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New results from the lattice on the theoretical inputs to the hadronic τ determination of V_{us}

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Recent sum rule determinations of $|V_{us}|$, employing flavor-breaking combinations of hadronic τ decay data, are significantly lower than either expectations based on 3-family unitarity or determinations from $K_{\ell 3}$ and $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$. We use lattice data to investigate the accuracy/reliability of the OPE representation of the flavor-breaking correlator combination entering the τ decay analyses. The behavior of an alternate correlator combination, constructed to reduce problems associated with the slow convergence of the $D = 2$ OPE series, and entering an alternate sum rule requiring both electroproduction cross-section and hadronic τ decay data, is also investigated. Preliminary updates of both analyses, with the lessons learned from the lattice data in mind, are also presented.

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1. Background

The determination of $|V_{us}|$ from analyses of flavor-breaking (FB) combinations of hadronic τ decay data [1, 2] proceeds via finite energy sum rules (FESRs), generically

$$\int_0^{s_0} w(s)\rho(s)\,ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} w(s)\Pi(s)\,ds\ , \tag{1.1}$$

the $|V_{us}|$ determination involving the FB difference $\Delta\Pi_\tau \equiv \left[\Pi^{(0+1)}_{V+A;ud} - \Pi^{(0+1)}_{V+A;us}\right]$, with $\Pi^{(J)}_{V/A;ij}(s)$ the spin $J=0,1$ components of the flavor ij, vector (V) or axial vector (A) current-current 2-point functions. The spectral functions, $\rho^{(J)}_{V/A;ij}$, of $\Pi^{(J)}_{V/A;ij}(s)$, and hence that, $\Delta\rho_\tau$, of $\Delta\Pi_\tau$, are related to the normalized differential distributions, $dR_{V/A;ij}/ds$, of flavor ij V- or A-current-induced τ decay widths, $R_{V/A;ij} \equiv \Gamma[\tau^- \to \nu_\tau\,\mathrm{hadrons}_{V/A;ij}(\gamma)]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]$, by [3]

$$\frac{dR_{V/A;ij}}{ds} = c^{EW}_\tau |V_{ij}|^2 \left[w_\tau(s)\rho^{(0+1)}_{V/A;ij}(s) - w_L(s)\rho^{(0)}_{V/A;ij}(s)\right] \tag{1.2}$$

with $w_\tau(s)$, $w_L(s)$ and c^{EW}_τ all known, and V_{ij} the flavor ij CKM matrix element. With $|V_{ud}|$ from other sources, $\Delta\rho_\tau(s)$ is expressible in terms of experimental data and $|V_{us}|$. $|V_{us}|$ is then obtained by using the OPE for $\Delta\Pi_\tau$ on the RHS and data on the LHS of Eq. (1.1).

The use of FESRs involving the $J=0+1$ combination $\Delta\Pi_\tau$ is necessitated by the very bad behavior of the integrated $J=0$, $D=2$ OPE series at scales kinematically accessible in τ decay [4]. Fortunately, the dominant such $dR_{ud,us}/ds$ contributions are from the π and K poles, whose strengths are accurately known. The remaining $J=0$ contributions, which are doubly chirally suppressed, are obtainable phenomenologically [5, 6]. With $J=0$ contributions subtracted from $dR_{ud,us}/ds$, one obtains $\rho^{(0+1)}_{V/A;ud,us}(s)$, allowing the LHS of Eq. (1.1) to be formed for any $w(s)$ and s_0. Defining the re-weighted $J=0+1$ spectral integrals $R^w_{V+A;ij}(s_0) = \int_0^{s_0} ds\, w(s) dR^{(0+1)}_{V+A;ij}(s)/ds$,

$$|V_{us}| = \sqrt{R^w_{V+A;us}(s_0)/\left[\frac{R^w_{V+A;ud}(s_0)}{|V_{ud}|^2} - \delta R^{w,OPE}_{V+A}(s_0)\right]}\ , \tag{1.3}$$

where $\delta R^w_{V+A}(s_0) = \frac{R^w_{V+A;ud}(s_0)}{|V_{ud}|^2} - \frac{R^w_{V+A;us}(s_0)}{|V_{us}|^2}$. $|V_{us}|$ should be independent of $w(s)$ and s_0, providing tests of the reliability of the OPE treatment and input data employed. Recent determinations [8], which yield $|V_{us}| \sim 3\sigma$ lower than 3-family-unitarity expectations[1], show non-trivial $w(s)$- and s_0-dependence, suggesting shortcomings in the experimental data and/or OPE representation.

