Amalgamation of Transition Sequences in the PEPA Formalism

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Abstract

This report presents a formal approach to reducing sequences in PEPA components. Performing the described amalgamation procedure we may remove, from the underlying Markov chain, those states for which local detailed balance equations cannot be formulated. This transformation may lead to a simpler model with product form solution. Some classes of reduced models preserve those performance measures which we are interested in and, moreover, the steady state solution is much easier computationally.

Keywords

PEPA, decomposability, reversible Markov chains, insensitive stochastic processes

1 Introduction

Construction and solution of a large Markov model of a real system is not a trivial task since the number of reachable states is very large and transitions between them do not follow an easily predictable pattern. The modeller faces the problem of creating and solving an infinitesimal generator matrix Q whilst coping with limited time/space computational resources and numerical precision. Using appropriate tools, such as Stochastic Process Algebras (for example: PEPA [5], TIPP [4], EMPA [1]) or the Stochastic Automata Network method [10], may help decompose the modelling task and express the model as a multidimensional Markov chain. Each of these methods lacks the versatility to deal with an arbitrary model, but each can treat some classes of models efficiently. The advantage of the modular approach to defining a Markov chain may be exploited not only when writing its description (the modeller can look at each sub-model separately) but

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also when solving it because possible state reductions, or even calculations, may be performed for processes smaller than the global one.

A class of models for which compositionality can be fully exploited are the so-called *product form* models. Such models are expressed as interactions of simple submodels and their steady state solution is a function of partial solutions of these elementary components. At the Markov chain level, fluxes between states fulfil local balance equations, which are more convenient to formulate and to solve than global balance equations, but which place restrictions on how models may be constructed. For example, for reversible models [8], detailed balance must exist between each pair of states, i.e. for every pair of states x_i and x_j, $\pi(x_i)q_{i,j} = \pi(x_j)q_{j,i}$, where $\pi(\cdot)$ is the steady state probability and $q_{i,j}$ is the instantaneous transition rate. This subclass of product form models has been characterised syntactically in PEPA [6], meaning that the satisfaction of the detailed balance condition can be recognised without recourse to the global state space. However, the restrictions result in structures which are rarely encountered when modelling real systems.

Nevertheless, product form models are also useful as targets for transformation and given a particular performance measure, a "close" product form one may be substituted to bound the desired measure. In [3], van Dijk presents his approach towards transforming an initial Markov chain into one which has product form solution and for which a chosen performance measure is bounded from above or below. In [11] this approach is applied to some examples, which do not have product form solution, in order to find upper and lower bounds of an arbitrary chosen performance measure, using reversible PEPA models as the target for transformation. In this paper we give a more formal description of transforming a Markov chain into another, with product form solution, at the PEPA component level. Our goal is to remove Markov chain states for which detailed local balance equations are not satisfied and to bound performance measures using the new Markov chain.

The next section contains a short description of the PEPA formalism. Section 3 contains descriptions of formal tools we will use to perform activity amalgamation and discusses how the amalgamation procedure affects steady state probabilities in some cases. An extended version of this paper is presented in [7].

2 Outline of the PEPA Formalism

Entities of Performance Evaluation Process Algebra (PEPA) [5] are termed components. A component P can perform action a, $a = (\alpha, r)$, $a \in Act(P)$, where α is an activity type, $\alpha \in A(P)$, and r is a transition rate according to an exponential distribution, $r \in \mathsf{R}^+$, and R^+ is a set of positive real numbers together with the symbol $\top$ indicating unspecified transition rate. The set of types enabled in all derivatives of P, i.e. within the life cycle of P, is termed $\vec{A}(P)$. Components can interact by the use of the PEPA operations: $(\alpha, r).P$, $P + Q$, $P \underset{L}{\bowtie} Q$, P/H, $\stackrel{def}{=}$, where $L \subseteq A(P)$ is a set of types over which cooperation between components is

performed, $H \subseteq A(P)$ is a set of types which are replaced in the component P by the undefined type τ. The listed PEPA combinators are named: prefix, choice, cooperation, hiding, definitional equality, respectively. Alternatively, $P \underset{\emptyset}{\bowtie} Q$ may be written as $P \parallel Q$. Details of the PEPA semantics are presented in [5].

We now define some terminology which we will use later in the paper.

