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# MIMO Capacity Improvement in the Presence of Antenna Mutual Coupling

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**Abstract**— Applying MIMO technology in small wireless devices leads to closely spaced antennas, which results in antenna mutual coupling (MC) and highly correlated signals. In this paper, we investigate the effect of the terminal load impedance of the antennas on the MIMO capacity in the presence of the mutual coupling. We use a new concept of receiving mutual impedances to model the MC effect, which has been shown to have a much better performance than the conventional open-circuit voltage method in different applications of array antennas. Simulation results for a  $2 \times 2$  MIMO system with half-wavelength dipoles in different scattering distribution scenarios, show that in our proposed method, an optimum value of the terminal load impedance  $Z_L$  of the antennas can be obtained to maximise the capacity for all scattering distributions, whereas the conventional methods need to different  $Z_L$  in different scattering scenarios.

**Keywords**— MIMO; compact; capacity; impedance matching; optimization; mutual coupling; receiving mutual impedance.

## 1. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) wireless systems, by using multiple antennas at both transmit and receive wireless link ends, offer a better quality of service and a linear increase in data rate with the number of antennas in a rich scattering environment [1]. However, integration of multiple antennas into small personal communication devices is faced by the well-known problem of antenna mutual coupling (MC) that degrades the MIMO communication performance. Hence, characterising the behaviour of antennas and the antenna mutual coupling is necessary to determine the compact MIMO performance [2].

Different studies have modelled the coupled received and/or transmitted signals of MIMO array antennas [2–9]. Most of them are based on the open-circuit voltage method, suggested by Gupta and Ksienski [10]. This method characterises the mutual coupling between two antennas by the conventional mutual impedance, which

is defined as the ratio of induced open-circuit voltage of one antenna to the exciting terminal current of the other antenna [10, 11]. It is shown that the conventional definition of mutual impedance is not an accurate approach to model the MC in realistic situations [12–14]. Some others [2] use the scattering-parameter (S-parameter) to describe the MC among the transmitting and receiving array antennas. In [12] it is claimed that by using this method, only the transmitting array is properly modelled to compensate the MC effect, and it fails to correctly model the receiving array.

Here, we use a new concept of the mutual coupling modeling, called *receiving mutual coupling impedance method* [14–16] which has been shown to have a better performance than the conventional method in several antenna array applications such as direction finding [13, 15], adaptive nulling [17], and in magnetic resonance imaging [16].

Recent works [2, 5, 7] have shown that using a simple matching network at the receive side can give a significant improvement for MIMO performance in the presence of MC. This paper investigate the terminal load impedance effect on MIMO capacity in the presence of the mutual coupling effect, by using the receiving mutual coupling impedance method.

## 2. RECEIVING MUTUAL IMPEDANCE METHOD

In the conventional method, the mutual impedance between two antennas is the ratio of the induced open-circuit voltage of one antenna to the current supplied to the other antenna when the first one is open-circuited [10, 11]. For instance, the mutual impedance  $Z_{12}$  can be calculated as

$$Z_{12} = \frac{V_{oc1}}{I_2} \Big|_{I_1=0} \quad (1)$$

where  $V_{oc1}$  is the voltage induced across the open-circuited terminal of antenna 1 (excited one), and  $I_2$  is the current of the terminal of antenna 2 (exciting one) when antenna 1 is open-circuited. It is clear that the conventional method does not model a receiving array properly, because one antenna should be in the transmitting mode and the others in the receiving mode, and the current distribution of antennas in transmit and re-

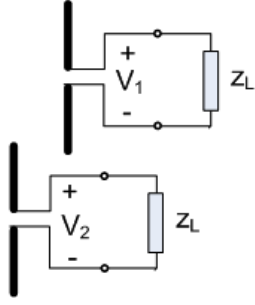


Fig. 1: Two dipole antennas in the receiving mode.

ceive modes are different [14]. Another problem with the conventional impedance method is that the terminal load effect is not taken into account, whereas it affects the antenna current distribution and subsequently the calculated mutual impedance.

