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The meson spectrum in large-N QCD

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We present lattice results on the meson spectrum and decay constants in large-N QCD. The results are obtained in the quenched approximation for $N = 2,3,4,5,6,7$ and 17 and extrapolated to $N = \infty$.

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1. Introduction

Quantum Chromodynamics (QCD), the theory of strong interactions is a non-Abelian quantum field theory characterized by local SU(N) gauge invariance where $N = 3$ denotes the number of "colours". The adjoint gauge bosons (gluons) couple n_f "flavours" of fermionic matter fields in the fundamental representation (quarks).

The standard non-perturbative definition of QCD is based on lattice regularization [1], which makes the theory mathematically well-defined and amenable to analytical as well as numerical studies. Theoretical progress, algorithmic innovation and ever more powerful computers have allowed some teams to derive non-perturbative low energy properties like the spectrum of QCD with $n_f = 2$, $n_f = 2+1$ and $n_f = 2+1+1$ sea quarks at realistic values of the physical parameters, see, e.g., ref. [2] for a recent review.

A different non-perturbative approach to QCD is based on an expansion in powers of $1/N$ of the inverse number of colour charges at fixed n_f [3]. When N is taken to infinity, and the gauge coupling g is sent to zero, keeping the product g^2N as well as n_f fixed (the 't Hooft limit), the theory reveals striking mathematical simplifications, see refs. [4] for recent reviews. For instance, all amplitudes of physical processes are determined by a particular subset of Feynman diagrams (planar diagrams), the low-energy spectrum consists of stable meson and glueball states and the scattering matrix becomes trivial. One may study the physical $N = 3$ case, expanding around the large-N limit in terms of $1/N$. Interestingly, the non-flavour-singlet spectra of QCD with sea quarks and quenched QCD agree within 10 % [2], which may indicate both n_f/N and $1/N^2$ corrections to be small.

The large-N limit also plays a vital role in the chiral effective theory approach where the N-dependence of low-energy constants is known [5] and, within this framework, in studies of properties of unstable resonances, see, e.g., refs. [6, 7]. (Un)fortunately, even in the large-N limit QCD is far from trivial.

Another non-perturbative approach — that, unlike lattice regularization, does not break the Euclidean spacetime symmetry — to low-energy properties of strongly coupled non-Abelian gauge theories is based on the conjectured correspondence between (supersymmetric) large-N gauge and string theories in the classical gravity limit in an anti-de-Sitter spacetime (AdS/CFT correspondence) [8]. During the last decade, many studies have used techniques based on this correspondence to construct models which reproduce the main features of the meson spectrum of QCD [9].

Recently, the dependence of various quantities on N was studied in lattice simulations. For instance, pseudoscalar and vector meson masses (among other observables) were determined in refs. [10 – 13]. Here we improve upon and extend these studies, reducing the quark masses and increasing N, the statistics, the number of states studied and the volume.

In view of the above discussion, it is important to determine the meson spectrum of large-N QCD to constrain effective field theory parameters and to enable comparison with AdS/CFT and AdS/QCD predictions. We perform our simulations in the quenched approximation to QCD, neglecting sea quark loops. Therefore, we only encounter $1/N^2$ corrections to the large-N limit, rather than n_f/N corrections. This allows for a more constrained $N \to \infty$ extrapolation, at the same time reducing the computational effort. We remark, however, that the naive cost of including sea quarks into the update only scales like N^2 while the pure gauge operations scale like N^3: in the large-N limit not only the quenched theory becomes unitary and identical to the un-quenched theory but so also does the computational effort, which is quite substantial at $N = 17$.

Finally, we aim at clarifying a discrepancy between the results of refs. [10 – 12], which at large N favour a value of the vector meson mass close to that of real-world QCD, and those obtained in ref. [13], reporting a value approximately twice as large.

2. Simulation details

Our simulation strategy is to tune the lattice couplings, keeping the square root of the string tension $\sqrt{\sigma}a \approx 0.2093$ in lattice units a fixed. We employ the Wilson gauge and fermionic action and are not yet in the position to perform a continuum limit extrapolation. Lattice artefact terms will have the same functional large-N scaling as the dominant continuum limit terms and hence basically should not affect the size of $1/N^2$ corrections. Our experience with the present action [10, 11] leads us to expect systematic errors on mass ratios of about 5 % at the present lattice spacing.

We define the decay constant of a meson X in the large N limit as

$$F_X^\infty = \lim_{N\to\infty} \sqrt{\frac{3}{N}} F_X \,, \tag{2.1}$$

where $F_X = f_X/\sqrt{2}$. We distinguish between $F_\pi \approx 92$ MeV at physical quark masses and $F = F_\pi(m_q = 0)$.

