowns (π_{11}, π_{10}, π_{01} and π_{00}) to give the steady state proportions of each type in the population.

5.2 The steady state income distribution

Once we have established the steady state distribution of types we can easily define the steady state income which each type receives. We assume that incomes are paid to each type according to the probability that they are talented conditional on their observables. So the income of a type 11 individual will be given by,[8]

$$Y_{11} = P(\tau = 1 | s_r = 1, s_t = 1) = \frac{[\alpha_t + \mu (1 - \alpha_t)] A}{\pi_{11}} \tag{20}$$

By the same logic,

$$Y_{10} = P(\tau = 1 | s_r = 1, s_t = 0) = \frac{(1 - \alpha_t)(1 - \mu) A}{\pi_{10}} \tag{21}$$

$$Y_{01} = P(\tau = 1 | s_r = 0, s_t = 1) = \frac{[\alpha_t + \mu (1 - \alpha_t)](K - A)}{\pi_{01}} \tag{22}$$

$$Y_{00} = P(\tau = 1 | s_r = 0, s_t = 0) = \frac{(1 - \alpha_t)(1 - \mu)(K - A)}{\pi_{00}} \tag{23}$$

Any solution to equations 16 through 23 gives a full characterisation of a steady state distribution of income (π_{11}, π_{10}, π_{01}, π_{00}, Y_{11}, Y_{10}, Y_{01}, Y_{00}). Equations 20 through 23 can also be used to show that average income is equal to K[9].

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[8]The probability that a type 11 individual is talented is given by,

$$
\begin{aligned}
P(\tau = 1 | s_r = 1, s_t = 1) &= \frac{P(s_r = 1, s_t = 1 | \tau = 1) P(\tau = 1)}{P(s_r = 1, s_t = 1)} \\
&= \frac{P(s_r = 1 | \tau = 1) P(s_t = 1 | \tau = 1) K}{\pi_{11}} \\
&= \frac{\left[\sum_{rt} P\left(s_r = 1 | \tilde{Y}_{rt}\right) P\left(\tilde{Y}_{rt} | \tau = 1\right)\right] [\alpha_t + \mu (1 - \alpha_t)] K}{\pi_{11}} \\
&= \frac{\left[\sum_{rt} R_{rt} S_{rt} \pi_{rt}\right] [\alpha_t + \mu (1 - \alpha_t)]}{\pi_{11}} \\
&= \frac{[\alpha_t + \mu (1 - \alpha_t)] A}{\pi_{11}}
\end{aligned}
$$

[9]Average income, $\overline{Y}$, is given by $Y_{11}\pi_{11} + Y_{10}\pi_{10} + Y_{01}\pi_{01} + Y_{00}\pi_{00} = A + (K - A) = K$
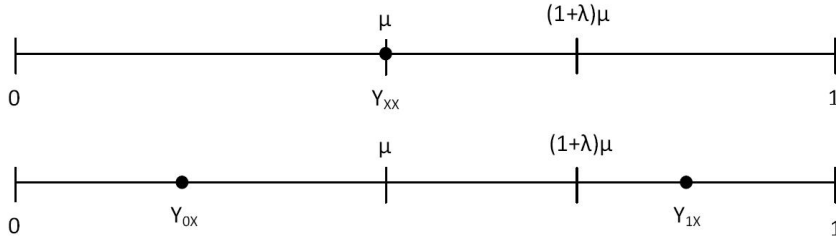
Figure 9: The steady states of the economy when α_t equals zero. There are two possibilities: (a) everyone has the same probability of being advantaged and so everyone receives the same income; (b) some people are more likely to be advantaged than others and the advantaged are paid more.

5.3 An Example

The goals of this paper, and the focus of the normal model, were two-fold: to demonstrate the multiplier effect whereby raising income inequality improved information on workers' quality and caused firms to pay more varied wages. Where there was sufficient inheritance this led to multiplicity; and secondly, to demonstrate that a more meritocratic society, defined as one in which firms are better informed of worker quality, displays lower mobility. When both of these points are combined, we see that a society in which firms are better informed will lead to both greater inequality and less mobility in a natural way. We now plan to demonstrate these points within our discrete model, without having infinite income variance in the high steady state.

5.3.1 The feedback mechanism

We begin by setting α_t equal to zero (with $\lambda > 0$). This means that the signal on talent is randomly distributed amongst society and hence uncorrelated with actual talent. It is disregarded by the firm. This is the situation that was previously described in section 2. I will use an X subscript to indicate that a signal is irrelevant so, for example, Y_{1X} is the income of an individual who is advantaged when the talent signal is not being used by the firm. In this situation, there can be at most two income levels where firms discriminate amongst workers based on whether or not they are advantaged.

