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# Pecuniary Externality through Credit Constraints: 

 Two Examples without UncertaintyJohn Moore (Edinburgh University and LSE)

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Pecuniary Externality through Credit Constraints:

Two Examples without Uncertainty*

by

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*I thank Hans Gersbach, Nobuhiro Kiyotaki, Guido Lorenzoni and Jean-Charles Rochet for their thoughtful comments, and I am particularly grateful to Jordi Gali for his encouragement to write up these two examples. The examples arose from opportunities I had to discuss Gersbach and Rochet's paper "Aggregate Investment Externalities and Macro-Prudential Regulation" at a JMCB conference held in the Studienzentrum, Gerzensee, Switzerland, 2021 October 2011, and to discuss Lorenzoni's paper "Inefficient Credit Booms" at a Research Workshop on the Economics of Financial Stability held in the Oesterreichische Nationalbank, Vienna, Austria, 7-8 July 2008.


#### Abstract

This paper is a contribution to the growing literature on constrained inefficiencies in economies with financial frictions. The purpose is to present two simple examples, inspired by the stochastic models in Gersbach-Rochet (2012) and Lorenzoni (2008), of deterministic environments in which such inefficiencies arise through credit constraints. Common to both examples is a pecuniary externality, which operates through an asset price. In the second example, a simple transfer between two groups of agents can bring about a Pareto improvement.

In a first best economy, there are no pecuniary externalities because marginal productivities are equalised. But when agents face credit constraints, there is a wedge between their marginal productivities and those of the non-credit-constrained agents. The wedge is the source of the pecuniary externality: economies with these kinds of imperfections in credit markets are not second-best efficient. This is akin to the constrained inefficiency of an economy with incomplete markets, as in Geanakoplos and Polemarchakis (1986).


## INTRODUCTION

This paper is a contribution to the growing literature on second-best inefficiencies in economies with financial frictions. ${ }^{1}$ My purpose is to present two examples of deterministic environments in which pecuniary externalities arise through credit constraints.

The first example is inspired by Gersbach-Rochet (2012). I simplify and modify their stochastic model to show that the second-best inefficiency they identify is present in a twoperiod environment without uncertainty. The welfare measure I use to demonstrate constrained inefficiency is aggregate surplus - for which, of course, it is easier to exhibit a welfare improvement than if, say, the Pareto criterion were used.

The second example is inspired by Lorenzoni (2008). I simplify and modify his stochastic model to show that uncertainty is not necessary to his analysis. But here the welfare criterion I use is more demanding: I demonstrate that a simple transfer between two groups of agents can bring about a Pareto improvement.

Common to both examples is a pecuniary externality, which operates through an asset price. In a first best economy, we know that there are no pecuniary externalities because marginal productivities are equalised. But in both my examples, certain agents face credit constraints. On account of these credit constraints, there is a wedge between their marginal productivities and those of the non-credit-constrained agents. The wedge is the source of the pecuniary externality: economies with these kinds of imperfections in credit markets are not even second-best efficient. This is akin to the constrained inefficiency of an economy with incomplete markets. See, for example, Geanakoplos and Polemarchakis (1986).

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## EXAMPLE 1

There are two days, 0 and 1, and two groups of agents, sheep farmers and wine makers. Production occurs overnight, between days 0 and 1 . Consumption is on day 1. The agents' respective outputs, wool and wine, are perfect substitutes as consumption goods. Measuring them so as to be one-for-one in consumption, we can take wool and wine to be a common unit of account, "output", the numeraire.

Besides their own non-tradable labour, sheep farmers and wine makers both use land as an input to production. Land can be traded on day 0 , in exchange for a promise to deliver day 1 output, wool or wine. But - and this is the key to the model - the promise has to be credible. The only trade in this economy is the exchange of land on day 0 for a credible promise to deliver day 1 output.

Land is variably productive. Think of land in thin contours, or strips, in a valley whose cross-section has the shape of an inverted Gothic arch; see Figure 1. Along each contour, parallel to the valley floor, the land is at a constant altitude and is homogeneous. Take the overall width of the valley, perpendicular to the valley floor, to be 2 , and suppose the altitude (above sea-level, not the valley floor) of a contour horizontal distance K from the valley floor is given by the constant elasticity function

$$
\eta(a+1-K)^{\eta-1} \quad \text { where } a>0 \text { and } 0<\eta<1 .
$$

Productivity - both for sheep rearing and vine growing measured per (tiny) unit-width strip varies across strips. It can depend on the altitude of the strip as well as on which side of the valley the strip is on.