Using the *ad hoc* value $\sigma = 1$ GeV/fm, our lattice spacing corresponds to $a \approx 0.093$ fm or $a^{-1} \approx 2.1$ GeV. Strictly speaking, we can only predict ratios of dimensionful quantities. In the real world where experiments are performed, $n_f > 0$, $N = 3 \neq \infty$ and even the string tension is not well defined. This means that any absolute scale setting in physical units will be arbitrary and is just meant as a rough guide. Nevertheless, we notice that other ways of setting the scale give similar results, indicating that the $N = \infty$ world is not far removed from $N = 3$ QCD with sea quarks. For instance, in the chiral limit we find $F^\infty = 0.22(2)\sqrt{\sigma} = 96(9)$ MeV, in qualitative agreement with the real QCD value [14] $F = 85.9(1.2)$ MeV. Moreover, we obtain $m_\rho = 1.638(7)\sqrt{\sigma} = 728(3)$ MeV at large N, quite close to the experimental ρ-meson mass of 775 MeV. This is remarkable, in particular since this resonance has a decay width of almost 150 MeV.

The string tension was computed in ref. [15] for $N = 2, 3, 4, 6$ and 8 and in ref. [16] for $N = 5$ and 7. For $N = 17$ no value is known and our lattice volume (see below) is too small for a reliable determination from torelon correlators. Therefore, we estimate

$$\Lambda \approx a^{-1} \exp\left[-\frac{1}{2\beta_0\alpha(a^{-1})}\right] \left[\beta_0\alpha(a^{-1})\right]^{-\frac{\beta_1}{2\beta_0^2}} \left[1 + \frac{1}{2\beta_0^3}\left(\beta_1^2 - \beta_2^L\beta_0\right)\alpha(a^{-1})\right] \tag{2.2}$$

and extrapolate the Λ-parameter obtained at $N \leq 8$ for $a\sqrt{\sigma} = 0.2093$ as a polynomial in $1/N^2$ to $N = 17$. The lattice coupling is defined as $\beta = 2N^2/\lambda = N/(2\pi\alpha)$, where $\lambda = Ng^2$ is the 't Hooft parameter in the lattice scheme. We obtain the central value $\beta_{17} = 208.45^{+59}_{-29}$ from a $1/N^2$ fit to the $N \geq 6$ data, with systematics estimated by varying the fit range and allowing for a quartic term. Our β-values and simulated volumes are summarized in table 1. Note that the lattice 't Hooft couplings deviate by $1/N^2$ terms from a constant along our trajectory of fixed σa^2. We remark that incidentally ref. [13] simulated SU(17) at $\beta = 208.08$ which is almost identical to our value

N	$N_s^3 \times N_t$	β	λ	$10^5\kappa$	$n_{\rm conf}$
2	$16^3 \times 32$			14581, 14827, 15008, 15096	400
	$24^3 \times 48$	2.4645	3.246	14581, 14827, 15008, 15096, 15195.9 ,15249.6	200
	$32^3 \times 64$			14581, 14827, 15008, 15096, 15195.9 ,15249.6	100
3	$16^3 \times 32$			15002, 15220, 15380, 15458	200
	$24^3 \times 48$	6.0175	2.991	15002, 15220, 15380, 15458, 15563.8, 15613	200
	$32^3 \times 64$			15002, 15220, 15380, 15458, 15563.8, 15613	100
4	$16^3 \times 32$	11.028	2.902	15184, 15400, 15559, 15635	200
	$24^3 \times 48$			15184, 15400, 15559, 15635, 15717.3, 15764	200
5	$16^3 \times 32$	17.535	2.851	15205, 15426, 15592, 15658	200
	$24^3 \times 48$			15205, 15426, 15592, 15658, 15754.8, 15835.5	200
6	$16^3 \times 32$	25.452	2.829	15264, 15479, 15636, 15712	200
	$24^3 \times 48$			15264, 15479, 15636, 15712, 15805.1, 15884.5	200
7	$16^3 \times 32$	34.8343	2.813	15281.6, 15496.7, 15654.7, 15733.9	200
	$24^3 \times 48$			15281.6, 15496.7, 15654.7, 15733.9, 15827.3, 15906.2	200
17	$12^3 \times 24$	208.45	2.773	15298, 15521, 15684, 15755, 15853.1, 15931	80

Table 1: Simulation parameters: $n_{\rm conf}$ is the number of configurations analysed for each set of parameters. All configurations were separated by 200 combined heatbath and overrelaxation Monte Carlo sweeps and found to be effectively statistically independent.