Definition 1 *The* apparent rate *of action type* α *in component* P *is denoted* $r_\alpha(P)$ *and is given by:*

$r_\alpha((\beta,r).P) = \begin{cases} r, & \text{if } \alpha = \beta \\ 0, & \text{if } \alpha \neq \beta \end{cases}$	$r_\alpha(P+Q) = r_\alpha(P) + r_\alpha(Q)$
$r_\alpha(P/H) = \begin{cases} r_\alpha(P), & \text{if } \alpha \notin H \\ 0, & \text{if } \alpha \in H \end{cases}$	$r_\alpha(P \underset{L}{\bowtie} Q) = \begin{cases} r_\alpha(P) + r_\alpha(Q), & \text{if } \alpha \notin L \\ \min(r_\alpha(P), r_\alpha(Q)), & \text{if } \alpha \in L \end{cases}$

For cooperation, if α is in the cooperation set, the slowest participant determines the rate of the shared action.

Definition 2 *If* $P_0 \xrightarrow{(\alpha,r)} P_1$, *then* P_1 *is a (one-step)* derivative *of* P_0. *More generally, if* $P_0 \xrightarrow{(\alpha_0,r_0)} P_1 \cdots \xrightarrow{(\alpha_{n-1},r_{n-1})} P_n$, *then* P_n *is a* derivative *of* P_0.

Definition 3 *The* derivative set *of a PEPA component* P *is denoted* $\mathrm{ds}(P)$ *and defined as the smallest set of components such that if* $P \stackrel{\text{def}}{=} P_0$ *then* $P_0 \in \mathrm{ds}(P)$*; and if* $P_i \in \mathrm{ds}(P)$ *and there exists* $a \in Act(P_i)$ *such that* $P_i \xrightarrow{a} P_j$ *then* $P_j \in \mathrm{ds}(P)$.

Syntactically, PEPA components are divided into two groups, model components M and sequential components S (where X denotes a constant which is a sequential component):

$$S ::= (\alpha,r).S \mid S+S \mid X \qquad M ::= S \mid M \underset{L}{\bowtie} M \mid M/H$$

3 Amalgamation of Sequences

In this section we describe a transformation procedure for PEPA models. Firstly, we define *local sequences* in PEPA sequential components building up a model. Secondly, we describe how the activity type is chosen to label the amalgamated transition. Thirdly, we define a surjective function which describes the amalgamation procedure. Finally, we propose a classification, discuss computation of the new transition rate and give examples of local sequences according to the introduced classifying scheme.

3.1 Local Sequences inside a PEPA Component

Firstly, we define an internal structure of a PEPA component which destroys its potential reversibility.

Definition 4 *A pair of activity types* (α_P, α_S) *form a* local sequence *in a component* X_0 *iff, for all* X_{j_1}, X_{j_2}, *such that* $X_{j_1}, X_{j_2} \in \mathrm{ds}(X_0)$, $X_{j_1} \neq X_{j_2}$, $X_{j_1} \xrightarrow{(\alpha_P, r_{j_{12}})} X_{j_2}$ *and* X_{j_2} *is only an one-step derivative of* X_{j_1}, *there exists a unique* X_{j_3}, *such that* $X_{j_3} \in \mathrm{ds}(X_0)$, $X_{j_2} \neq X_{j_3}$, $X_{j_2} \xrightarrow{(\alpha_S, r_{j_{23}})} X_{j_3}$, X_{j_3} *is the only one-step derivative of* X_{j_2}.
In a local sequence (α_P, α_S), α_P is termed the *predecessor* and α_S the *successor*.

Figure 1 depicts a local sequence in X, (τ, α_S). In this case the undefined type τ is the predecessor and α_S is the successor. For the state X_k we cannot write a local balance equation and we do not want to include this state in our calculation. Thus we replace the component X by another, X', in which $X'_j \xrightarrow{(\xi, R)} X'_l$. The states X'_j and X'_l are images of X_j and X_l respectively in a transforming function. However, we have to decide what type ξ is and what is the transition rate.

3.2 Choice of Activity Type

The synchronising actions in a PEPA model cannot be of undefined type. If no reward is associated with the type of an independent action it may be hidden without loss of information. Thus, when only one type of the local sequence (α_P, α_S) is a cooperating one, we pick it as the type of the new transition.

In the case when both actions are either synchronised or independent we assume that the succeeding activity type of a sequence absorbs the preceding one. Our assumption is a direct result of an emphasis on observation.