On the dark side of the valley, the left side of Figure 1, the sun seldom shines and so the productivity of growing vines is less than on the right, the sunny side. But the productivity of growing vines is not affected by altitude: it is a constant, $\mathrm{R}_{\ell}>0$, on the left; and a constant, $\mathrm{R}_{\mathrm{h}}>\mathrm{R}_{\ell}$, on the right.

Rearing sheep is affected by altitude - sheep thrive in higher altitudes - but is unaffected by light. Supposing that productivity is a linear function of altitude, we can measure altitude so as to make them equal. Thus, for example, if all the land on one side of

productivity of vine growing $\left\{\begin{array}{l}\text { increasing with light } \\ \text { unaffected by altitude }\end{array}\right.$
productivity of sheep rearing $\left\{\begin{array}{l}\text { increasing with altitude (equal to!) } \\ \text { unaffected by light }\end{array}\right.$

Figure 1
the valley, above a contour horizontal distance K from the valley floor, were devoted to sheep rearing, then output from this land would be

$$
(a+1-K)^{\eta}-a^{\eta} \equiv F(1-K), \text { say. }
$$

The derivative, $F^{\prime}(1-K)$, of this function is the altitude $\eta(a+1-K)^{\eta-1}-$ equal to the productivity of rearing sheep - of a strip horizontal distance K from the valley floor.

Crucially, on day 0 wine makers can credibly pledge at most a fraction $\theta<1$ of their day 1 output. We will see that, because of this, they face a "credit constraint". (Note that we could also assume, for symmetry, that on day 0 sheep farmers too are unable to credibly pledge more than a fraction $\theta$ of their day 1 output, but that would not substantively affect our analysis. So let us not make this assumption.)

Wine makers are endowed with all the lowest strips of land, horizontal distance up to $\mathrm{W}_{\ell}$ from the valley floor on the dark side, and up to $\mathrm{W}_{\mathrm{h}}$ on the sunny side. Sheep farmers are endowed with all the remaining land. The values of $\mathrm{W}_{\ell}$ and $\mathrm{W}_{\mathrm{h}}$ are small enough that at the boundaries, the productivity of sheep rearing is strictly less than $\theta$ times that of wine making:

$$
\mathrm{F}^{\prime}\left(1-\mathrm{W}_{\ell}\right)<\theta \mathrm{R}_{\ell}
$$

and

$$
\mathrm{F}^{\prime}\left(1-\mathrm{W}_{\mathrm{h}}\right)<\theta \mathrm{R}_{\mathrm{h}} .
$$

### 1.1. Implementation of the First-Best

The initial endowment of land is not too inefficient: the higher land (land more productive for sheep rearing) belongs to the sheep farmers; and the lower land (land more productive for vine growing) belongs to the wine makers. But to reach first-best requires equating productivities at the boundaries on each side of the valley:

$$
\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\ell}{ }^{\mathrm{FB}}\right)=\mathrm{R}_{\ell}
$$

and

$$
\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\mathrm{h}}{ }^{\mathrm{FB}}\right)=\mathrm{R}_{\mathrm{h}}
$$

where $\mathrm{K}_{\ell}{ }^{\mathrm{FB}}$ is the horizontal distance from the valley floor, in the first-best allocation, of the boundary between sheep rearing and vine growing on the dark side of the valley, and $\mathrm{K}_{\mathrm{h}}{ }^{\mathrm{FB}}$ is similarly defined for the sunny side.

As a preliminary observation: implementation of this first-best allocation can be decentralised as a market equilibrium outcome if $\theta$ is equal to 1 , so that wine makers face no credit constraint.