Quantifying the OPE uncertainty and, from this, the theoretical error on $|V_{us}|$, is complicated by the slow convergence, at the correlator level, of the leading $D=2$ OPE series $[\Delta\Pi_\tau]^{OPE}_{D=2}$. To four loops, with $\bar{a} = \alpha_s(Q^2)/\pi$, and $\alpha_s(Q^2)$, $m_s(Q^2)$ the running coupling and strange quark mass in the $\overline{MS}$ scheme, and neglecting $m_{u,d}$ relative to m_s, one has, from Ref. [9][2]

$$\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}_{D=2} = \frac{3}{2\pi^2}\frac{m_s(Q^2)}{Q^2}\left[1 + \frac{7}{3}\bar{a} + 19.93\bar{a}^2 + 208.75\bar{a}^3 + d_4\bar{a}^4 + \cdots\right]\ . \tag{1.4}$$

[1] A recent update of the kinematic weight, $s_0 = m_\tau^2$ analysis, e.g., quotes the result $|V_{us}| = 0.2173(20)_{exp}(10)_{th}$ [7].

[2] We use the estimate $d_4 = 2378$ of Ref. [9] for the at-present-unknown 5-loop coefficient d_4.

Since $\bar{a}(m_\tau^2)\simeq 0.1$, convergence at the spacelike point on the contour $|s|=s_0$ is marginal at best, and conventional error estimates may significantly underestimate the $D=2$ truncation uncertainty. The alternate fixed-order (FOPT) and contour-improved (CIPT) schemes for the truncated integrated series[3], e.g., despite differing only by contributions beyond the common truncation order, yield $|V_{us}|$ whose difference not only significantly exceeds such estimates, but increases steadily (from ~ 0.0010 to ~ 0.0020) as one moves from 3- to 5-loop truncation.

With problems in the FB $\Delta\Pi_\tau$ FESRs due, to at least some extent, to slow $D=2$ OPE convergence, FESRs having reduced $D=2$ OPE contributions at the correlator level are highly desirable. In Ref. [11], FB combinations of $\Pi^{(0+1)}_{V/A;ud}$, $\Pi^{(0+1)}_{V+A;us}$ and the EM correlator, Π_{EM}, (whose spectral function, ρ_{EM}, is determined by the bare $e^+e^-\rightarrow$ *hadrons* cross-sections) were constructed having vanishing leading $O(\alpha_s^0)$ $D=2$ OPE contributions. The unique such combination in which $\Pi^{(0+1)}_{V+A;us}$ appears with the same normalization as in $\Delta\Pi_\tau$ is

$$\Delta\Pi_{\tau,EM}=9\Pi_{EM}-6\Pi^{(0+1)}_{V;ud}+\Delta\Pi_\tau \tag{1.5}$$

whose $D=2$ OPE series is

$$\frac{-3}{2\pi^2}\,\frac{\bar{m}_s}{Q^2}\left[\frac{1}{3}\bar{a}+4.38\bar{a}^2+44.9\bar{a}^3+\cdots\right]\ . \tag{1.6}$$

The higher order coefficients in this series are also significantly smaller than those for $\Delta\Pi_\tau$. The $D=4$ series is also, fortuitously, suppressed, the results of Ref. [10] leading to the form

$$\frac{m_s\langle\bar{s}s\rangle - m_\ell\langle\bar{\ell}\ell\rangle}{Q^4}\sum_k c_k\bar{a}^k\ , \tag{1.7}$$

with $(c_0,c_1,c_2)=(-2,-2,-26/3)$ for $\Delta\Pi_\tau$ and $(0,8/3,59/3)$ for $\Delta\Pi_{\tau,EM}$. The analogue of Eq. (1.3) for $|V_{us}|$, based on the $\Delta\Pi_{\tau,EM}$ rather than $\Delta\Pi_\tau$ FESR, is thus expected to have a much smaller OPE contribution, and hence much reduced theoretical uncertainty. Lattice data will be used to check whether or not this expected OPE suppression is realized below.