$\beta_{17} = 208.45$, ruling out the hypothesis that our $N < 17$ simulation points are not in the "continuum phase of the large-N theory" [13]. In fact at our lattice spacing we will find the $7 \geq N \geq 3$ spectra to be in almost perfect agreement with the $N = 17$ results.

Different lattice sizes are investigated to exclude finite volume effects spoiling the large-N extrapolation, in particular at the lighter quark masses. As expected, these become irrelevant at large N, thereby justifying the use of a relatively small lattice at $N = 17$.

The so-called hopping parameter κ is related to the lattice quark mass m_q via

$$am_q = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right). \tag{2.3}$$

κ_c denotes the critical value, corresponding to a massless quark. The additive constant is given by $\kappa_c^{-1} = 8 + \mathscr{O}(\lambda)$ and we will determine this non-perturbatively. The κ-values shown in table 1 were selected to keep one set of pion masses approximately constant across the different SU(N) theories. We vary the "pion" mass down to $m_\pi/\sqrt{\sigma} \approx 0.5$ for groups with $N \geq 5$, and to $m_\pi/\sqrt{\sigma} \approx 0.75$ for $N < 5$. We also simulated a smaller quark mass for SU($N < 5$) but found significant numbers of "exceptional configurations" [17] (up to 15 % of the total); we leave these data out of this work. For $N = 5$, at the lowest quark mass, only two exceptional configurations were encountered that we removed from the analysis.

Our code is based on the Chroma suite [18], which we have adapted to work for generic N values. We compute correlation matrices between differently smeared interpolators, allowing us not only to extract the ground states but also giving us access to excitations in many channels. Details of the analysis can be found in ref. [19].

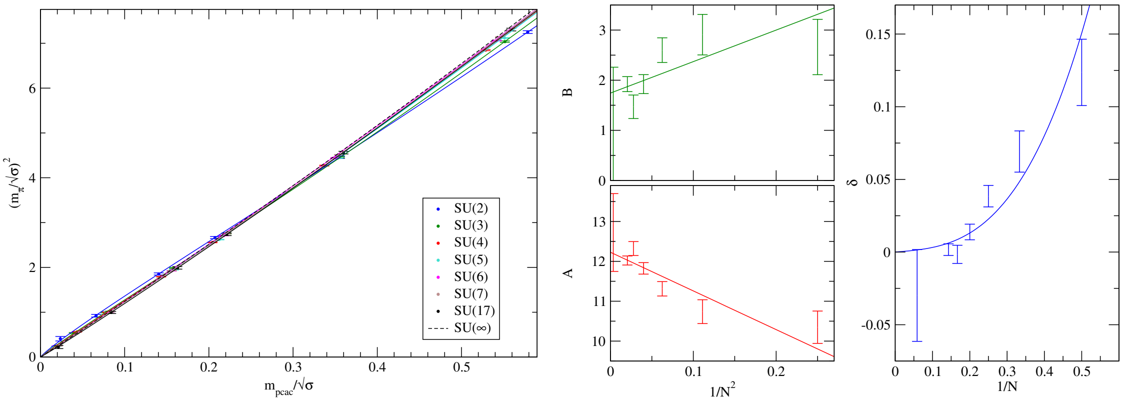

Figure 1: Pion mass vs. PCAC mass eq. (3.2) (left). N dependence of the fit parameters eq. (3.3) (right).

3. Results

We employ the lattice quark mass m_{PCAC}, defined through the axial Ward identity, as our reference mass, avoiding the additive renormalization (κ_c^{-1}) of the quark mass m_q defined in eq. (2.3). These two masses are related by a combination of renormalization constants,

$$a m_{\mathrm{PCAC}} = \frac{Z_P}{Z_A Z_S}\left(1 + b\, a m_{\mathrm{PCAC}} + \cdots\right)\frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right), \tag{3.1}$$

where the bam-term parameterizes the leading lattice correction. All three fit parameters κ_c, b and $Z_P/(Z_A Z_S)$ are well described by constants plus $1/N^2$-corrections. We find the latter ratio of renormalization constants to vary between 0.68 ($N = 2$) and 0.83 ($N = 17$), with the SU(3)-value 0.75, which is consistent with the non-perturbative result 0.81(7) [20] obtained at $\beta = 6.0$ — close to our value $\beta = 6.0175$. Motivated by the weak N-dependence of this result — which is also supported by perturbation theory — we will use the non-perturbative SU(3)-values for the renormalization factors of quark bilinears for all gauge groups, allowing for a 8 % systematic uncertainty, due to this approximation. These factors are needed to determine the decay constants below.