Market equilibrium pricing, in the first-best when $\theta$ equals 1 , is as follows. On the dark side of the valley, all strips of land up to horizontal distance $\mathrm{K}_{\ell}{ }^{\mathrm{FB}}$ from the valley floor are uniformly priced at $\mathrm{R}_{\ell}$; each higher strip, horizontal distance $\mathrm{K}>\mathrm{K}_{\ell}{ }^{\mathrm{FB}}$ from the valley floor, is priced at $\mathrm{F}^{\prime}(1-\mathrm{K})$. And on the sunny side, all strips of land up to horizontal distance $\mathrm{K}_{\mathrm{h}}{ }^{\mathrm{FB}}$ from the valley floor are uniformly priced at $\mathrm{R}_{\mathrm{h}}$; each higher strip, horizontal distance $\mathrm{K}>\mathrm{K}_{\mathrm{h}}{ }^{\mathrm{FB}}$ from the valley floor, is priced at $\mathrm{F}^{\prime}(1-\mathrm{K})-$ as on the dark side.

The point is that, when $\theta$ equals 1 , competition drives the land price up to the zeroprofit level across the valley.
1.2. Laissez faire equilibrium with credit constraints

When the wine makers' credit constraint binds (requiring that $\theta<1$ ), the nature of the land pricing is similar. On the dark side of the valley, for some $\mathrm{P}_{\ell}$, land horizontal distance up to $\mathrm{K}_{\ell}$ from the value floor is priced at $\mathrm{P}_{\ell}$, where $\mathrm{K}_{\ell}$ solves

$$
\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\ell}\right)=\mathrm{P}_{\ell} ;
$$

and each higher strip, horizontal distance $\mathrm{K}>\mathrm{K}_{\ell}$ from the valley floor, is priced at $\mathrm{F}^{\prime}(1-\mathrm{K})$. And on the sunny side, for some $\mathrm{P}_{\mathrm{h}}$, land horizontal distance up to $\mathrm{K}_{\mathrm{h}}$ from the value floor is priced at $\mathrm{P}_{\ell}$, where $\mathrm{K}_{\mathrm{h}}$ solves

$$
\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\mathrm{h}}\right)=\mathrm{P}_{\mathrm{h}} ;
$$

and each higher strip, horizontal distance $\mathrm{K}>\mathrm{K}_{\mathrm{h}}$ from the valley floor, is priced at $\mathrm{F}^{\prime}(1-\mathrm{K})$.

Sheep farmers compete to buy all the higher land, the land priced above $\mathrm{P}_{\ell}$ and $\mathrm{P}_{\mathrm{h}}$ on the dark and sunny sides respectively. They make zero profit.

The equilibrium values of $\mathrm{P}_{\ell}$ and $\mathrm{P}_{\mathrm{h}}$ are determined by the behaviour of the wine makers. We will see that, in equilibrium, wine makers use more land than their aggregate endowment; i.e. $\mathrm{K}_{\ell}>\mathrm{W}_{\ell}$ and $\mathrm{K}_{\mathrm{h}}>\mathrm{W}_{\mathrm{h}}$.

Consider a typical wine maker, endowed with land worth y. (The argument is the same whichever side of the valley his endowment lies: it may be $\mathrm{y} / \mathrm{P}_{\ell}$ units on the dark side, or $\mathrm{y} / \mathrm{P}_{\mathrm{h}}$ units on the sunny, or a convex combination. He can realise y by selling his endowment and then investing in land either on the dark or sunny side of the valley.) If he invests on the dark side, the scale of his investment k will be governed by a flow-of-funds constraint

$$
\mathrm{P}_{\ell} \mathrm{k} \leq \mathrm{y}+\theta \mathrm{R}_{\ell} \mathrm{k},
$$

given that he cannot credibly pledge more than a fraction $\theta$ of his day 1 return $R_{\ell} k$. His day 1 net payoff is maximised when the constraint binds. That is, his maximum payoff from levered investment on the dark side of the valley is

$$
\frac{(1-\theta) R_{\ell} y}{P_{\ell}-\theta R_{\ell}}
$$

By a similar argument, his maximum payoff from levered investment on the sunny side of the valley is

$$
\frac{(1-\theta) R_{h} y}{P_{h}-\theta R_{h}}
$$

In equilibrium, these rates of return on levered investment must be equalized (so as to have vine growing on both sides of the valley):

$$
\frac{(1-\theta) R_{\ell}}{P_{\ell}-\theta R_{\ell}}=\frac{(1-\theta) R_{h}}{P_{h}-\theta R_{h}}
$$

Here the denominators are the downpayments required per unit of land purchased; the numerators are the output that cannot be pledged.