Consider component X shown in Figure 1. Starting from state X_j, the component performs its internal action (τ, λ) and an observer cannot say of what type the activity is. When the internal action finishes, he can recognise the activity type of the consecutive action (assuming it is of "public" type). From his perspective, the change of state to X_k is of little interest. He may suppose that the time of passing from X_j to X_l has a distribution defined by a Coxian distribution equal to the convolution of the "private" and "public" exponential distributions with parameters λ and r_S, respectively and the type of this transition is the type of the visible transition, i.e. α_S. This reasoning suggests that the activity type ξ (Section 3.1) resulting from the amalgamation procedure should be the successor type α_S.

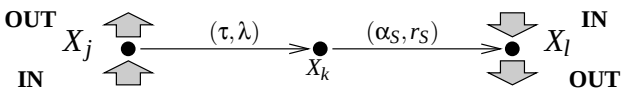


Figure 1: A local sequence in component X with internal type τ as predecessor

3.3 Amalgamation Procedure

To begin, we state precisely how "an internal element" (such as X_k in Figure 1), which will be removed due to the amalgamation procedure, may be identified.

Definition 5 *A derivative X^* is an* internal sequential derivative *of a local sequence (α_P, α_S) in X if its enabled activity multi-set is $\{|\,(\alpha_S, r_S)\,|\}$ and it is an α_P-derivative of a component whose enabled activity multi-set is $\{|\,(\alpha_P, r_P)\,|\}$.*

A function which amalgamates local sequences and removes their internal sequential derivative is defined as follows:

Definition 6 *A function $f : \mathrm{ds}(X) \longrightarrow \mathrm{ds}(X')$ is a* sequential epimorphism *from X to X' over a local sequence (α_P, α_S) if it is a surjective function such that*

1. *if $X^* \in \mathrm{ds}(X)$ and X^* is not an internal sequential derivative of local sequence (α_P, α_S), then X^* and $f(X^*)$ are identical,*
2. *for any $a \in Act(X^*)$, not part of local sequence (α_P, α_S), the set of a-derivatives of $f(X^*)$ equals the f-image of the set of a-derivatives of X^*,*
3. *for activities (α_P, r_P) and (α_S, r_S), such that $X_0^* \xrightarrow{(\alpha_P, r_P)} X_1^* \xrightarrow{(\alpha_S, r_S)} X_2^*$, there is exactly one derivative of $f(X_0^*)$, which may be reached by performing (α_S, R) such that $f(X_0^*) \xrightarrow{(\alpha_S, R)} f(X_2^*)$. The expected delay between $f(X_0^*)$ and $f(X_2^*)$ is the same as the expected delay between X_0^* and X_2^*.*

Sequential epimorphism is a generalisation of weak isomorphism defined in [5].

3.4 Classification of Sequences to be Amalgamated

Let $M = \triangle(X_0^{(i)})$, $i = 0, 1, \ldots, n-1$, be a PEPA model made up of n sequential components, the $X_0^{(i)}$. The $\triangle$ operator composes the $X_0^{(i)}$ using PEPA's composition and hiding combinators. Let α be an activity type which may occur in M, i.e. $\alpha \in \vec{A}(M)$. We use shorthands as stated below:

M: the set of all the components $X_0^{(i)}$, $i = 0, 1, \ldots, n-1$,

$M^{\alpha,\emptyset}$: the set of all components $X_0^{(i)}$ in which no action of type α occurs,

$M^{\alpha,*}$: the set of all components $X_0^{(i)}$ in which an action of type α may occur but these components do not cooperate over α,

M^{α}: the set of all components $X_0^{(i)}$ cooperating over the activity type α.

Clearly, $M = M^{\alpha,\emptyset} \cup M^{\alpha,*} \cup M^{\alpha}$ and $M^{\alpha,\emptyset} \cap M^{\alpha,*} = M^{\alpha,*} \cap M^{\alpha} = M^{\alpha,\emptyset} \cap M^{\alpha} = \emptyset$. Let a pair of activity types (α_P, α_S) be a local sequence in some sequential components $X_0^{(i)}$ of the model M. We restrict ourselves to the cases listed in Table 1 for which $M^{\alpha_P,\emptyset} = M^{\alpha_S,\emptyset} = \emptyset$. If these sets were not empty our classification would be still valid as long as their elements were cyclic components. We also assume that each sequence (α_P, α_S) appears at most once in each sequential component.