Next, we determine the dependence of the pseudoscalar mass m_π on the quark mass m_{PCAC}:

$$\frac{m_\pi^2}{\sigma} = A\left(\frac{m_{\mathrm{PCAC}}}{\sqrt{\sigma}}\right)^{\frac{1}{1+\delta}} + B\,\frac{m_{\mathrm{PCAC}}^2}{\sigma}\,. \tag{3.2}$$

A quenched chiral log is expected at small N-values, parameterized by δ. We also include a subleading term to prevent interference between the larger mass data and the chiral log. The fits for the different N are depicted in figure 1. δ is expected to be suppressed by a factor [5] $1/N$, with $1/N^3$ corrections. We find the parameter values

$$A = 12.23(0.10) - \frac{9.7(1.6)}{N^2}\,, \quad B = 1.74(0.13) + \frac{6.3(2.2)}{N^2}\,, \quad \delta = \frac{0.021(19)}{N} + \frac{1.12(21)}{N^3}\,, \tag{3.3}$$

see the right panel of figure 1. $\delta \gtrsim 0.05$ at $N = 3$ is coherent with expectations but the $1/N$ coefficient is statistically compatible with zero. In principle, the determination of δ may be obscured by

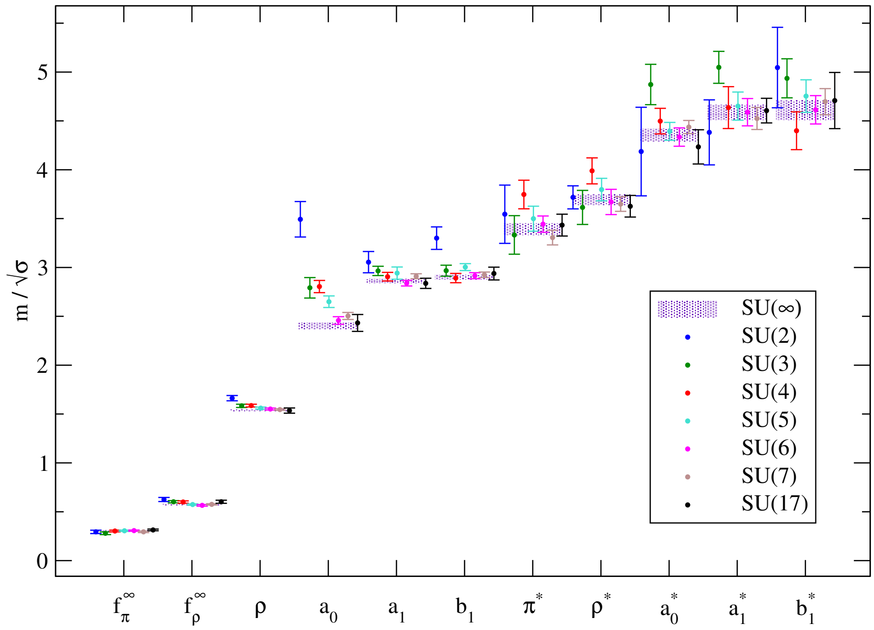

Figure 2: The spectrum, extrapolated to the chiral limit. The error bands correspond to $N = \infty$.

the possibility of a non-zero pion mass at $m_{\rm PCAC} = 0$, at finite lattice spacings. However, the linear disappearance of $m_\pi^2 \propto m_{\rm PCAC}$ for $N \geq 5$ indicates chiral symmetry to be broken only mildly.

We choose to parameterize the quark mass dependence of all our results through $m_{\rm PCAC}$ which can be determined more precisely and reliably than the pseudoscalar mass m_π. The relation $m_\pi^2(m_{\rm PCAC})$ above enables the translation of a functional dependence on $m_{\rm PCAC}$ into a dependence on m_π. Nevertheless, we quote the $N \to \infty$ result

$$\frac{m_\rho(m_\pi)}{m_\rho(0)} = 1 + 0.375(64)\left(\frac{m_\pi}{m_\rho(0)}\right)^2 + \cdots\,, \tag{3.4}$$

to allow for a direct comparison with the prediction, e.g., of ref. [21].

We display the chirally extrapolated spectrum in figure 2 for the different SU(N) groups. The $N \to \infty$ values are shown as horizontal error bands. Note that the $N = 17$ values are perfectly consistent with the $N < 17$ results, ruling out the twice as large ρ-meson mass obtained at almost the same lattice coupling in ref. [13]. This may be an artefact of the method used in that reference, e.g., due to excited state pollutions.