To overcome the aforementioned problems, the new receiving mutual impedance method is introduced [15]. Let us consider a closely spaced receive array of two dipole antennas as shown in Figure 1. Both dipoles are passive and connected to a terminal impedance  $Z_L$ . The correlation coefficient of the voltage received at the antenna terminals is affected by the mutual coupling. The voltage across the terminal load of any of these antennas consists of two components [15]: the voltage due to the arrived signal alone, and the induced voltage due to the current distribution of the other antenna. The relationship between the terminal voltages  $V_1$  and  $V_2$ , and the voltages due to the signal alone,  $U_1$  and  $U_2$ , can be written as

$$V_1 = Z_{12}I_2 + U_1 \quad (2a)$$

$$V_2 = Z_{21}I_1 + U_2 \quad (2b)$$

where  $Z_{12}$  and  $Z_{21}$  are the mutual impedances between the antennas, and  $I_1$  and  $I_2$  are the terminal currents flowing through the terminal loads  $Z_L$  of the antennas. The terminal voltages and currents are related by

$$V_1 = Z_L I_1 \quad (3a)$$

$$V_2 = Z_L I_2 \quad (3b)$$

where  $Z_L$  is the terminal load impedance. It should be noted that in this new method, equations (2a) and (2b) are different from the conventional concept and they do not use the open-circuit voltages which require the knowledge of self-impedance of antennas. Here, the mutual impedance  $Z_{12}$  is defined as the ratio of the induced voltage across the terminal load of antenna 1 due to the terminal current  $I_2$  flowing through the terminal load of antenna 2 to this terminal current, i.e.,  $Z_{12} = (V_1 - U_1)/I_2$ . Similarly,  $Z_{21}$  can be defined by changing the position of antenna 1 and antenna 2 in the previous expression for  $Z_{12}$ .

In this method, instead of having one antenna in transmitting mode and exciting the others by the field of this antenna, a plane wave excitation from the horizontal direction in the far field is used to obtain an estimated

current distribution over all the antennas. This approximation is accurate for the signals with low elevation angles related to the horizon [14,15]. Substituting (3a)-(3b) into (2a)-(2b) results in

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{Z_{12}}{Z_L} \\ \frac{Z_{21}}{Z_L} & 1 \end{bmatrix}^{-1} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \quad (4)$$

Thus, by having the receiving mutual impedances, and the coupled terminal voltages of antennas, the uncoupled terminal voltages  $U_i$  can be obtained.

### 3. MIMO MODEL

Consider a MIMO system with  $M_T$  transmit and  $M_R$  receive antennas including antenna mutual coupling at both sides, shown in Figure 2. The receive array can be considered as a coupled  $M_R$  port network with  $M_R$  terminals, where  $\mathbf{V}_R = [V_{R1}, V_{R2}, \dots, V_{RM_R}]^T$  is the vector of terminal voltages including antenna mutual coupling, and  $\mathbf{U}_R = [U_{R1}, U_{R2}, \dots, U_{RM_R}]^T$  represents the terminal voltages at the receive array without the mutual coupling effect. Using the new concept of receiving mutual impedances, the relation between  $\mathbf{V}_R$  and  $\mathbf{U}_R$  can be written as

$$\mathbf{V}_R = \mathbf{Z}_R^{-1} \mathbf{U}_R \quad (5)$$

where

$$\mathbf{Z}_R = \begin{bmatrix} 1 & \frac{Z_{1,2}^R}{Z_L} & \dots & \frac{Z_{1,M_R}^R}{Z_L} \\ \frac{Z_{2,1}^R}{Z_L} & 1 & \dots & \frac{Z_{2,M_R}^R}{Z_L} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{Z_{M_R,1}^R}{Z_L} & \frac{Z_{M_R,2}^R}{Z_L} & \dots & 1 \end{bmatrix} \quad (6)$$

where  $Z_L$  is the terminal load impedance of antennas, and  $Z_{i,j}^R$  ( $i, j = 1, 2, \dots, M_R$ ) is the receiving mutual impedance between the  $i$ th and  $j$ th receiving antennas, as defined in the previous section.