To remove ambiguity, in the context of the proposed classification, we give details of the reduction of local sequences in each case. In some cases, amalgamation is only possible when certain *boundary conditions* are met, i.e. all components involved can be guaranteed to enter or exit the local sequence simultaneously.

a)	$M^{\alpha_P,*} = M$	I	$M^{\alpha_S,*} = M$		
		II	$M^{\alpha_S} = M$		
		III	$M^{\alpha_S,*} \neq M \wedge M^{\alpha_S} \neq M$		
b)	$M^{\alpha_P} = M$	I	$M^{\alpha_S,*} = M$		
		II	$M^{\alpha_S} = M$		
		III	$M^{\alpha_S,*} \neq M \wedge M^{\alpha_S} \neq M$		
c)	$M^{\alpha_P,*} \neq M$ $\wedge$ $M^{\alpha_P} \neq M$	I	$M^{\alpha_S,*} = M$		
		II	$M^{\alpha_S} = M$		
		III	$M^{\alpha_S,*} \neq M$ $\wedge$ $M^{\alpha_S} \neq M$	$\star$	$M^{\alpha_P} = M^{\alpha_S}$
				$\star\star$	$M^{\alpha_P} \cap M^{\alpha_S} = \emptyset$
				$\star\star\star$	otherwise

Table 1: Possible manifestations of local sequences in the model M

Definition 7 *A local sequence* (α_P, α_S) *in component X, within model M, is a* beginning sequence *iff* α_P *is not a successor in any local sequence in X and a component whose* α_P*-derivative is an internal sequential derivative of* (α_P, α_S) *in X, is obtained only after cooperation by all components belonging to the set* M^{α_S}.

Definition 8 *A local sequence* (α_P, α_S) *in component X, within model M, is an* ending sequence *iff* α_S *is not a predecessor in any local sequence in X and from an* α_S*-derivative of an internal sequential component, all outgoing actions require cooperation of all components belonging to the set* M^{α_P}.

Based upon these two definitions, illustrated in Figure 2, we are able to define local sequences which are made up of exactly two actions labelled with activity types α_P and α_S, respectively.

Definition 9 *A local sequence* (α_P, α_S) *in X included in a model M is a* strict sequence *in M if it is a beginning and ending sequence in M.*

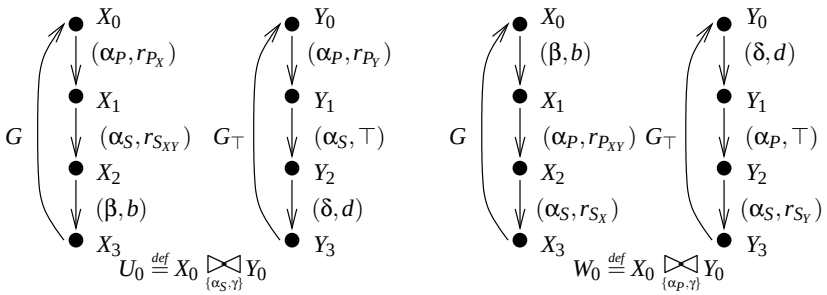


Figure 2: Examples of beginning ((α_P, α_S) in U_0) and ending ((α_P, α_S) in W_0) sequences; shorthands: $G = (\gamma, g)$, $G_\top = (\gamma, \top)$

Finally, we point out that we assume no reward function is associated with actions of type α_P or α_S for these actions will vanish during the amalgamation process. The choice of transition rate R will be discussed in the next subsection.

3.4.1 Phase-type Distribution

When calculating rates for amalgamated transitions we first attribute rates to α_P and α_S actions in the global model. Synchronised actions are performed simultaneously in a group of components, and will have a rate dictated by the semantics of cooperation. Other actions are executed independently and their instances interleave; they will have a rate which has a phase-type distribution. Such distributions are characterised by the time to absorption in a Markov chain, itself characterised by a matrix Q; *absorption* occurs when all copies of the action have completed. We construct the Markov chain (and the phase-type distribution) representing the interleaving as shown in Figure 3 for the 3-dimensional case. The state $(1,1,1)$ is its absorbing state. For n independent components, the matrix Q is obtained by a tensor (Kronecker) sum [2] $Q = \bigoplus_{i=0}^{n-1} Q^{(i)}$ which has structure: $Q = \left[\begin{array}{c|c} Q^* & q \\ \hline O & 0 \end{array}\right]$, where Q^* is an upper triangular matrix, $\dim(Q^*) = n-1$ and q is a column vector of $n-1$ elements. The distribution, $F(t)$, of the time to complete the interleavings