It follows from this equation that there must exist some $\propto$ such that

$$
\mathrm{P}_{\ell}=\alpha \mathrm{R}_{\ell}
$$

and

$$
P_{h}=\alpha R_{h}
$$

Note that $\alpha$ is strictly greater than $\theta$ (the required downpayments are positive). And $\propto$ is strictly less than 1 (the rates of return on levered investment exceed 1 ).

### 1.3 Planner's constrained optimum

$$
\begin{aligned}
& \text { Think now of a planner choosing } \mathrm{K}_{\ell} \text { and } \mathrm{K}_{\mathrm{h}} \text { to maximise aggregate output } \\
& \qquad \mathrm{R}_{\ell} \mathrm{K}_{\ell}+\mathrm{F}\left(1-\mathrm{K}_{\ell}\right)+\mathrm{R}_{\mathrm{h}} \mathrm{~K}_{\mathrm{h}}+\mathrm{F}\left(1-\mathrm{K}_{\mathrm{h}}\right)
\end{aligned}
$$

subject to the constraint

$$
\left[\mathrm{K}_{\ell}-\mathrm{W}_{\ell}\right] \mathrm{F}^{\prime}\left(1-\mathrm{K}_{\ell}\right)+\left[\mathrm{K}_{\mathrm{h}}-\mathrm{W}_{\mathrm{h}}\right] \mathrm{F}^{\prime}\left(1-\mathrm{K}_{\mathrm{h}}\right) \leq \theta \mathrm{R}_{\ell} \mathrm{K}_{\ell}+\theta \mathrm{R}_{\mathrm{h}} \mathrm{~K}_{\mathrm{h}}
$$

The logic here is that, in choosing $\mathrm{K}_{\ell}$ and $\mathrm{K}_{\mathrm{h}}$, the planner is in effect choosing prices $\mathrm{P}_{\ell}=\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\ell}\right)$ and $\mathrm{P}_{\mathrm{h}}=\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\mathrm{h}}\right)$. The planner's problem is constrained by the need to
respect the (aggregate of the) wine makers' credit constraints, assuming that they and the sheep farmers take $\mathrm{P}_{\ell}$ and $\mathrm{P}_{\mathrm{h}}$ as parametric prices.

Form the Lagrangian L, with multiplier $\lambda$. The first-order conditions (FOCs) are:

$$
\begin{aligned}
\frac{\partial \mathrm{L}}{\partial K_{\ell}} & =\mathrm{R}_{\ell}-\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\ell}\right)-\lambda\left\{\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\ell}\right)-\left[\mathrm{K}_{\ell}-\mathrm{W}_{\ell}\right] \mathrm{F}^{\prime \prime}\left(1-\mathrm{K}_{\ell}\right)-\theta \mathrm{R}_{\ell}\right\} \\
& =0 \\
\frac{\partial \mathrm{~L}}{\partial K_{h}} & =\mathrm{R}_{\mathrm{h}}-\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\mathrm{h}}\right)-\lambda\left\{\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\mathrm{h}}\right)-\left[\mathrm{K}_{\mathrm{h}}-\mathrm{W}_{\mathrm{h}}\right] \mathrm{F}^{\prime \prime}\left(1-\mathrm{K}_{\mathrm{h}}\right)-\theta \mathrm{R}_{\mathrm{h}}\right\} \\
& =0
\end{aligned}
$$

The solution, $\left(\mathrm{K}_{\ell}{ }^{\mathrm{SB}}, \mathrm{K}_{\mathrm{h}}{ }^{\mathrm{SB}}\right)$ say, to these FOCs are the second-best values of $\mathrm{K}_{\ell}$ and $\mathrm{K}_{\mathrm{h}}$ - the constrained efficient allocation of land across sheep rearing and vine growing.