The mutual coupling in the transmit side can be taken into account in a similar way. The terminal voltages at the transmitting antennas with and without the mutual coupling, denoting by  $\mathbf{V}_T$  and  $\mathbf{U}_T$  respectively, are related by

$$\mathbf{V}_T = \mathbf{Z}_T^{-1} \mathbf{U}_T \quad (7)$$

where

$$\mathbf{Z}_T = \begin{bmatrix} 1 & \frac{Z_{1,2}^T}{Z_A} & \dots & \frac{Z_{1,M_T}^T}{Z_A} \\ \frac{Z_{2,1}^T}{Z_A} & 1 & \dots & \frac{Z_{2,M_T}^T}{Z_A} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{Z_{M_T,1}^T}{Z_A} & \frac{Z_{M_T,2}^T}{Z_A} & \dots & 1 \end{bmatrix} \quad (8)$$

where  $Z_A$  is the input impedance of the antennas, and  $Z_{i,j}^T$  ( $i, j = 1, 2, \dots, M_T$ ) is the transmitting mutual impedance between the  $i$ th and  $j$ th transmitting anten-

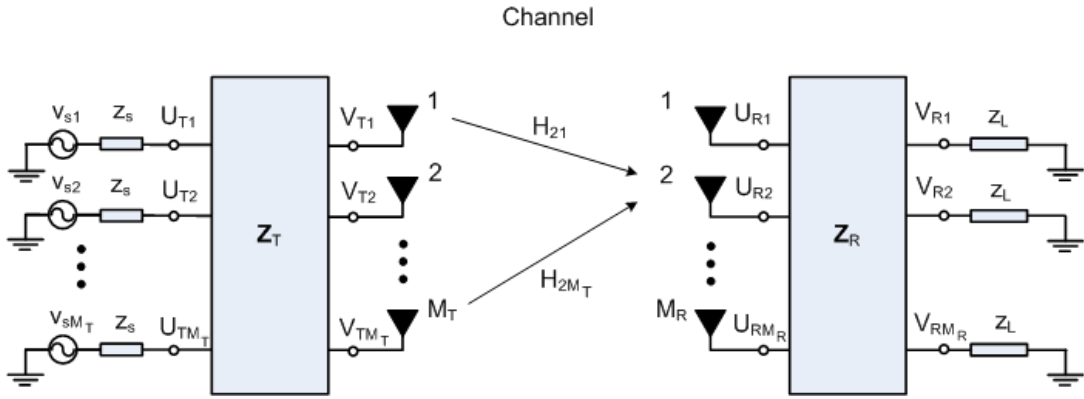


Fig. 2: Block diagram of a  $M_T \times M_R$  MIMO system with antenna mutual coupling.

nas. As an example,  $Z_{1,2}^T$  is defined as [18]

$$Z_{1,2}^T = \frac{V_{T1}}{I_2} \Big|_{V_{s1}=0} \quad (9)$$

It should be noted that the coupled voltage  $V_{T1}|_{V_{s1}=0}$  is the voltage across the antenna input impedance rather than the open-circuit voltage as in the conventional method [18]. Considering (5) and (7), the transmit and receive correlation matrices (including spatial correlation and mutual coupling) can be expressed as

$$\mathbf{R}_T = \mathcal{E}\{\mathbf{V}_T \mathbf{V}_T^H\} = \mathbf{Z}_T^{-1} \mathcal{E}\{\mathbf{U}_T \mathbf{U}_T^H\} (\mathbf{Z}_T^{-1})^H \quad (10a)$$

$$\mathbf{R}_R = \mathcal{E}\{\mathbf{V}_R \mathbf{V}_R^H\} = \mathbf{Z}_R^{-1} \mathcal{E}\{\mathbf{U}_R \mathbf{U}_R^H\} (\mathbf{Z}_R^{-1})^H \quad (10b)$$

where  $\mathcal{E}\{\cdot\}$  is the expectation operator over all multipath scattering directions. We note that the correlation matrices of the uncoupled terminal voltages at the transmit and receive side, denoted as  $\mathcal{E}\{\mathbf{U}_T \mathbf{U}_T^H\}$  and  $\mathcal{E}\{\mathbf{U}_R \mathbf{U}_R^H\}$  respectively, only take the spatial correlation into account, whereas  $\mathbf{R}_T$  and  $\mathbf{R}_R$  include the mutual coupling as well.

For the MIMO system shown in Figure 2, the input-output relation between  $\mathbf{U}_T$  and  $\mathbf{V}_R$  can be written as

$$\mathbf{V}_R = \mathbf{H}_{mc} \mathbf{U}_T + \mathbf{n} \quad (11)$$

where  $\mathbf{H}_{mc}$  is the channel matrix including the antenna mutual coupling, and  $\mathbf{n}$  is the additive noise in the receiver. In the case of independent fading statistics for the transmitter and receiver, this correlation matrix can be created by the Kronecker product of two separated matrices [19]. A common way to include the correlation among the channel matrix entries is to use the Kronecker model [19–21] as

$$\mathbf{H}_{mc} = \mathbf{R}_R^{1/2} \mathbf{H}_w \mathbf{R}_T^{1/2} \quad (12)$$

where  $\mathbf{H}_w$  is the i.i.d. Rayleigh fading channel model, i.e., its entries are independent, zero-mean, unit-variance circularly symmetric complex Gaussian random variables [22, 23].