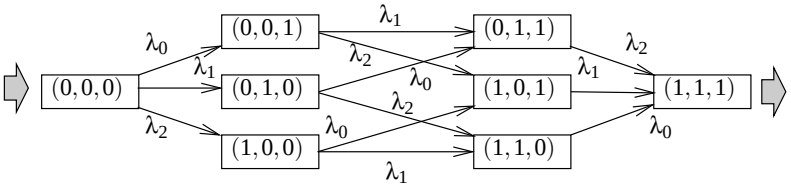


Figure 3: An example of PH-distribution, 3-lattice

in n components is equal to [9]:

$$F(t) = 1 - \alpha e^{Q^* t}\underline{1} \quad \text{for} \quad t \geq 0 \qquad \text{and} \quad F(t) = 0 \quad \text{for} \quad t < 0,$$

where $\underline{1}$ is a column vector of size $n-1$ all of whose elements are equal to 1 and α is a row vector containing initial probabilities of lattice states. The pair (α, Q^*) defines the phase-type distribution. We set the initial vector α to $(1,0,\ldots,0)$ since we always start from the state in which all instances of the action are newly enabled (state $(0,0,0)$ in Figure 3). The mean m of this distribution may be calculated by the formula

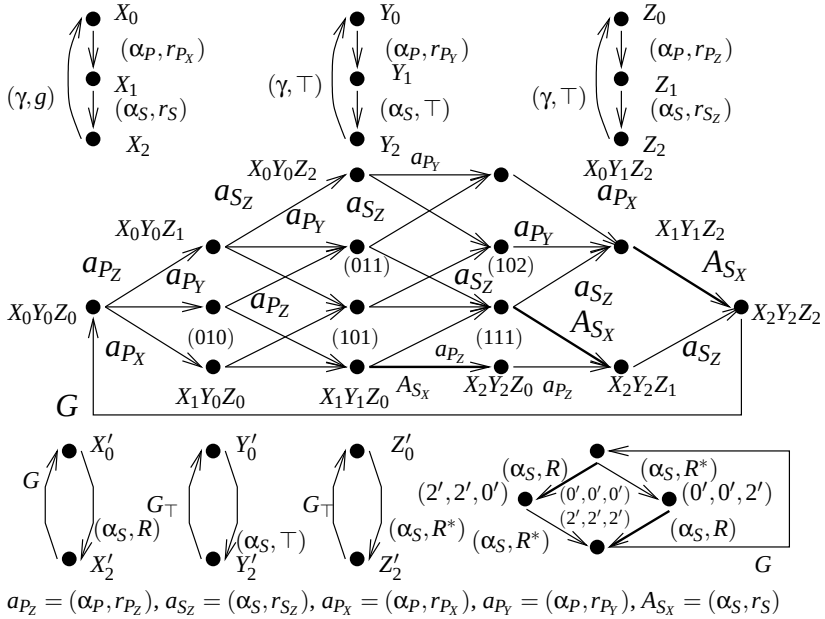
$$m = -\alpha(Q^*)^{-1}\underline{1}. \tag{1}$$

3.4.2 Review of Sequence Cases

We review the cases listed in Table 1. Diagrams illustrating the cases are complex and for this reason we apply some abbreviations, mostly concerning types of a local sequence. Independent actions performed in a component X are written as a_{P_X} or a_{S_X} depending on whether they are of α_P or α_S type. For synchronised actions we introduce similar shorthand terms, for example $A_{P_{XY}}$, $A_{S_{XUZ}}$, where the lowest subscript is a list of components which cooperate over either α_P or α_S type.

Case a.I: None of the α_P or α_S actions are synchronised. Consequently sequence amalgamation may be performed entirely at the PEPA component level. No "boundary condition" is required. (This coincides with weak isomorphism when both elements of the local sequence have type τ.) The new transition rate $R^{(i)}$ in a PEPA sequential component $X^{(i)}$ is equal to $\frac{r_P^{(i)} r_S^{(i)}}{r_P^{(i)} + r_S^{(i)}}$, $i = 0, 1, \ldots, n-1$.

Case a.II: All α_P actions are performed independently, all α_S actions are synchronised. Here we are restricted to beginning sequences only. The activities of type α_P form an n-lattice in the model. The average time $\frac{1}{\Lambda_P}$ of passing through this structure is given by Eq. (1). The effective cooperation rate R_S in the global chain is equal to $\min_i(r_S^{(i)})$. The effective rate of the replacement activity for the entire strict local sequence is equal to $R = \frac{R_S \Lambda_P}{R_S + \Lambda_P}$.