### 1.4 Constrained inefficiency of laissez-faire

To compare these values $\mathrm{K}_{\ell}{ }^{\mathrm{SB}}$ and $\mathrm{K}_{\mathrm{h}}{ }^{\mathrm{SB}}$ with the laissez-faire equilibrium

$$
\left\{\begin{array}{l}
\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\ell}\right)=\propto \mathrm{R}_{\ell} \\
\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\mathrm{h}}\right)=\propto \mathrm{R}_{\mathrm{h}}
\end{array}\right.
$$

we substitute these values of $\mathrm{K}_{\ell}$ and $\mathrm{K}_{\mathrm{h}}$ into the left-hand sides of the above first-order conditions:

$$
\frac{1}{R_{\ell}} \frac{\partial \mathrm{L}}{\partial K_{\ell}}=1-\propto-\lambda\left\{\propto-\theta+\left[\frac{K_{\ell}-W_{\ell}}{1-K_{\ell}}\right] \propto(1-\eta)\right\}
$$

and

$$
\frac{1}{R_{\mathrm{h}}} \frac{\partial \mathrm{~L}}{\partial K_{\mathrm{h}}}=1-\propto-\lambda\left\{\propto-\theta+\left[\frac{K_{h}-W_{h}}{1-K_{h}}\right] \propto(1-\eta)\right\}
$$

The crucial point is that, typically, these expressions for $\frac{\partial \mathrm{L}}{\partial K_{\ell}}$ and $\frac{\partial \mathrm{L}}{\partial K_{\mathrm{h}}}$ cannot both be equal to zero, because typically

$$
\left[\frac{K_{\ell}-W_{\ell}}{1-K_{\ell}}\right] \neq\left[\frac{K_{h}-W_{h}}{1-K_{h}}\right]
$$

Thus, typically, the laissez-faire equilibrium does not maximise aggregate output subject to the wine makers' credit constraints. The laissez-faire equilibrium is typically not constrained efficient.

For example, in the special case where $W_{\ell}=W_{h}$, since $\mathrm{K}_{\ell}<\mathrm{K}_{\mathrm{h}}$,

$$
\frac{\partial L}{\partial K_{h}}<0<\frac{\partial L}{\partial K_{\ell}}
$$

- evaluated at the allocation of the laissez-faire equilibrium with credit constraints. In this special case, relative to this laissez-faire allocation, the planner wants to raise $\mathrm{K}_{\ell}$ (raise $\mathrm{P}_{\ell}$ ) and lower $\mathrm{K}_{\mathrm{h}}$ (lower $\mathrm{P}_{\mathrm{h}}$ ). With $W_{\ell}=W_{h}$, a small increase in the price $\mathrm{P}_{\ell}$, offset by an equal reduction in the price $\mathrm{P}_{\mathrm{h}}$, makes no difference to the wine makers' (credit-constrained) aggregate output, but shifts a little sheep rearing from the less productive boundary on the dark side of the valley, productivity $\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\ell}\right)$, to the more productive boundary on the sunny side of the valley, productivity $\mathrm{F}^{\prime}\left(1-\mathrm{K}_{\mathrm{h}}\right)$.

The disparity between the planner's second-best allocation and the laissez-faire allocation arises because the planner internalises the effect his choice of $\mathrm{K}_{\ell}{ }^{\mathrm{SB}}$ and $\mathrm{K}_{\mathrm{h}}{ }^{\mathrm{SB}}$ has on the prices $\mathrm{P}_{\ell}$ and $\mathrm{P}_{\mathrm{h}}$, whereas the market does not. Absent the wine makers' credit constraint, and the resultant wedge between their productivity and the marginal productivity of the sheep rearers - i.e., in the first-best - this would not matter. But it does matter in the second-best.

## EXAMPLE 2

The example follows the model in Lorenzoni (2008), but with important differences, which will be pointed out.

There are three dates, $t=0,1,2$, and two types of agent, entrepreneurs and consumers. There is a consumption good, fruit, that is perishable, and a capital good that perishes at $\mathrm{t}=2$. Unlike in Lorenzoni (2008), there is no uncertainty. And, critically, there is no financial contracting, because there is no collateral available to secure borrowing.

