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Corrigendum to "Path regularity and explicit convergence rate for BSDE with truncated quadratic growth" [Stochastic Process. Appl. 120 (2010) 348-379]*

Peter Imkeller

Dept. of Mathematics
Humboldt University Berlin
Unter den Linden 6
10099 Berlin
imkeller@math.hu-berlin.de

Gonçalo Dos Reis

CMAP
École Polytechnique
Route de Saclay
91128 Palaiseau Cedex
dosreis@cmap.polytechnique.fr

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This short note is written to replace the defective proof of Theorem 5.5-(ii) in [1]. The result remains true without any additional assumptions. For this reason we literally adopt all the notations, assumptions and equation numbers used in [1]. The error in the mentioned proof originates in a misapplication of Hölder's inequality in the estimate of I_1 (see page 370 of [1] two lines after (29)).

Theorem 5.5 Part (ii) in [1]: Under HX1 and HY1, the FBSDE system (1), (2) has a unique solution $(X, Y, Z) \in \mathcal{S}^{2p} \times \mathcal{S}^\infty \times \mathcal{H}^{2p}$ for all $p \geq 1$. Moreover, the following holds true:

(ii) For all $p \geq 1$ there exists a constant $C_p > 0$ such that for any partition π of $[0, T]$ with N points and mesh size $|\pi|$

$$\sum_{i=0}^{N-1} \mathbb{E}\Big[\Big(\int_{t_i}^{t_{i+1}} |Z_t - Z_{t_i}|^2 \mathrm{d}t\Big)^p\Big] \leq C_p |\pi|^p.$$

Proof. Throughout fix $p \geq 1$. Theorem 5.3 states that $Z \in \mathcal{S}^{2p}$ and therefore, using Jensen's inequality and Fubini's theorem we are able to write

$$\mathbb{E}\Big[\Big(\int_{t_i}^{t_{i+1}} |Z_t - Z_{t_i}|^2 \mathrm{d}t\Big)^p\Big] \leq |\pi|^{p-1} \int_{t_i}^{t_{i+1}} \mathbb{E}\big[\,|Z_t - Z_{t_i}|^{2p}\,\big]\mathrm{d}t.$$

Using Theorem 5.2 and the representation formulas of Theorem 2.9 we can rewrite the difference inside the expectation as $Z_t - Z_{t_i} = J_1 + J_2 + J_3$ with $J_1 = [\nabla Y_t - \nabla Y_{t_i}](\nabla X_{t_i})^{-1}\sigma(X_{t_i})$, $J_2 = \nabla Y_t[(\nabla X_t)^{-1} - (\nabla X_{t_i})^{-1}]\sigma(X_{t_i})$ and $J_3 = \nabla Y_t(\nabla X_t)^{-1}[\sigma(X_t) - \sigma(X_{t_i})]$ (with $t \in [t_i, t_{i+1}]$).

Estimates for J_2 and J_3 are easy to obtain since they rely mainly on the fact that $\nabla Y \in \mathcal{S}^q$ for all $q \geq 2$ and the known estimates for SDEs found for instance in Section 2.5. We give details for J_2 and hints on how to deal with J_3, remarking that its treatment is very similar.

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Hölder's inequality combined with the growth condition of σ produce for $t \in [t_i, t_{i+1}]$

$$\mathbb{E}[\,|J_2|^{2p}] \le C\|\nabla Y\|^{2p}_{\mathcal{S}^{6p}}\,\mathbb{E}\Big[\,|(\nabla X_t)^{-1} - (\nabla X_{t_i})^{-1}|^{6p}\Big]^{\frac{1}{3}}\,(1+\|X\|^{2p}_{\mathcal{S}^{6p}}) \le C|\pi|^{3p\frac{1}{3}} = C|\pi|^p.$$

Where in the last line we used (4), (8) and $\|\nabla Y\|_{\mathcal{S}^q} < \infty$ for any $q \ge 2$. For J_3, the method is similar: instead of (4) and (8) one uses (5) and (7) combined with HX0.