The figure does not only give the masses of the excited and ground state mesons but also illustrates the decay constants f_π and f_ρ to show the expected scaling behaviour $\propto \sqrt{N}$, with small corrections. The analysis of the scalar (a_0) correlation function at low N is complicated by the presence of ghost states, due to the unitarity violation of the quenched model. Subtracting these contributions results in large errors.

Of phenomenological interest is not only the spectrum at $m_q = 0$ but are also the spectra at $m_q = m_{ud}$ and at $m_q = m_s$ where m_{ud} and m_s denote the physical (isospin-averaged) light quark and strange quark masses, respectively. We fix the former, imposing the value [14]

$$\frac{F_\pi(m_{ud})}{F} = 1.073(15)\,, \tag{3.5}$$

		$m_\infty/\sqrt{\sigma}$			m_∞/F^∞		
Particle	J^{PC}	$m_q=0$	$m_q=m_{ud}$	$m_q=m_s$	$m_q=0$	$m_q=m_{ud}$	$m_q=m_s$
π	0^{-+}	0	0.417(100)	1.62(10)	0	1.92(46)	7.46(48)
ρ	1^{--}	1.5382(65)	1.6382(66)	1.9130(79)	7.08(10)	7.54(11)	8.80(13)
a_0	0^{++}	2.401(31)	2.493(31)	2.755(32)	11.04(21)	11.47(22)	12.67(23)
a_1	1^{++}	2.860(21)	2.938(21)	3.158(22)	13.16(21)	13.51(21)	14.53(23)
b_1	1^{+-}	2.901(23)	2.978(23)	3.197(23)	13.35(21)	13.70(22)	14.71(23)
π^*	0^{-+}	3.392(57)	3.462(57)	3.659(58)	15.61(34)	15.93(35)	16.83(36)
ρ^*	1^{--}	3.696(54)	3.756(54)	3.928(54)	17.00(34)	17.28(35)	18.07(36)
a_0^*	0^{++}	4.356(65)	4.420(65)	4.603(66)	20.04(41)	20.33(41)	21.18(42)
a_1^*	1^{++}	4.587(75)	4.646(75)	4.816(77)	21.10(46)	21.38(46)	22.15(47)
b_1^*	1^{+-}	4.609(99)	4.673(99)	4.85(10)	21.20(54)	21.50(55)	22.33(56)
f_π^∞	—	0.3074(43)	0.3271(44)	0.3784(56)	$\sqrt{2}$	1.505(29)	1.741(36)
f_ρ^∞	—	0.5721(49)	0.5855(50)	0.6196(64)	2.632(43)	2.694(44)	2.850(50)

Table 2: The $N=\infty$ meson spectrum and decay constants in units of the square root of the string tension $\sqrt{\sigma}$ and in units of the (normalized) chiral pion decay constant $F_\infty = F\sqrt{3/N}$ for three different values of the quark mass. A systematic error of 5 % needs to be added, due to the missing continuum limit extrapolation. Because of the non-perturbative $N=3$ rather than $N=\infty$ renormalization, an extra 8 % error should be added to the last three columns and to the last two rows of the table, with the exception of the f_π^∞/F^∞-ratios where this factor cancels.

at $N=3$, keeping $m_{ud}/\sqrt{\sigma}$ constant for $N\neq 3$. The renormalization constant and N-dependence cancel from the ratio. The strange quark mass is obtained by fixing the ratio of a (fictitious) strange-antistrange pion over the $\varphi(1020)$ vector particle at $N=3$ to the experimental value

$$\frac{m_\pi(m_s)}{m_\rho(m_s)} = \frac{686.9}{1019.5}, \tag{3.6}$$

where $(m_{K^\pm}^2 + m_{K^0}^2 - m_{\pi^\pm}^2)^{1/2} \approx 686.9\,\mathrm{MeV}$. We display the results at the different quark masses in table 2. The normalization in units of F^∞ should be particularly useful for chiral perturbation theory applications [6, 7].

4. Summary

We have determined the decay constants as well as the ground and first excited state masses of mesons in the large-N limit of QCD by lattice simulation of the $N=2,3,4,5,6,7$ and 17 quenched theories. In almost all but the scalar channels $1/N^2$ corrections are found to be tiny for $N\geq 3$. In a forthcoming publication [19] we will compare our findings to model expectations. We find the scalar to be by factors of about 1.5 heavier than the vector particle at any quark mass smaller than the strange quark, which is of particular relevance to the phenomenology of scalar mesons.

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