$a_{P_Z} = (\alpha_P, r_{P_Z})$, $a_{S_Z} = (\alpha_S, r_{S_Z})$, $a_{P_X} = (\alpha_P, r_{P_X})$, $a_{P_Y} = (\alpha_P, r_{P_Y})$, $A_{S_X} = (\alpha_S, r_S)$

Figure 4: An example of the **a.III** case, $X_0 \underset{\{\alpha_S, \gamma\}}{\bowtie} Y_0 \underset{\{\gamma\}}{\bowtie} Z_0$

Case a.III: All α_P actions are performed independently, some of the α_S actions are synchronised. Again, we restrict to beginning sequences. All local sequences are replaced by actions of type α_S. For components in M^{α_S} the new transition rate is computed as $\frac{\min_i(r_S^{(i)})\Lambda}{\min_i(r_S^{(i)}) + \Lambda}$, where Λ is an inversion of the mean passage time through the n-lattice formed by α_P actions from components in M^{α_S}. For components $X^{(i)}$ in $M - M^{\alpha_S}$ the amalgamation procedure is performed individually; the transition rates are $R^{(i)*} = \frac{r_{P_Z} r_{S_Z}}{r_{P_Z} + r_{S_Z}}$. (See example in Figure 4).

Case b.I: All α_P actions are synchronised and all α_S actions are independent. This case is dual to **a.II**, so it concerns ending sequences only: firstly an action of type α_P is performed in all the sequential components, secondly the components independently execute their own actions of type α_S. As for **a.II** we set the transition rate of the new action, R, as an inversion of the mean of the Coxian distribution of two exponential distributions with parameters R_P and Λ_S.

Case b.II: All model components cooperate over both α_P and α_S. Since both elements of the local sequence are synchronised they automatically establish "boundary conditions" for launching and terminating the execution of the sequence. Thus, in this case a local sequence may be amalgamated even if it is neither a beginning nor ending sequence. We assume that all sequential components in the model contain the local sequence and that when α_P is enabled in a derivative no other actions are also enabled. These conditions guarantee that the components must perform each element of the sequence with probability 1.0 (Figure 5). Under these conditions, and calculating R as above, we can show that

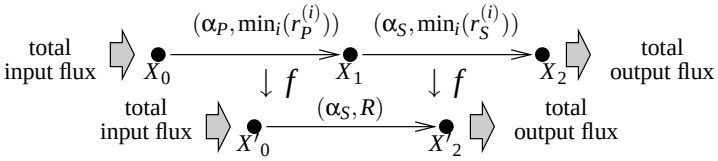


Figure 5: Amalgamation of a sequence for the **b.II** case, all components building up a model contain the local sequence (α_P, α_S); states of the global Markov chain, $X_j = (X_j^{(0)}, \ldots, X_j^{(N-1)})$, $j = 0,1,2$, $X'_k = (X'^{(0)}_k, \ldots, X'^{(N-1)}_k)$, $k = 0,2$

$\pi(X_0) + \pi(X_1) = \pi(X'_0)$, and all other steady state probabilities are preserved.

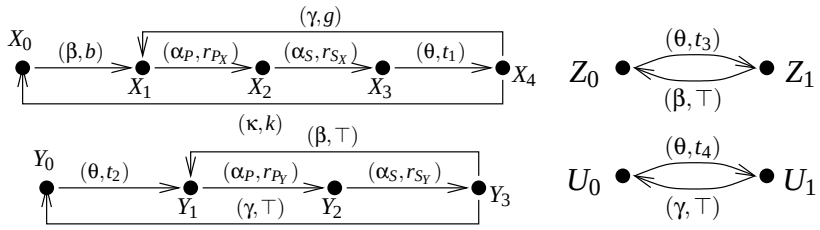


Figure 6: An example of the **b.II** model, $((X_1 \underset{\{\alpha_P,\alpha_S,\beta,\gamma\}}{\bowtie} Y_1) \underset{\{\ beta\}}{\bowtie} Z_0) \underset{\{\gamma\}}{\bowtie} U_0$

We have found that the reduced model preserves throughput of activities, other than the removed α_P, even if some components of the model do not contain the amalgamated local sequence (see [7] for details). Figure 6 shows an example model. The initial model consists of 24 states, the amalgamated one, 20. Assuming that all the initial model's transition rates are equal to 1.0 we calculate