### 3.1 MIMO Capacity

The ergodic capacity (Mean capacity) for the MIMO system described by (12) (when the channel is unknown at the transmitter) is given by

$$C = \mathcal{E}_{\mathbf{H}} \left\{ \log_2 \det \left[ \mathbf{I}_{M_R} + \frac{\rho}{M_T} \mathbf{H}_{mc} \mathbf{H}_{mc}^H \right] \right\} \quad (13)$$

where  $\rho = P_T / \sigma_n^2$  is the average SNR at the receiver for each antenna ( $P_T$  is the total transmitted power).

## 4. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we consider a  $2 \times 2$  MIMO system of identical dipole antennas with identical source and terminal load impedances, i.e.,  $Z_{s1} = Z_{s2} = Z_s$  and  $Z_{L1} = Z_{L2} = Z_L$ . For the sake of simplicity, we assume that the transmit antennas are separated far enough to ignore the mutual coupling at the transmit side, i.e.,  $\mathbf{V}_T = \mathbf{U}_T$ , and  $\mathcal{E}\{\mathbf{U}_T \mathbf{U}_T^H\} = \mathbf{I}$  is considered. Another point, as is mentioned in previous works [5, 8], is that in practical MIMO applications the voltage across the real part of the terminal load  $Z_L$  is considered as the received signal rather than the voltage across the complex load  $Z_L$ . So, we modify the channel matrix relation in (12) by replacing  $Z_L$  with  $\text{real}(Z_L)$  in the  $\mathbf{Z}_R$  definition.

The dimension of the dipole antennas are length =  $\lambda/2$ , wire radius = 5 mm, at the frequency of 1800 MHz and antenna spacing  $d = 0.05\lambda$ . The receiving mutual impedances for different values of the terminal load impedance  $Z_L = R_L + jX_L$  were obtained from FEKO software [24] based on the method described in [14].

Some recent works [5, 7, 9], by using the conventional mutual impedance method, have investigated the effect of the terminal load  $Z_L$  on the MIMO performance. It is shown that in the presence of the mutual coupling effect, MIMO performance metrics (signal correlation, received power, or capacity) can be improved by choosing a proper  $Z_L$  to optimise the desired metric.

Here, we calculate the ergodic capacity for both mutual coupling models (conventional and receiving mutual impedance methods) for different complex values of  $Z_L$ . The ergodic capacity is calculated from (13) by averaging

over 3000 realisations of  $\mathbf{H}_w$ . Assume  $\alpha$  be the uncoupled terminal voltages correlation defined by

$$\alpha = \frac{\mathcal{E}\{U_{R1}U_{R2}^H\}}{\sqrt{\mathcal{E}\{U_{R1}U_{R1}^H\}\mathcal{E}\{U_{R2}U_{R2}^H\}}} \quad (14)$$

In our simulation, three different scenarios are considered for  $\alpha$  (and for the open-circuit voltages in the conventional method): uniform distribution, and two Laplacian distribution cases with the similar amplitudes but different phases. In the case of the uniform distribution,  $\alpha = J_0(2\pi d/\lambda)$ , where  $J_0(\cdot)$  is the zeroth-order Bessel function of the first kind. To generalise our result to more realistic cases, we have considered a Laplacian distribution which is described by

$$p(\phi) = c_1 \exp\left[-\frac{\sqrt{2}|\phi - \phi_0|}{\sigma}\right]$$

where  $\phi_0$  and  $\sigma$  are the mean and the standard deviation of the distribution, respectively.  $c_1$  is a normalization factor such that the integral of  $p(\phi)$  over the azimuth plane, i.e. over the interval  $[-180^\circ, 180^\circ)$ , is equal to 1. In the Laplacian distribution case, two different sets of  $(\phi_0, \sigma) = \{(0^\circ, 33.5^\circ), (90^\circ, 58^\circ)\}$  corresponding to  $\alpha = 0.99\angle 0^\circ$  and  $\alpha = 0.99\angle 167.7^\circ$  are considered. For all scattering distribution scenario, a reference SNR of 20 dB is assumed.