When α_t is equal to zero, the simplest steady state occurs where both income levels lie below the income threshold $(1 + \lambda) K$. This implies that all individuals have the same probability of being advantaged, and so being advantaged or not is not a useful signal to the firm. They disregard it, along with the talent signal, and pay everyone the same income, K. This common income level can be shown to equal μ since,

$$K = P\left(\tau = 1\right) = P\left(\tau = 1 | \tilde{Y} = K\right) P\left(\tilde{Y} = K\right) = \mu$$

This steady state is illustrated in figure 9(a) and essentially describes the same steady state as the zero variance one in section 2. This steady state will always exist.

There is an alternative steady state which can be reached where there are two groups of individuals with different probabilities of being advantaged. For this to be a steady state, it must be that the two incomes are sufficiently different so that one group is more likely to be both advantaged and talented, implying advantage is a useful signal of talent. This implies we require that $Y_{1X} >$
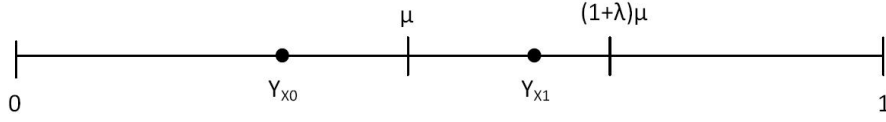
19

Figure 10: The steady states of the economy when α_t is larger than zero but less than $\frac{\lambda\mu}{1-\mu}$

$(1 + \lambda)K$ and that $Y_{0X} < K$. It is relatively easy to show that should such a steady state exist there would be a proportion μ of the population who are advantaged and a proportion μ who are talented. The level of incomes would be given by,

$$Y_{1X} = \alpha_r\alpha_s + \mu\left(1 - \alpha_r\alpha_s\right)$$
$$Y_{0X} = \mu\left(1 - \alpha_r\alpha_s\right)$$

This steady state is illustrated in figure 9(b). When the advantaged group is more likely to be talented, they are paid more, implying they are more likely to be talented. It is similar to the high steady state in section 2. In particular, this steady state will not always exist (the advantaged group need to be paid *sufficiently* more) but is more likely the greater is the inheritability of talent through parental income.[10]

5.3.2 The curse of meritocracy I

We now turn our attention to demonstrating how improvements in meritocracy may lead to increased inequality and falling mobility. To do so, we will focus on the low steady state illustrated in figure 9(a). We will increase meritocracy by increasing α_t, the parameter which governs the information which firms have on an individual's talent. The resulting steady state is illustrated in figure 10.

As we increase α_t above zero, we produce an economy in which there are two groups of individuals, one of which is more likely to be talented and hence more likely to be promising. The probability of being advantaged is the same for everyone. We can see that the proportion in the promising group in steady state is given by,

$$P\left(s_t = 1\right) = \pi_{X1} = \alpha_t K + \mu\left(1 - \alpha_t\right)$$

Similarly we can show that the proportion of talented individuals is given by,

$$P\left(\tau = 1\right) = K = \alpha_s\pi_{X1} + \mu\left(1 - \alpha_s\right)$$

Solving these two equations we find that a proportion μ of the population is promising and that a proportion μ of the population is talented. The correlation between the membership of these two groups will determine the extent to which the signal is useful to the firm. This is determined by α_t. It is easy to show that the incomes which the firms will pay are given by,

$$Y_{X1} = P\left(\tau = 1 | s_t = 1\right) = \alpha_t + \mu\left(1 - \alpha_t\right)$$
$$Y_{X0} = P\left(\tau = 1 | s_t = 0\right) = \mu\left(1 - \alpha_t\right)$$

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[10]This steady state exists if $\lambda < \frac{\alpha_r\alpha_s(1-\mu)}{\mu}$, so a higher value of α_s will improve the likelihood of multiplicity as in section 2.

μ   (1+λ)μ

0   Y_{00} Y_{10} Y_{01} Y_{11} 1
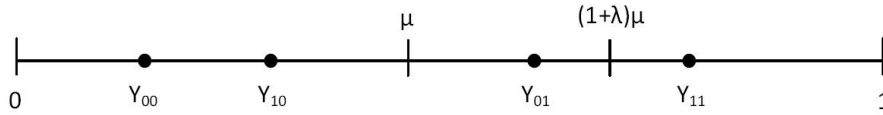
Figure 11: One steady states of the economy when α_t is equal to $\frac{\lambda\mu}{1-\mu}$