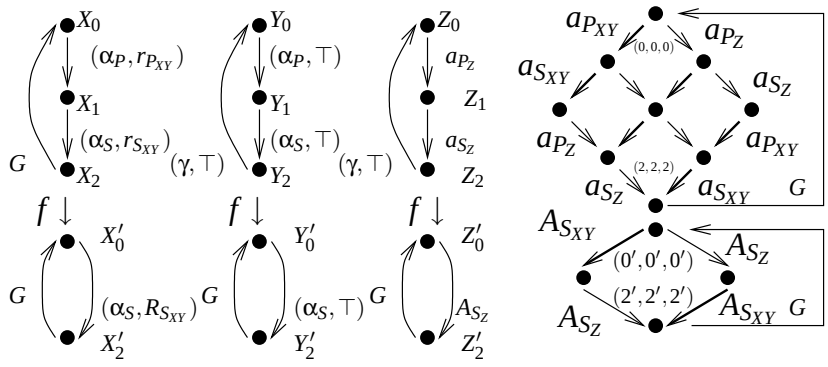
throughput of γ and α_S as: $\vartheta_\gamma = 0.11$ and $\vartheta_{\alpha_S} = 0.22$ both before and after reduction.

Case b.III: All α_P actions are synchronised and some α_S actions are synchronised. Amalgamation in this case may be performed for ending sequences only. For the submodel consisting of $X_0^{(i)}$ components in the set M^{α_P} set whose indices are the same as those of elements of the set M^{α_S}, the conditions of the **b.II** case are fulfilled. The other submodel, the components of which belong to $M^{\alpha_{S,*}}$, satisfies the conditions of the **b.I** case. The apparent rate of the synchronised α_P action is calculated from rates in the components $X^{(i)}$, $R_P = \min_i(r_P^{(i)})$. The rate of the synchronised α_S action is equal to $R_S = \min_{i:X_0^{(i)} \in M^{\alpha_S}}(r_S^{(i)})$. The total transition rate of the α_S action we find using Eq. (1) for α_S activities of $X_0^{(i)} \in M^{\alpha_{S,*}}$ together with the "cooperation rate" R_S.

Case c.I: Some α_P actions are synchronised, all α_S actions are performed independently. The amalgamation procedure may be performed for ending sequences only. In the components whose indices belong to M^{α_P} the new activity type is set as α_P, in the others as α_S. For the first group, the rate of the new action is $R = \frac{\min_i(r_P^{(i)})\Lambda}{\min_i(r_P^{(i)})+\Lambda}$, $X^{(i)} \in M^{\alpha_P}$, where Λ is the inversion of the mean computed according to formula (1). For the latter group the transition rates are evaluated for each component $X^{(i)}$ separately, $R^{(i)*} = \frac{r_P^{(i)} r_S^{(i)}}{r_P^{(i)}+r_S^{(i)}}$, $X^{(i)} \in M - M^{\alpha_P}$.

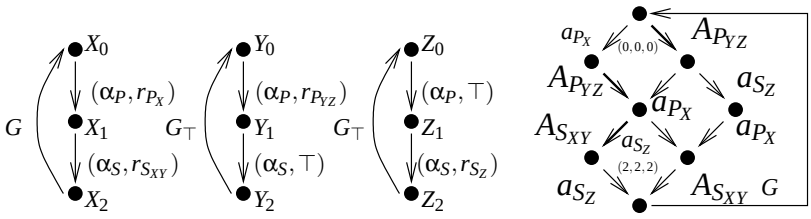
Case c.II: Some α_P actions are synchronised, all α_S actions are synchronised. This case consists of instances of the **a.II** and **b.I** cases so only beginning sequences may be amalgamated. The rate of the synchronised α_S action is equal to $R_S = \min_i(r_S^{(i)})$. The rate of the α_P action synchronised in components in M^{α_P} is $R_P = \min_i(r_P^{(i)})$ and this value is included in the computation of total transition rate of passing through the *n*-lattice, according to Eq. (1).

Case c.III: Some components of the model cooperate over α_P, some of them over α_S. This case has to be considered more carefully, by three sub-cases:
$\star$ If $M^{\alpha_P} = M^{\alpha_S}$ we deal with **a.I** and **b.II** applied to two disjoint sets of sequential components and amalgamation is possible when done independently (Fig. 7).
$\star\star$ If $M^{\alpha_P} \cap M^{\alpha_S} = \emptyset$ the model splits into two sub-models depending on the activity type, α_P or α_S, over which synchronisation is carried out. One of them fulfils case **a.II** conditions, the other those of **b.I**.
$\star\star\star$ Some model components cooperate only over α_P, some only over α_S, some over both types. Amalgamation is not possible as presented in Figure 8 by an example in which $M^{\alpha_P} = \{Y_0, Z_0\}$, $M^{\alpha_{P,*}} = \{X_0\}$, $M^{\alpha_S} = \{X_0, Y_0\}$, $M^{\alpha_{S,*}} = \{Z_0\}$.