Start with a representative entrepreneur. At $\mathrm{t}=0$, he is endowed with n units of fruit, but thereafter he has no endowment. He can convert fruit into capital, one-for-one: $\mathrm{k}_{0}$ capital held overnight between $\mathrm{t}=0$ and $\mathrm{t}=1$ yields him ak fruit at $\mathrm{t}=2$ (not at $\mathrm{t}=1$ as in Lorenzoni (2008)), where $\mathrm{a}>1$. Neither capital nor fruit have collateral value, hence no borrowing is possible. The entrepreneur will choose to invest all his endowment in capital: $\mathrm{k}_{0}=\mathrm{n}$.

At $\mathrm{t}=1$, the capital stock requires maintenance (otherwise it perishes), using up $\gamma \mathrm{k}_{0}$ fruit, where $\gamma<1$. Hence the entrepreneur needs to raise funds, by selling $\mathrm{k}_{0}-\mathrm{k}_{1}$ newlymaintained capital, at price q (in terms of fruit), where $\gamma<\mathrm{q}<1$. $\mathrm{k}_{1}$ units of capital held overnight between $\mathrm{t}=1$ and $\mathrm{t}=2$ yields him again $\mathrm{Ak}_{1}$ fruit at $\mathrm{t}=2$, where $\mathrm{A}>1$. Given that there is no borrowing, the maintenance costs have to be wholly met from asset sales:

$$
\gamma \mathrm{k}_{0}=\mathrm{q}\left(\mathrm{k}_{0}-\mathrm{k}_{1}\right)
$$

or

$$
\mathrm{k}_{1}=(\mathrm{q}-\gamma) \mathrm{k}_{0} / \mathrm{q}
$$

At $\mathrm{t}=2$, the entrepreneur eats fruit $\mathrm{ak}_{0}+\mathrm{Ak}_{1}$, his utility.
Now turn to a representative consumer. She has a large endowment of fruit at each date $t=0,1$ and 2 . She eats fruit $c_{t}$ each date: her utility is $c_{0}+c_{1}+c_{2}$. She can also produce fruit by holding capital overnight between $t=1$ and $t=2$ : in aggregate, $\mathrm{k}_{0}-\mathrm{k}_{1}$
capital acquired by consumers at $t=1$ yields $F\left(k_{0}-k_{1}\right)$ fruit at $t=2$. Specifically, we suppose

$$
\mathrm{F}\left(\mathrm{k}_{0}-\mathrm{k}_{1}\right) \equiv 2\left(\mathrm{k}_{0}-\mathrm{k}_{1}\right)^{1 / 2}
$$

The first-order conditions for (non-discounted) profit maximisation:

$$
\mathrm{F}^{\prime}\left(\mathrm{k}_{0}-\mathrm{k}_{1}\right)=\mathrm{q}
$$

imply that at $\mathrm{t}=1$ the consumers' demand for capital, $\mathrm{k}_{0}-\mathrm{k}_{1}$, equals $1 / q^{2}$ and that their maximised profit (utility), $\pi$, from fruit production equals $1 / q$.

### 2.1. Equilibrium

In the market for capital at $\mathrm{t}=1$ :

$$
\begin{array}{ll}
\text { supply } & =\gamma \mathrm{k}_{0} / \mathrm{q} \\
\text { demand } & =1 / \mathrm{q}^{2}
\end{array}
$$

Notice that the supply schedule is a downward-sloping function of the price. However the demand schedule is less steeply downward-sloping, which ensures that the equilibrium is unique and stable. See Figure 2.

The competitive equilibrium price q is

$$
\mathrm{q}^{\mathrm{CE}}=1 / \gamma \mathrm{k}_{0}
$$

which equals $1 / \gamma \mathrm{n}$ given that the entrepreneurs choose $\mathrm{k}_{0}=\mathrm{n}$ at $\mathrm{t}=0$.


Figure 2

The price $q^{\text {CE }}$ lies strictly between $\gamma$ and 1 if we assume:

$$
1 / \gamma<\mathrm{n}<1 / \gamma^{2}
$$

(Assumption 1)

In equilibrium, the consumer profit $\pi$ is

$$
\pi^{\mathrm{CE}}=1 / \mathrm{q}=\gamma \mathrm{k}_{0}
$$

which equals $\gamma \mathrm{n}$ given $\mathrm{k}_{0}=\mathrm{n}$. And the entrepreneurs' capital holding $\mathrm{k}_{1}$ between $\mathrm{t}=1$ and $t=2$ is

$$
\mathrm{k}_{1}^{\mathrm{CE}}=(\mathrm{q}-\gamma) \mathrm{k}_{0} / \mathrm{q}=\left(1-\gamma^{2} \mathrm{k}_{0}\right) \mathrm{k}_{0}
$$

which, given $\mathrm{k}_{0}=\mathrm{n}$, equals $\left(1-\gamma^{2} \mathrm{n}\right) \mathrm{n}$. This is positive by Assumption 1.