2. Lattice vs OPE results for $\Delta\Pi_\tau$ and $\Delta\Pi_{\tau,EM}$

The $\Pi^{(J)}_{V/A;ud}(Q^2)$ for spacelike $Q^2=-q^2>0$ also enter the decomposition of the Euclidean space V and A 2-point functions, and hence are measurable on the lattice. We report here on comparisons of OPE expectations for the combinations $\Delta\Pi_\tau(Q^2)$ and $\Delta\Pi_{\tau,EM}(Q^2)$ with results obtained on $n_f=2+1$ domain wall fermion ensembles with $1/a=1.37$ *GeV* and $m_\pi=171$ and 248 *MeV*. Full details of the simulations are given in Ref. [12]. Expanded comparisons to results from finer $1/a=2.28$ *GeV*, $m_\pi=289$, 345 and 394 *MeV* ensembles [13], will be considered elsewhere. Here, by keeping momentum components $\leq 1/8^{th}$ of the lattice maximum, $Q^2\sim 4.6\ GeV^2$ can be reached. The OPE-lattice comparisons are designed to explore (i) the accuracy of the OPE representation for different $D=2$ truncation orders, (ii) the question of whether the fixed-scale or

[3]In FOPT, one first integrates with fixed renormalization scale μ, then resums logs through the "fixed-scale" choice $\mu^2=s_0$; in CIPT logs are instead resummed point by point along the contour before integration via the "local-scale" choice $\mu^2=Q^2$.

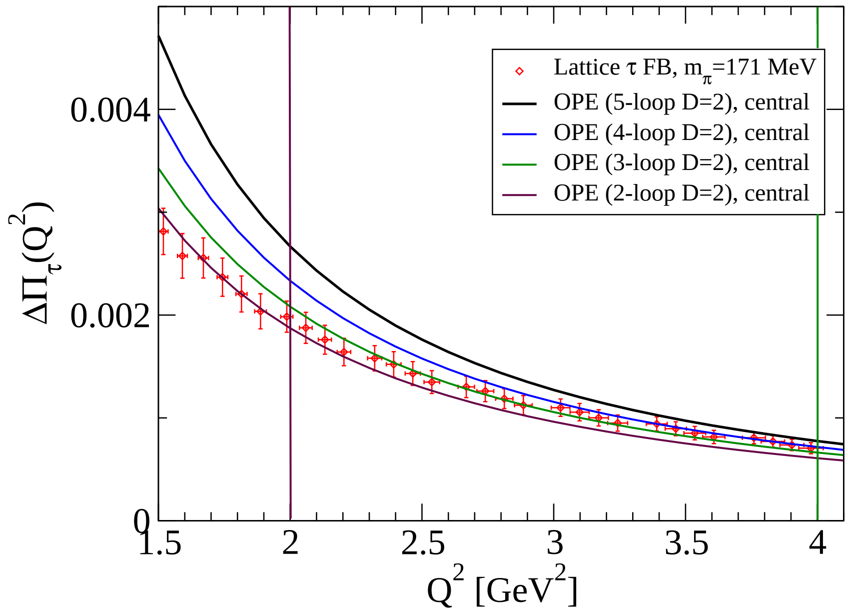

Figure 1: Lattice data vs. the OPE for $\Delta\Pi_\tau(Q^2)$

local-scale representation best describes the Q^2-dependence of the lattice data, and (iii) whether the data bears out the strong suppression of $\Delta\Pi_{\tau,EM}$ relative to $\Delta\Pi_\tau$ suggested by the truncated OPE representations. Since results for $m_\pi = 171, 248\ MeV$ are qualitatively identical, we show results for the former only.

