



# THE UNIVERSITY *of* EDINBURGH

## Edinburgh Research Explorer

### Precautionary Demand for Money in a Monetary Business Cycle Model

**Citation for published version:**

Telyukovay, IA & Visschers, L 2013, 'Precautionary Demand for Money in a Monetary Business Cycle Model' *Journal of Monetary Economics*, vol. 60, no. 8, pp. 900–916. DOI: 10.1016/j.jmoneco.2013.08.006

**Digital Object Identifier (DOI):**

[10.1016/j.jmoneco.2013.08.006](https://doi.org/10.1016/j.jmoneco.2013.08.006)

**Link:**

[Link to publication record in Edinburgh Research Explorer](#)

**Document Version:**

Peer reviewed version

**Published In:**

*Journal of Monetary Economics*

**Publisher Rights Statement:**

NOTICE: this is the author's version of a work that was accepted for publication in *Journal of Monetary Economics*. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version will subsequently published in *Journal of Monetary Economics*.

Telyukovay, I. A., & Visschers, L. (2013). Precautionary Demand for Money in a Monetary Business Cycle Model. *Journal of Monetary Economics*, 60(8), 900–916. 10.1016/j.jmoneco.2013.08.006

**General rights**

Copyright for the publications made accessible via the Edinburgh Research Explorer is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

**Take down policy**

The University of Edinburgh has made every reasonable effort to ensure that Edinburgh Research Explorer content complies with UK legislation. If you believe that the public display of this file breaches copyright please contact [openaccess@ed.ac.uk](mailto:openaccess@ed.ac.uk) providing details, and we will remove access to the work immediately and investigate your claim.



# Precautionary Demand for Money in a Monetary Business Cycle Model\*

Irina A. Telyukova<sup>†</sup>  
University of California, San Diego

Ludo Visschers  
Universidad Carlos III de Madrid

November 2, 2012

## Abstract

Precautionary demand for money is significant in the data, and may have important implications for business cycle dynamics of velocity and other nominal aggregates. Accounting for such dynamics is a standing challenge in monetary macroeconomics: standard business cycle models that have incorporated money have failed to generate realistic predictions in this regard. In those models, the only uncertainty affecting money demand is aggregate. We investigate a model with uninsurable *idiosyncratic* uncertainty about liquidity need. The resulting precautionary motive for holding money produces substantial improvements in accounting for business cycle behavior of nominal variables, at no cost to real variables.

**JEL classification:** E21, E32, E41, E52

**Keywords:** precautionary money demand, velocity of money, business cycle, idiosyncratic risk

---

\*First version: June 2008. For helpful comments, we thank Alexander Wolman and an anonymous referee, as well as Jonathan Chiu, Makoto Nakajima, Fernando Alvarez, Manuel Amador, Paul Beaudry, Miquel Faig, Belén Jerez, Allen Head, Amartya Lahiri, Fernando Martin, Martin Schneider and Shouyong Shi as well as seminar participants at UCLA, UCSD, UC Irvine, UC Riverside, Stanford, U. Carlos III Madrid, U. of Chicago, UBC, SFU, Zürich, Leicester and conference participants at EERTEC Florence, 2009 Philadelphia Fed Workshop on Monetary and Macroeconomics, LAEF Payments and Networks Conference at UCSB, CMSG Annual Meetings 2008, Chicago Fed Summer Workshop on Money, Banking and Payments, NES Moscow and IHS Vienna. We completed a previous version of this paper while visiting the Einaudi Institute for Economics and Finance. Visschers acknowledges SFU's President's Research Grant, UC3M's Instituto de Economia, a Spanish Ministry of Science Grant and a grant from the Juan de la Cierva Program. ©2012 I. A. Telyukova and L. Visschers

<sup>†</sup>Corresponding author. 9500 Gilman Drive, 0508, La Jolla CA, 92093-0508. (858) 822-2097. itelyukova@ucsd.edu

# 1 Introduction

In monetary macroeconomics, it is an outstanding challenge to account for business cycle behavior of nominal aggregates and their interaction with real aggregates. Previous business cycle models that have tried to incorporate money explicitly through, for example, cash-in-advance constraints, have done so while assuming that agents face only aggregate risk, which has resulted in the demand for money being largely deterministic, so that the cash-in-advance constraint almost always binds. Such models have unrealistic implications for the dynamics of nominal variables, as well as for interaction between real and nominal variables, when compared to data (Cooley and Hansen, 1995, Hodrick et al, 1991).