We suppose that maintenance costs $\gamma \mathrm{k}_{0}$ are large enough that, in the neighbourhood of n , a fall in $\mathrm{k}_{0}$ would lead to a rise in $\mathrm{k}^{\mathrm{CE}}$. Specifically, we assume

$$
\gamma>1 / \sqrt{ }(2 \mathrm{n}) \quad \text { or } \quad 1 / 2 \gamma^{2}<\mathrm{n} \quad \text { (Assumption 2) }
$$

Assumption 2 ensures that

$$
\mathrm{k}_{1}^{\mathrm{CE}}=\left(1-\gamma^{2} \mathrm{k}_{0}\right) \mathrm{k}_{0}
$$

would go $u p$ if $\mathrm{k}_{0}$ went down. Note that Assumptions 1 and 2 are compatible. They can be amalgamated:

$$
\max \left\{1 / \gamma, 1 / 2 \gamma^{2}\right\}<\mathrm{n}<1 / \gamma^{2}
$$

### 2.2. Welfare

Consider the following experiment. At $\mathrm{t}=0$, suppose the entrepreneurs are obliged to transfer a small amount of fruit, $\tau_{0}=\varepsilon$, to the consumers.

As a result, the entrepreneurs' capital investment $\mathrm{k}_{0}$ is reduced by $\varepsilon-$ down from n to $\mathrm{n}-\varepsilon$ :

$$
\Delta \mathrm{k}_{0}=-\varepsilon<0
$$

This reduction in $\mathrm{k}_{0}$ lowers the entrepreneurs' maintenance costs $\gamma \mathrm{k}_{0}$ at $\mathrm{t}=1$, shifts down their capital supply schedule in the market, and hence raises the equilibrium price $q^{C E}$ from $1 / \gamma \mathrm{n}$ to $1 / \gamma(\mathrm{n}-\varepsilon)$ :

$$
\Delta q \approx \varepsilon / \gamma n^{2}>0 .
$$

The price rise reduces the consumers' profit $\pi^{\mathrm{CE}}$ from $\gamma \mathrm{n}$ to $\gamma(\mathrm{n}-\varepsilon)$ :

$$
\Delta \pi=-\gamma \varepsilon<0,
$$

but, since $\gamma<1$, the consumers are more than compensated by the initial payment $\tau_{0}=\varepsilon$. And the entrepreneurs' capital holding $\mathrm{k}_{1}{ }^{\mathrm{CE}}$ between $\mathrm{t}=1$ and $\mathrm{t}=2$ rises from $\left(1-\gamma^{2} \mathrm{n}\right) \mathrm{n}$ to $\left(1-\gamma^{2} n+\gamma^{2} \varepsilon\right)(n-\varepsilon):$

$$
\Delta \mathrm{k}_{1} \approx\left(2 \gamma^{2} \mathrm{n}-1\right) \varepsilon,
$$

which is strictly positive by Assumption 2. The effect on the entrepreneurs' consumption (utility) at $\mathrm{t}=2$ is

$$
\mathrm{a} \Delta \mathrm{k}_{0}+\mathrm{A} \Delta \mathrm{k}_{1} \approx-\mathrm{a} \varepsilon+\mathrm{A}\left(2 \gamma^{2} \mathrm{n}-1\right) \varepsilon,
$$

which is strictly positive if A is enough larger than a. Specifically, we assume

$$
\mathrm{A} / \mathrm{a}>1 /\left(2 \gamma^{2} \mathrm{n}-1\right)
$$

(Assumption 3)

Here, the right hand side is strictly greater than 1 given Assumptions 1 and 2. Hence Assumption 3 implies A>a.

Under Assumptions 1-3, then, the consumers and the entrepreneurs are strictly better off as a result of the experiment. We have a strict Pareto improvement!