$a_{P_Z} = (\alpha_P, r_{P_Z})$, $a_{S_Z} = (\alpha_S, r_{S_Z})$, $a_{P_{XY}} = (\alpha_P, r_{P_{XY}})$, $a_{S_{XY}} = (\alpha_S, r_{S_{XY}})$, $A_{S_{XY}} = (\alpha_S, R_{S_{XY}})$, $A_{S_Z} = (\alpha_S, R_{S_Z})$

Figure 7: An example of the **c.III.**$\star$ case, $(X_0 \underset{\{\alpha_P, \alpha_S, \gamma\}}{\bowtie} Y_0) \underset{\{\gamma\}}{\bowtie} Z_0$



$a_{P_X} = (\alpha_P, r_{P_X})$, $A_{P_{YZ}} = (\alpha_P, r_{P_{YZ}})$, $a_{S_Z} = (\alpha_S, r_{S_Z})$, $A_{S_{XY}} = (\alpha_S, r_{S_{XY}})$

Figure 8: An example of the **c.III.**$\star\star\star$ case, $(X_0 \underset{\{\alpha_S, \gamma\}}{\bowtie} Y_0) \underset{\{\alpha_P, \gamma\}}{\bowtie} Z_0$

3.4.3 Performance Values Results for the Reviewed Cases

Models of each case investigated in the previous subsection were computed in order to find throughput of transitions not involved in the amalgamation procedure. Depending on whether the underlying process was insensitive or not, the results were exact or approximate. In particular for our examples, cases **a.II, b.I, b.III, c.II** were found to be insensitive and results were exact, whereas cases **a.I, a.III, b.II, c.I, c.III.**$\star$**, c.III.**$\star\star$ led to approximate results (see [7]).

4 Conclusions and Further Work

In this paper we have investigated the amalgamation of sequences in PEPA components in order to obtain a smaller model whilst preserving some features of the original. The resulting action type is chosen, depending upon whether the amalgamated actions are synchronised or not and an observational assumption. We have defined *local sequences* suitable for amalgamation and a classification of their manifestations. For each case we gave an explanation of how to calculate the transition rate of the new action.

Sequence amalgamation may lead to a model with a reversible Markov chain.

Further investigation will focus on when the reduced model has this feature and could be used to efficiently compute bounds of some performance measures. We will also study when the reduced chain is insensitive leading to exact measures, relative to the original chain. From a theoretical standpoint, we would like to establish an equivalence relation based on the notion of *sequential epimorphism*.

References

[1] BERNARDO, M., AND GORRIERI, R. A Tutorial on EMPA: A Theory of Concurrent Processes with Nonderteminism, Probabilities and Time. In *Theoretical Computer Science* (July 1998).

[2] DAVIO, M. Kronecker Products and Shuffle Algebra. *IEEE Trans. on Comp. 30*, 2 (1981), 116–125.

[3] DIJK, N. *Queueing Networks and Product Forms: A Systems Approach*. John Wiley & Sons, 1993.

[4] HERMANNS, H., AND RETTELBACH, M. Syntax, Semantics, Equivalences, and Axioms for MTIPP. In *Conference Proceedings, PAPM'94* (1994).

[5] HILLSTON, J. *A Compositional Approach to Performance Modelling.* Cambridge University Press, 1996.

[6] HILLSTON, J., AND THOMAS, N. A Syntactical Analysis of Reversible PEPA Models. In *Conference Proceedings, PAPM'98* (Nice, 1998).

[7] HILLSTON, J., AND TOMASIK, J. Amalgamation of Transition Sequences in the PEPA Formalism. Technical report, LFCS, March 2000.

[8] KELLY, F. P. *Reversibility and Stochastic Networks.* John Wiley & Sons, Chichester, New York, Brisbane, Toronto, 1987.

[9] NEUTS, M. *Matrix-Geometric Solution in Stochastic Models — An Algorithmic Approach.* The John Hopkins University Press, 1981.

[10] PLATEAU, B. *De l'evaluation du parallelisme et de sa synchronisation*. Thèse d'état, Université de Paris-Sud, Centre d'Orsay, Novembre 1984.

[11] TOMASIK, J., AND HILLSTON, J. Transforming PEPA Models to Obtain Product Form Bounds. Technical report, LFCS, February 2000.

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