We begin with the lattice-OPE comparison for the case where the dominant $D = 2$ OPE contribution is evaluated using the local-scale prescription (the analogue of the FESR CIPT prescription, used in essentially all FESR $|V_{us}|$ determinations in the literature). Fig. 1 shows the OPE results for 2-, 3-, 4- and 5-loop $D = 2$ truncation. An apparently asymptotic behavior is found for the integrated $D = 2$ OPE series, the terms decreasing in magnitude with increasing order until the smallest term is reached, and increasing in magnitude thereafter. To the right of the right-most vertical line, the 4-loop contribution is smallest, and, interpreting the behavior as that of a conventional asymptotic series, 4-loop truncation would be favored. Between the two vertical lines the 3-loop contribution is smallest, favoring 3-loop truncation, while to the left of the left-most vertical line, where it is the 2-loop contribution which is smallest, 2-loop truncation is favored.

In Fig. 2, we compare the Q^2-dependences of the lattice data and OPE representation, the $D = 2$ contribution to the latter being truncated at 4-loops and evaluated using both local-scale and fixed-scale prescriptions. For the fixed-scale case, we use $\mu^2 = 4\ GeV^2$. The lattice data is evidently represented considerably better by the fixed-scale version (whose use generates the FOPT version of the FESR integrals).

Fig. 3 compares the lattice data and OPE representation for $\Delta\Pi_{\tau,EM}(Q^2)$, with the analogous

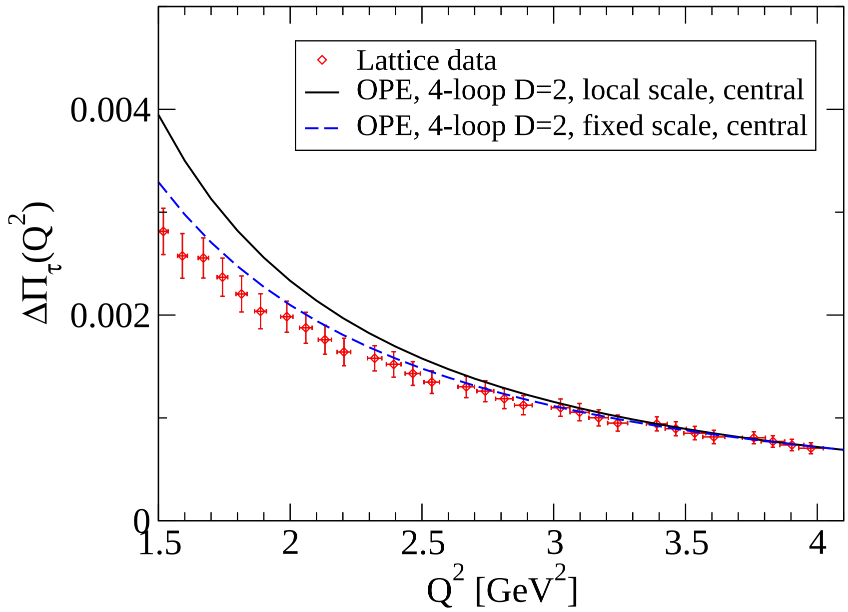

Figure 2: Lattice data vs. the OPE with either fixed-scale or local-scale versions of the $D=2$ contribution to $\Delta\Pi_\tau(Q^2)$

$\Delta\Pi_\tau(Q^2)$ results included for comparison. The lattice data clearly confirms the strong suppression in $\Delta\Pi_{\tau,EM}$ relative to $\Delta\Pi_\tau$ suggested by the OPE representation, the numerical extent of the suppression being even greater than suggested by the central OPE result. One thus expects very small theoretical errors on the $|V_{us}|$ obtained from mixed τ-electroproduction FESRs.