At this point it is fairly easy to see that

$$\sum_{i=0}^{N-1} |\pi|^{p-1} \int_{t_i}^{t_{i+1}} \mathbb{E}\big[\,|J_2|^{2p} + |J_3|^{2p}\,\big]\mathrm{d}s \le \sum_{i=0}^{N-1} |\pi|^{p-1}\,(t_{i+1}-t_i)\,C\,|\pi|^p = C\,T\,|\pi|^{2p-1}.$$

To handle the term J_1 one needs to proceed with more care. Let us start with a simple trick:

$$\mathbb{E}\Big[\,|(\nabla Y_t - \nabla Y_{t_i})(\nabla X_{t_i})^{-1}\sigma(X_{t_i})|^{2p}\Big] = \mathbb{E}\Big[\mathbb{E}\big[\,|\nabla Y_t - \nabla Y_{t_i}|^{2p}\big|\mathcal{F}_{t_i}\big]\,|(\nabla X_{t_i})^{-1}\sigma(X_{t_i})|^{2p}\Big].$$

Writing the BSDE for the difference $\nabla Y_t - \nabla Y_{t_i}$ for $t_i \le t \le t_{i+1}$ we have for some positive constant C

$$\begin{aligned}\mathbb{E}\Big[\,|\nabla Y_t - \nabla Y_{t_i}|^{2p}\big|\mathcal{F}_{t_i}\Big] &\le C\,\mathbb{E}\Big[\,\Big|\int_{t_i}^{t}\big\langle(\nabla f)\big(r,\Theta(r)\big),(\nabla\Theta)(r)\big\rangle\mathrm{d}r\Big|^{2p} + \Big|\int_{t_i}^{t}\nabla Z_r\mathrm{d}W_r\Big|^{2p}\,\Big|\mathcal{F}_{t_i}\Big]\\ &\le C\,\mathbb{E}\Big[\Big(\int_{t_i}^{t_{i+1}}|(\nabla f)(r,\Theta_r)|\,|\nabla\Theta_r|\mathrm{d}r\Big)^{2p} + \Big(\int_{t_i}^{t_{i+1}}|\nabla Z_r|^2\mathrm{d}r\Big)^{p}\,\Big|\mathcal{F}_{t_i}\Big].\end{aligned}$$

Here we used the conditional Burkholder-Davis-Gundy inequality and maximized over the time interval $[t_i, t_{i+1}]$. For convenience of notation we define the sum of the integrals inside the conditional expectation by $\widehat{J}_{[t_i,t_{i+1}]}$.

Combining these last two inequalities and observing that since ∇X_{t_i} and $\sigma(X_{t_i})$ are $\mathcal{F}_{t_i}$-adapted we can drop the conditional expectation. This way for some positive constant C we obtain

$$\begin{aligned}&|\pi|^{p-1}\sum_{i=0}^{N-1}\int_{t_i}^{t_{i+1}}\mathbb{E}\Big[\mathbb{E}\big[\,|\nabla Y_t - \nabla Y_{t_i}|^{2p}\big|\mathcal{F}_{t_i}\big]\,|(\nabla X_{t_i})^{-1}\sigma(X_{t_i})|^{2p}\Big]\mathrm{d}t\\ &\le C\,|\pi|^{p-1}\sum_{i=0}^{N-1}|\pi|\,\mathbb{E}\big[\,\widehat{J}_{[t_i,t_{i+1}]}\,|(\nabla X_{t_i})^{-1}\sigma(X_{t_i})|^{2p}\,\big]\\ &\le C\,|\pi|^p\,\mathbb{E}\big[\sup_{0\le t\le T}|(\nabla X_t)^{-1}\sigma(X_t)|^{2p}\sum_{i=0}^{N-1}\widehat{J}_{[t_i,t_{i+1}]}\,\big]\\ &\le C\,|\pi|^p\,\mathbb{E}\Big[\sup_{0\le t\le T}|(\nabla X_t)^{-1}\sigma(X_t)|^{2p}\Big\{\Big(\int_0^T|(\nabla f)(r,\Theta_r)|\,|\nabla\Theta_r|\mathrm{d}r\Big)^{2p} + \Big(\int_0^T|\nabla Z_r|^2\mathrm{d}r\Big)^{p}\Big\}\Big]\\ &\le C\,|\pi|^p.\end{aligned}$$

The last line follows from a combination of inequality (25), assumption HY1 (namely the growth conditions for the derivatives of f) and the fact that for any $q \ge 2$ we have: $X, \nabla X, (\nabla X)^{-1} \in \mathcal{S}^q$, $Y, Z, \nabla Y \in \mathcal{S}^q \cap \mathcal{H}^q$ and $\nabla Z \in \mathcal{H}^q$.

Collecting now the estimates on J_1, J_2 and J_3 we obtain the desired result. □

References

[1] P. Imkeller and G. Dos Reis. Path regularity and explicit convergence rate for BSDE with truncated quadratic growth. *Stochastic Process. Appl.*, 120(3):348–379, 2010. ISSN 0304-4149. doi: 10.1016/j.spa.2009.11.004.