### 2.3. Intuition for the Pareto improvement

Between $\mathrm{t}=1$ and $\mathrm{t}=2$, the entrepreneurs' return on fruit equals

$$
\mathrm{A} / \mathrm{q}>1
$$

whereas consumers' (marginal) return on fruit only equals

$$
\mathrm{F}^{\prime}\left(\mathrm{k}_{0}-\mathrm{k}_{1}\right) / \mathrm{q}=1
$$

This differential in returns reflects the entrepreneurs' borrowing constraint at $\mathrm{t}=1$. Ceteris paribus, it would therefore be efficient to inject funds (fruit) into the hands of the entrepreneurs at $\mathrm{t}=1$.

Unfortunately, consumers cannot commit at $\mathrm{t}=0$ to pay anything to the entrepreneurs at $t=1$ : in effect the consumers, too, cannot "borrow" at $t=0$ because they have no collateral. Also, the entrepreneurs cannot store (perishable) fruit between $t=0$ and $t=1$.

However, there is an indirect method of injecting funds. Namely, raise the price q of capital sold by the entrepreneurs at $\mathrm{t}=1$ - by reducing $\mathrm{k}_{0}$, so as to lower their maintenance costs $\gamma \mathrm{k}_{0}$ and thus shift down their capital supply schedule.

Individually, an entrepreneur cannot raise the price q , and so will not choose to reduce his private choice of capital investment at $\mathrm{t}=0$. But, collectively, the rise in q helps all entrepreneurs.

The price rise directly hurts the consumers, but the transfer of fruit at $\mathrm{t}=0$ more than compensates.

Via ak ${ }_{0}$, the reduction in $\mathrm{k}_{0}$ directly hurts the entrepreneurs' consumption at $\mathrm{t}=2$; but this is more than made up for via $\mathrm{Ak}_{1}$ and the increase in $\mathrm{k}_{1}$, provided A is enough larger than a.

Everyone is strictly better off. This strict Pareto improvement is implemented through a simple transfer $\boldsymbol{\tau}_{0}$ from the entrepreneurs to the consumers at $\mathfrak{t}=0$.

### 2.4. Summary

It may be worthwhile summing up example 2. Consider the effect on the group of entrepreneurs, E, of an upfront transfer from them to the group of consumers, C. The scale of group E's ex ante investment is reduced, which reduces their maintenance costs, thus reducing their need to sell assets at the interim date. The market-clearing asset price rises, which indirectly helps all group E: in effect, funds are injected at the interim date from group C to group E. The gain, to group E, from this injection of funds through the raised asset price, can outweigh their direct loss from the transfer. Meantime, group C indirectly lose from the rise in asset price, but this can be more than offset by their direct gain from the transfer. Overall, everyone can be a net gainer: the ex ante transfer can lead to a Pareto improvement.

Notice the somewhat paradoxical nature of the transfer: from the credit-constrained group E to the unconstrained group C . One might have expected the direction to be the other way: from the deep pockets (group C) to the shallow pockets (group E). The reason why the upfront transfer from group E to group C works well is that it facilitates an indirect subsidy back from group C to group E at the crucial interim date when, at the margin, disinvestment by group E is socially inefficient: group E's return on retaining a marginal unit of asset can greatly exceed the return to group C.

At the heart of this there lies a tension between the individual and the group. No individual entrepreneur would choose to curtail his ex ante investment, because he is too small to affect the asset price at the interim date. As a group, however, the entrepreneurs are better off if they reduce their ex ante investment - thereby, at the interim date, reducing their maintenance costs and raising the asset price.

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[^0]:    ${ }^{1}$ See, for example, Lorenzoni (2008), Jeanne and Korinek (2010), Korinek (2011), Davila (2012), Gersbach and Rochet (2012), He and Kondor (2012), Hart and Zingales (2013), Jeanne and Korinek (2013). For papers that are of a more dynamic stochastic general equilibrium (DSGE) nature, see, for example, Bianchi and Mendoza (2010), Jeanne and Korinek (2010). There are also papers on exchange rate externalities as opposed to asset price externalities; see, for example, Caballero and Krishnamurthy (2003), Korinek (2011). At the intersection of DSGE and exchange rate externalities, see, for example, Bianchi (2011).