We now perform preliminary updates of the $\Delta\Pi_\tau$ and $\Delta\Pi_{\tau,EM}$ FESR determinations of $|V_{us}|$, taking the lessons provided by the above comparisons into account. The flavor *us* V+A τ data used are obtained by rescaling the old ALEPH [14] results mode-by-mode for subsequent changes in the exclusive branching fractions. The updated branching fractions are those from the unitarity-constrained HFAG fit incorporating also $K_{\mu 2}$ and $\pi_{\mu 2}$ input, discussed in Ref. [15], further updated for the $B[\tau \to K^- n\pi^0 \nu_\tau]$ results of Ref. [16]. For the flavor *ud* V and A distributions, we employ the update of the OPAL distribution [18] detailed in Ref. [15], further modified by a small common global V and A rescaling, needed to restore unitarity after inclusion of the new $B[\tau \to K^- n\pi^0 \nu_\tau]$ results. This interim global rescaling will be replaced by a further-updated mode-by-mode rescaling once the results of Ref. [16] are finalized and can be incorporated into the global HFAG branching function fit. While details of the electroproduction cross-section data employed will be given elsewhere, we mention that the tension between τ and electroproduction results for the $\pi\pi$ contribution to $\rho^{I=1}_{EM}(s)$ is assumed to be accounted for by the long-distance EM $\rho - \gamma$ mixing effect identified in Ref. [17], implying that the τ $\pi\pi$ data is to be used for the $\pi\pi$ contribution to the $\Delta\Pi_{\tau,EM}$ FESR, where the effect of this long-distance EM contribution is not accounted for on the OPE side.

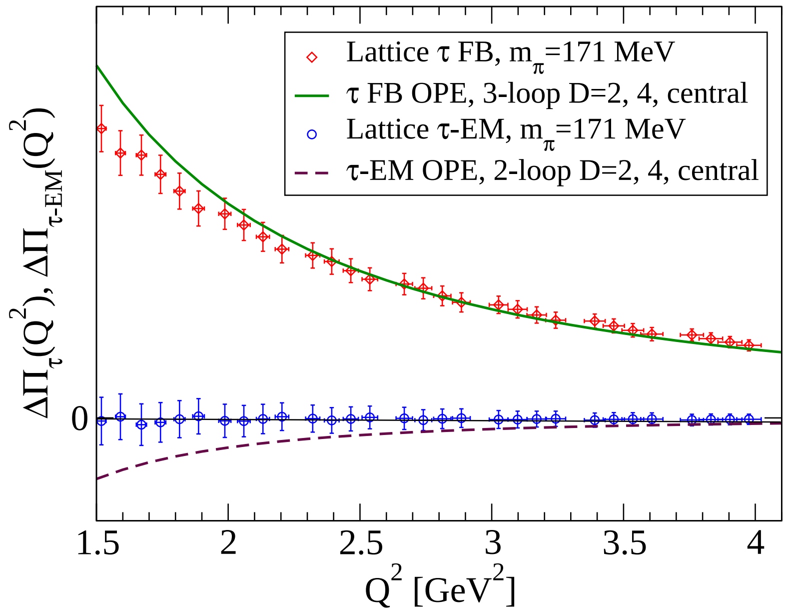

Figure 3: Lattice data vs. the OPE for $\Delta\Pi_\tau(Q^2)$, $\Delta\Pi_{\tau\ ,EM}(Q^2)$

For the $\Delta\Pi_\tau$ analysis, we employ the 3-loop $D=2$ truncation favored by the lattice data. The results obtained from FESRs involving the kinematic weight, w_τ, and two other weights used previously in the literature[4] are shown in Fig. 4, for both CIPT and FOPT prescriptions, though it is the latter which is, in fact, favored. For CIPT, we show results obtained using both the integrated correlator and same-order-truncated Adler function forms (the latter obtained by partial integration and re-truncation before integration). These again differ only by contributions beyond the common truncation order. For $s_0 = m_\tau^2$, $w = w_\tau$, shifting from the 5-loop-truncated $D=2$ CIPT+correlator to the 3-loop-truncated FOPT prescription favored by the lattice data raises $|V_{us}|$ by 0.0017. Significant $w(s)$-dependence, and, for w_τ, significant s_0-dependence, are evidently present, though the latter is reduced when FOPT, rather than CIPT, is used for the integrated $D=2$ series. These effects produce a contribution to the theoretical systematic uncertainty on $|V_{us}|$ already much larger than the total estimated theoretical uncertainty reported previously in the literature.