For comparison, the mean capacity surfaces of the new method and the conventional method, in the case of the uniform distribution, for different values of the real and imaginary parts of the terminal load  $Z_L = R_L + jX_L$  are shown in Figure 3 and 4, where  $R_L \in [0, 200]\Omega$  and  $X_L \in [-50, 100]\Omega$ . It can be seen that for any method, an optimal  $Z_L$  can be found to maximize the mean capacity.

We note that here we are looking for an optimum terminal load impedance  $Z_L$  that maximises the capacity rather than the

The maximum mean capacity (bits/s/Hz) and the corresponding  $Z_L(\Omega)$  values for the uniform and Laplacian  $(\phi_0, \sigma) = \{(0^\circ, 33.5^\circ), (90^\circ, 58^\circ)\}$  scattering distributions are shown in Table 1. It is clear that by using the conventional method for different scenarios of scattering distribution at the receive side, different values of the terminal load impedance  $Z_L$  are needed to compensate the mutual coupling effect and maximize the MIMO capacity. The new approach uses the receiving mutual impedance method to characterises the mutual coupling effect and compute an optimum value of  $Z_L$  for different scenarios. It shows that, the conventional method can not model the mutual coupling precisely, so for any case of the scattering distribution, we need to find an optimum value of  $Z_L$ , and it is not a practical way.

## 5. CONCLUSION

In this paper, we investigated the effect of the terminal load impedance  $Z_L$  of the antennas on the MIMO capacity by using a new method to characterise the antenna

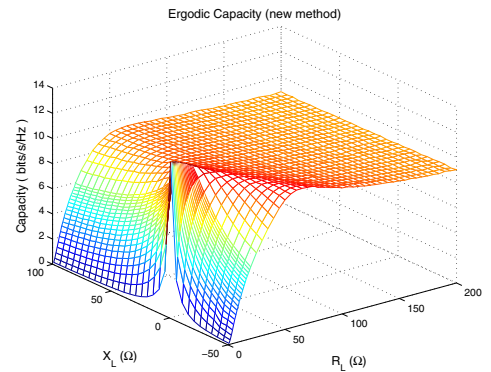


Fig. 3: Mean capacity based on the new receiving mutual impedance method and uniform scattering distribution. The max value of mean capacity is 12.4 b/s/Hz for  $Z_L = 1 + j0$ .

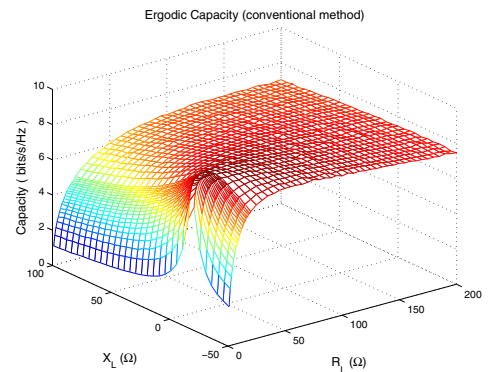


Fig. 4: Mean capacity based on the conventional method and uniform scattering distribution. The max value of mean capacity is 8.56 b/s/Hz for  $Z_L = 45 - j15$ .

Table 1: Optimized mean capacity (bits/s/Hz) and the corresponding  $Z_L(\Omega)$  values for the uniform and Laplacian  $(\phi_0, \sigma)$  scattering distributions.

Capacity (bits/s/Hz)	uniform	$(0^\circ, 33.5^\circ)$	$(90^\circ, 58^\circ)$
New method	12.4	12.56	12.18
Conventional method	8.56	7.54	7.73
Optimum $Z_L(\Omega)$	uniform	$(0^\circ, 33.5^\circ)$	$(90^\circ, 58^\circ)$
New method	$1 + j0$	$1 + j0$	$1 + j0$
Conventional method	$45 - j15$	$50 - j20$	$45 - j20$

mutual coupling in compact MIMO systems. We considered different scenarios for the scattering distribution at the receive side. The results showed that by using this new method, an optimum value of  $Z_L$  can be obtained to maximise the capacity for different scattering distribution scenarios, whereas the conventional methods need to different  $Z_L$  in different cases.

## 6. ACKNOWLEDGEMENT

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