This paper presents a theoretical and quantitative investigation of aggregate business cycle implications of *precautionary* demand for money. Precautionary motive for holding liquidity appears strong in the data. Telyukova (2012) documents that the median household has about 50% more liquidity than it spends on average per month, and that controlling for observables, consumption of goods requiring a liquid payment method (e.g. cash or check) exhibits volatility consistent with the presence of significant idiosyncratic risk. This risk and resulting precautionary money demand may have important implications for aggregate money demand and other nominal variables. The goal of this paper is to investigate whether precautionary demand for money can help account for business cycle dynamics of velocity of money, interest rates and inflation, and their interaction with real variables.

We study the relevant mechanisms qualitatively and quantitatively in a model that combines, in each period, two types of markets in a sequential manner, and where both aggregate and idiosyncratic uncertainty are present. The first-subperiod market, termed the “credit market”, is Walrasian. It is close to a standard real business cycle model, with the production function subject to aggregate productivity shocks, but has two distinguishing features. First, households have to decide how much money to carry out of this market for future cash consumption. Second, part of the output in the credit market is carried into the cash market by retail firms, who buy these goods on credit and subsequently transform them into cash goods. This introduces an explicit link between the real and monetary sectors of the economy, as credit-market capital becomes indirectly productive in the cash market.

The second market is also competitive, but in it, agents must consume with money; this is the “cash market”.<sup>1</sup> At the start of this subperiod, households are subject to uninsurable idiosyncratic preference shocks which determine how much of the cash good they want to consume, but the realization of the shock is not known at the time

---

<sup>1</sup>This setup is consistent with both cash-credit good models a la Lucas and Stokey (1987) and monetary search models in the style of Lagos and Wright (2005). In theory, money-search-style idiosyncratic matching shocks could be interpreted as a type of idiosyncratic preference shock (Wallace 2001). However, with matching shocks, agents spend all or none of their money, while a crucial part of the argument here is that a preference shock may cause a household to spend only part of their money holdings. The natural empirical counterparts of the two types of shocks are also different.

that agents make their portfolio decisions. This generates precautionary motive for holding money. We show analytically how the idiosyncratic shocks, and the resulting heterogeneity of households, result in amplified dynamics of velocity of money, and how in their absence, the model can produce counterfactual nominal dynamics under standard parameter values.

The calibration of the model is in itself a contribution. All the existing models of the type mentioned above that have looked at aggregate behavior of nominal variables have been calibrated to aggregate data. Instead, this paper also uses micro data on liquid consumption from the Consumer Expenditure Survey, like in Telyukova (2012), to calibrate idiosyncratic preference risk in the cash market. In general, in the few contexts where precautionary liquidity demand has appeared, it has been treated as a free parameter (e.g. Faig and Jerez, 2007). The use of micro data tightly disciplines measurement of the risk, and hence of the extent of precautionary money demand.

Once calibrated, the model's equilibrium is computed to investigate the effects of real productivity shocks and monetary policy shocks. The main finding is that precautionary demand for money makes a dramatic difference for a variety of dynamic moments of nominal aggregates in the data, relative to a version of the model with the idiosyncratic risk shut down. The key mechanism is in breaking of the link between aggregate money demand and aggregate consumption. In deterministic monetary business cycle models, the cash-in-advance constraint in practice always binds, so that aggregate money demand is equivalent to aggregate cash-good consumption. This tightly links the volatilities of money demand and aggregate consumption; the latter, in turn, is not volatile enough in the data to generate observed volatility of money demand (or its inverse, velocity) and other nominal aggregates. In contrast, in the model with precautionary money demand, agents generally hold more money than they spend, so that total money demand is now linked only to consumption of agents whose preference shock realizations make them spend all of their money in trade. Velocity of money is significantly more volatile in this heterogeneous-agent setting, thanks to the agents whose cash constraint does *not* bind, who are absent in models with only aggregate risk. In other words, idiosyncratic risk in this context does not average out in a way that can be captured by a representative agent model (see discussion in Hodrick et al (1991)). In addition, the magnitude of idiosyncratic volatility is much higher than aggregate volatility: the standard deviation of aggregate consumption is 0.5%; the standard deviation of household-level cash consumption turns out to be around 19%.

Introducing uninsurable idiosyncratic risk into the model also changes the nature of the inflation tax, and thus has an impact on welfare costs of inflation. In any cash-credit good model, the nominal interest rate drives a wedge between the marginal rate of transformation and the marginal rate of substitution between cash and credit goods. Without idiosyncratic shocks, this wedge affects all households equally, since all have a binding

cash constraint. The cost of a positive nominal interest rate is in having to hold the money to transact in the cash market, but the allocation equates the marginal utilities across households. Instead, with uninsurable idiosyncratic risk, the additional inefficiency is that the allocation of total cash-good consumption among households is also unequal. The agents whose idiosyncratic shock realization causes their cash constraint to bind have marginal utility that is higher than that of the unconstrained agents, and thus, *ex post*, bear more of the cost of inflation. This raises the welfare cost of inflation. The nature of the inflation tax in the model with idiosyncratic risk also depends on whether inflation increases are anticipated. A surprise increase in inflation can exacerbate this last distortion.