Fig. 5 shows the results for $|V_{us}|$ obtained from $\Delta\Pi_{\tau EM}$ FESRs employing a number of weights used in the earlier literature [19, 20]. There are two curves for each weight, one corresponding to an analysis in which (in keeping with the lattice results) OPE contributions are treated as negligible, one to an analysis using the 2-loop-truncated version of the $D=2$ OPE series. The results show much weaker s_0- and $w(s)$-dependence, and are in excellent agreement with the expectations of 3-family unitarity. We emphasize that these results are preliminary, and require further updating

[4] w_{20} was constructed to improve integrated $D=2$ CIPT convergence [19]; w_2 is a member of the family $w_N(y) = 1 - \frac{N}{N-1}y + \frac{1}{N-1}y^N$ constructed to keep higher D OPE contributions under control [20].

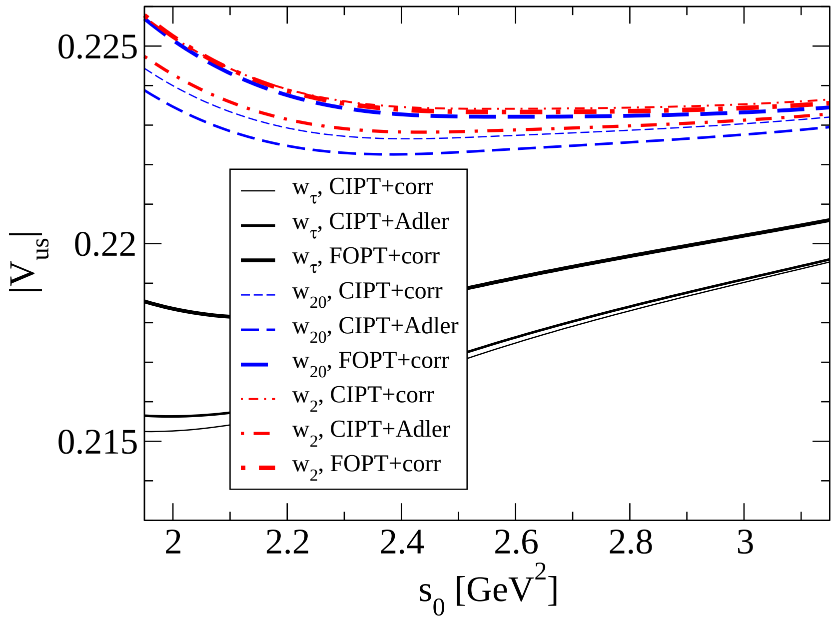

Figure 4: V_{us} vs. s_0 for the τ FB sum rule

once improved versions of the input experimental data become available.

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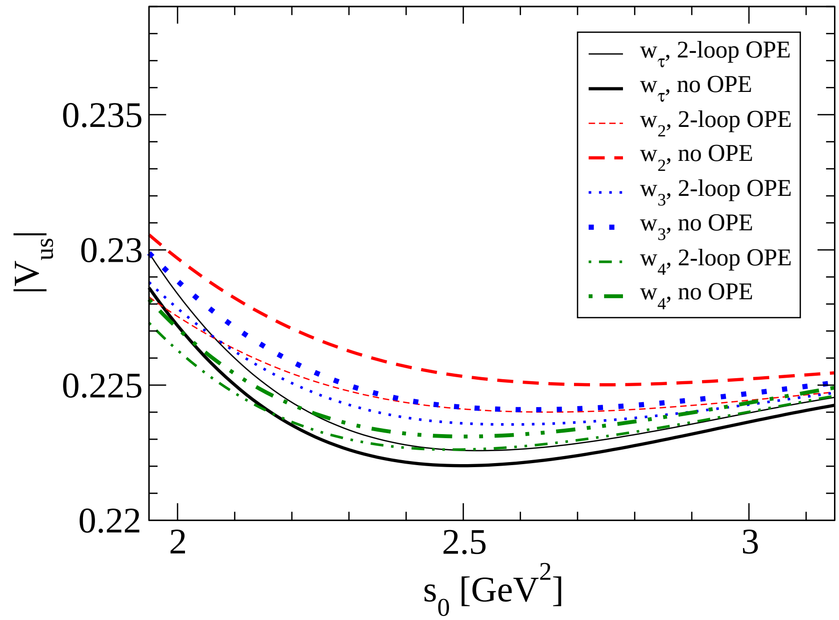

Figure 5: V_{us} vs. s_0 for the mixed τ-EM sum rule

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