The difference in the income paid to the two groups depends on the quality of the information available to the firm, α_t. As we increase α_t, Y_{X1} increases and Y_{X0} decreases. In addition, as a higher income is paid to those who are promising (those who are more likely to be talented), and those who are promising tend to come from richer backgrounds, this reduces mobility. The increase in income dispersion and fall in mobility due to increased use of the talent signal are the same effects as we saw in section 3.3. This steady state distribution of incomes exists for all α_t less than $\frac{\lambda\mu}{1-\mu}$.[11]

5.3.3 The curse of meritocracy II

Finally, we want to consider what happens when α_t reaches $\frac{\lambda\mu}{1-\mu}$. At this point, Y_{X1} reaches the threshold $(1+\lambda)\mu$ and the probability of being advantaged jumps from $\mu(1-\alpha_F)$ to μ for the children of the rich. The fact that some of those from rich backgrounds, who are more likely to be talented, are also now more likely to be advantaged, means that both signals are now useful to the firm. There are four income groups formed by each of our previous groups, Y_{X1} and Y_{X0}, splitting in two. The advantaged members of each group move to a higher steady state income, while the disadvantaged in each group move to a lower steady state income. At this point there are several steady states which the economy could jump to. I will focus attention on one. This is the one where Y_{01} and Y_{10} do not jump far from Y_{X1} and Y_{X0} respectively so that $Y_{10} < K < Y_{01} < (1+\lambda)K$.[12] This is illustrated in figure 11. K is again equal to μ.

As in the normal version of the model we observe complementarity between the two signals. As we improve the quality of the signal on talent, there will come a point where the firm begins to use the signal on parental income. This is shown in figure 11. At the point where firms begin using a person's advantage to place them in a job there will be a discrete fall in mobility. This is brought about by the increased inequality associated with improved meritocracy. The idea that meritocracy can simultaneously increase inequality and lower mobility was one of the central themes in the previous sections of this paper.

6 Conclusions

In our model, inequality leads to discrimination because people are more different and the signals they provide more informative. This discrimination feeds back into inequality. This feedback mechanism may lead to multiplicity of steady states, one with a relatively small amount of discrimination and inequality and one with a relatively large amount of discrimination and ever increasing inequality.

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[11] At this point Y_{X1} reaches the threshold at which the parental income signal starts to have some value. Although we did not restrict λ, we will assume that this threshold is less than one otherwise we would never leave this steady state.

[12] For this to be the steady state that is reached at $\alpha_t = \frac{\lambda\mu}{1-\mu}$ we require that $\alpha_s < min\left\{\frac{\lambda(1-\mu)}{\mu\alpha_r[\lambda^2+(1+\lambda)(1-\alpha_r)[1-\mu(1+\lambda)]]}, \frac{\lambda\mu}{\alpha_r[\lambda\mu+[1-\mu(1+\lambda)]^2]}\right\}.$

There exists a negative correlation between inequality and mobility, which was observed in Corak's "Great Gatsby Curve". We believe this may occur for two reasons: the increased discrimination which accompanies greater inequality results in firms being better able to identify talent, which is correlated with parental income; and it may result in them using family background information more. This move towards greater inequality and lower mobility may be started by an exogenous increase in the degree of meritocracy in society, which improves the quality of information available to firms, feeding into discrimination, inequality and further endogenous increases in meritocracy.

These conclusions suggests some policy implications. Suppose the government has a choice to make between providing funding for pre-school and university education. We would model the early intervention as a way of relieving the credit constraints on poorer families and ensuring a good platform of basic skills on which to build for *all* children. This should weaken the relationship between talent development and parental income, which in this model refers to a fall in α. Alternatively, the university investment would allow the best and the brightest to go on to university and provide a "Spence-type" signalling opportunity for them. This improved ability to signal their skills is what we modelled above as a fall in $\sigma^2_{\epsilon s_2}$. The implications from the model were that the university funding would lead to increased inequality and lower mobility. Pre-school investment would lead to a weakening of the relationship between talent and parental income, making the parental income signal less useful, lowering discrimination and in turn lowering inequality and raising mobility. As a result, to the extent that equality and intergenerational mobility are valued by society, we would support public funding of early childhood interventions over university education.

In terms of future research there are a number of avenues we are currently exploring. Firstly, we have removed the tournament (or relative) component of talent development to allow the economy to have growth in average income. Secondly, the discrimination element of our model is loosely based on the statistical discrimination model of Coate & Loury (1993) but lacks a parental investment. Including an optmising agent who chooses how much to invest in their child would give the model a forward-looking channel since parents would be concerned about the expected returns to investment in their child. This could lead to multiplicity of equilibria, if parents beliefs were self-fulfilling, in addition to multiplicity of steady states. This is the focus of our current research. We are also trying to get closer to describing a joint distribution of talent and advantages, which is an important element in models on the misallocation of talent and central to our understanding of inequality and mobility.

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