These results suggest that in many monetary contexts, especially those aimed at accounting for aggregate data facts, it is important not to omit idiosyncratic uncertainty that gives rise to precautionary demand for money. As one example, we demonstrate that omitting this empirically relevant mechanism may cause the standard practice of calibrating monetary models to the aggregate money demand equation, as has been done in many cash-in-advance models and monetary search models, to produce misleading results for parameters and counterfactual quantitative implications.

This paper is related to several strands of literature. On the topic of precautionary demand for liquidity,<sup>2</sup> the key mechanism in this model is close to Telyukova (2012), Telyukova and Wright (2008) and Faig and Jerez (2007). In Telyukova and Wright (2008) and Telyukova (2012), the idiosyncratic uncertainty about liquidity need is shown, respectively theoretically and quantitatively, to be relevant for household portfolio decisions to hold liquid assets and credit card debt simultaneously. Faig and Jerez (2007) look at the behavior of velocity and nominal interest rates over the long run. They find that with precautionary liquidity demand, the simulated time series of velocity over the last century, interpreted as a series of steady states, fits the empirical series well. Lagos and Rocheteau (2005) study steady state properties of a monetary economy with idiosyncratic preference shocks.<sup>3</sup> Wang and Shi (2006) are also interested in aggregate behavior of nominal variables, but with search intensity as the key mechanism behind velocity fluctuations over the business cycle.

Section 2 below describes the model. Section 3 demonstrates *analytically* the impact of precautionary demand for money on the dynamic behavior of money, velocity and interest rates, and discusses the inflation tax. Section 4 describes the calibration strategy for several versions of the model. In section 5 we discuss the

---

<sup>2</sup>The subject of precautionary money demand goes back to at least Keynes(1936), who defined its reason as “to provide for contingencies requiring sudden expenditure and for unforeseen opportunities of advantageous purchases”. Precautionary demand for money is often modeled in Baumol-Tobin-style inventory-theoretic models, from Whalen (1966) to fully dynamic stochastic models such as Alvarez and Lippi (2009). Uninsurable idiosyncratic liquidity shocks are also an essential element of models based on Diamond and Dybvig (1983). Lucas (1980) studies the equilibrium in a cash-in-advance model with precautionary demand for money.

<sup>3</sup>In another broadly related paper, Hagedorn (2008) shows that strong liquidity effects can arise when precautionary demand for money is taken into account in a cash-credit good model with banking.

*quantitative* role of precautionary liquidity demand, and show how omission of precautionary demand may lead model calibration and implications astray. Section 6 shows the effect of precautionary demand on welfare costs of anticipated and unanticipated inflation. Section 7 concludes.

## 2 Model

The economy is populated by measure 1 of infinitely-lived households, who rent labor and capital to firms, consume goods bought from the firms, and save. Two markets are open sequentially during the period. In the first subperiod, a Walrasian market opens, in which all parties involved in transactions are known and all trades can be enforced. In the second subperiod, the market is competitive, in the sense that all agents are price takers, but money is assumed essential in trade.<sup>4</sup> Since in the first subperiod households use cash or credit for consumption, and as discussed below, retail firms buy on credit, we will refer to this as the “credit market”,<sup>5</sup> while the second subperiod will be termed the “cash market”.

There are two types of firms in the economy. Production firms use capital and labor as inputs in production, and their output is used for consumption and capital investment in the credit market. Part of their output is also bought in the credit market by retail firms, who then transform the goods one-for-one into retail goods to be sold in the cash market.<sup>6</sup>

### 2.1 Households

Households maximize lifetime expected discounted utility,

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( U(c_t) - Ah_t + \vartheta_t u(q_{\vartheta,t}) \right) \right] \quad (1)$$

where  $0 < \beta < 1$ . While here for analytical tractability the utility function is assumed separable, in computation both separable and CES utility functions will be used.<sup>7</sup> Inada conditions on  $U(\cdot)$  and  $u(\cdot)$  ensure that consumers participate in both markets. Utility depends on consumption  $c_t$  and time spent working  $h_t \in [0, 1]$  in the first subperiod, and on consumption  $q_t$  and the preference shock  $\vartheta_t$  in the second. First-subperiod utility follows the Hansen-Rogerson specification of indivisible labor with lotteries. The taste shock  $\vartheta_t$  is realized when the credit

---

<sup>4</sup>Temzelides and Yu (2004) derive sufficient conditions under which money is essential in competitive markets. See also Levine (1991) and Rocheteau and Wright (2005).

<sup>5</sup>In the data, credit cards are a complex contract that for some households is a convenience device, while for others, a long-term revolving credit arrangement. Here, the credit market is a natural simplification not meant to capture credit cards tightly. Following survey data, cash consumption will be defined as goods which cannot be paid by credit.

<sup>6</sup>The retailers are not meant to correspond one-for-one to the retail sector in the data: some retailers in the data are better characterized as selling in the credit market, whereas the cash sector includes firms that are not retailers.

<sup>7</sup>Given that the leisure component of utility is linear, results along the same lines as presented go through for the CES case, at some notational cost.

market is already closed and money holdings can no longer be adjusted, as described below. This will lead to precautionary demand for money.

Households start the period with money holdings  $M_t$  and choose  $\tilde{M}_t$  money to bring into the cash market, before  $\vartheta_t$  occurs. Households also own capital  $k_t$  and nominal bonds  $B_t$  sold to them by retail firms, as detailed below.<sup>8</sup> Let wage, capital rent, and the return on nominal bonds be  $w_t, r_t, i_{t-1}$ . Let the nominal price of the credit good be  $P_t$ . The budget constraint, expressed in nominal terms, is

$$M_t + (1 + r_t - \delta)P_t k_t + w_t P_t h_t + B_t(1 + i_{t-1}) = P_t c_t + \tilde{M}_t + P_t k_{t+1} + B_{t+1} \quad (2)$$

Further, given nominal price  $\Psi_t$  of the cash good, consumption  $q_{\vartheta,t}$  in the cash market, conditional on the preference shock realization  $\vartheta_t$ , has to satisfy  $\Psi_t q_{\vartheta,t} \leq \tilde{M}_t$ . From now on, we will normalize household money and nominal bond holdings by the aggregate money supply  $\mathcal{M}_t$ , rendering them stationary, which will lead to the following normalizations:  $m_t = M_t/\mathcal{M}_t$ ,  $b_t = B_t/\mathcal{M}_t$ ,  $p_t = P_t/\mathcal{M}_t$  and  $\psi_t = \Psi_t/\mathcal{M}_t$ . In addition, to formulate the budget constraint in real terms below,  $\phi_t = 1/p_t$  will denote the value of one unit of normalized money in terms of the credit good.

## 2.2 Firms

The problem of the *production* firm is standard. Given a constant returns to scale production function  $y_t = e^{z_t} f(k_t, h_t)$ , where  $z_t$  is stochastic productivity level, the firm solves  $\max_{k_t, h_t} \{e^{z_t} f(k_t, h_t) - w_t h_t - r_t k_t\}$ . The solution is characterized by the usual first-order conditions.

*Retail* firms exist for two periods: they buy the goods in the credit market, selling nominal bonds to households to do so, sell the goods in the subsequent cash market, and settle their debt in the following credit market, before disbanding. Free entry in the retail market yields the following condition, expressed in nominal terms at time  $t$ :

$$\Pi_{rt} = \max_{q_t} \left( \frac{\psi_t q_t}{1 + i_t} - \frac{q_t}{\phi_t} \right) = 0. \quad (3)$$

All cash receipts from retail sales go towards repayment next period. The repayment equals the current nominal value for the  $q_t$  goods purchased in the credit market, and is discounted using the nominal interest rate. Since the cash market is competitive, retail firms sell all their goods in equilibrium,<sup>9</sup> and the cash market price is higher than the credit market price by a factor  $1 + i$ , adjusted for marginal utility.

<sup>8</sup>In principle, households can hold shares of firms as well. Since all firms make zero profits, however, shareholding is irrelevant. Alternatively, the economy can be written with firms selling shares instead of bonds; this leads to equivalent allocations, but involves more notation.

<sup>9</sup>We assume that goods are storable, so even at zero expected real interest rate, this is without loss of generality.

### 2.3 Monetary Policy and Aggregate Shocks

The monetary authority follows an interest rate feedback rule

$$\frac{1 + \bar{i}_{t+1}}{1 + \bar{i}} = \left( \frac{1 + \bar{i}_t}{1 + \bar{i}} \right)^{\xi_{ii}} \left( \frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\xi_{i\pi}} \left( \frac{y_t}{\bar{y}} \right)^{\xi_{iy}} \exp(\varepsilon_t^{mp}). \quad (4)$$

The variables with bars denote central bank's long-run target levels of output, inflation and the nominal interest rate. The term  $\varepsilon_t^{mp}$  denotes a stochastic monetary policy shock which is realized at the beginning of period  $t$ . At the end of the period, the government makes a lump sum transfer  $\varpi_t \mathcal{M}_t$ . The rate of money supply growth  $\varpi_t$  is adjusted by the central bank to make  $i_t$  arise as the equilibrium price.

The second, independent, aggregate shock process is on the productivity level  $z_t$ . As is standard in business cycle literature,  $z_t$  follows an AR(1) of the form  $z_{t+1} = \xi_{zz} z_t + \varepsilon_{t+1}^z$ .

### 2.4 Recursive Formulation of the Household Problem

From now on, we will conserve notation by omitting time subscripts, and using primes to denote  $t + 1$ . The aggregate state variables in this economy are  $S = (\mathcal{K}, z, i_{-1}, i, (1 + \varpi_{-1})\phi_{-1})$ : the aggregate capital stock, the technology shock, previous interest rate in the economy, current interest rate, and the previous period's post-injection real value of money, which households need to determine the current rate of inflation. The individual state variables at the beginning of the credit market are  $s = (k, m, b)$ : capital holdings, and money and bond holdings normalized by the money stock. The credit good is the numeraire. The household solves:

$$V(k, m, b, S) = \max_{c, h, \tilde{m}, k', b', \{q_\vartheta\}} \left\{ U(c) - Ah + \mathbb{E}_\vartheta \left[ \vartheta u(q_\vartheta) + \beta \mathbb{E}_{z', i'} V(k', m', \frac{b'}{1 + \varpi}, S') \right] \right\} \quad (5)$$

$$\text{s.t. } c + \phi \tilde{m} + k' + \phi b' = \phi m + \phi b(1 + i_{-1}) + (1 + r - \delta)k + wh \quad (6)$$

$$\psi q_\vartheta \leq \tilde{m} \quad (7)$$

$$\pi = \frac{(1 + \varpi_{-1})\phi_{-1}}{\phi} \quad (8)$$

$$m' = \frac{\tilde{m}}{1 + \varpi} - \frac{\psi q_\vartheta}{1 + \varpi} + \frac{\varpi}{1 + \varpi} \quad (9)$$

$$z' = \xi_{zz} z + \varepsilon_1' \quad (10)$$

$$(\widehat{1 + i'}) = \xi_{ii}(\widehat{1 + i}) + \xi_{i\pi}\hat{\pi} + \xi_{iy}\hat{y} + \varepsilon_2' \quad (11)$$

$\hat{x}$  refers to log-deviations of the variable  $x$  from its target level. Denote the policy functions of the household's problem by  $g(s, S)$ , with  $g_x(\cdot)$  as the policy function for the choice variable  $x$ . Assume that these functions, and the value functions, are stationary, i.e. not time-dependent.



**Definition 1.** A Symmetric Monetary Equilibrium is a set of pricing functions  $\phi(S)$ ,  $\psi(S)$ ,  $w(S)$ ,  $r(S)$ ; law of motion  $\mathcal{K}'(S)$ , value function  $V(s, S)$  and policy functions  $g_c(s, S)$ ,  $g_h(s, S)$ ,  $g_k(s, S)$ ,  $g_b(s, S)$ ,  $g_m(s, S)$ ,  $\{g_{q,\vartheta}(s, S)\}$ , all  $\vartheta$ , such that: (i) households optimize by solving (5), given prices and laws of motion; (ii) production and retail firms optimize, as in section 2.2; (iii) free entry of retailers implies  $\Pi_r = 0$ ; (iv) the aggregate law of motion follows from the aggregation of all individual decisions:  $\mathcal{K}'(S) = \int_0^1 g_k^i(s, S) di$ . Finally, (v) all markets clear:

$$\begin{aligned} \int_0^1 g_m^i(s, S) di &= 1 \\ \int_0^1 \phi(S) g_b^i(s, S) di &= \int_0^1 [\mathbb{E}_{\vartheta} g_{q,\vartheta}^i(s, S)] di \\ \int_0^1 g_h^i(s, S) di &= H(S) \\ (1 - \delta)\mathcal{K} + e^z f(H(S), \mathcal{K}) &= \int_0^1 g_c^i(s, S) di + \mathcal{K}'(S) + \int_0^1 [\mathbb{E}_{\vartheta} g_{q,\vartheta}^i(s, S)] di \end{aligned}$$

Appendix A presents the analysis of the equilibrium conditions of the problem, to show that the quasi-linear specification of the utility function allows equilibria in which the distribution of wealth created by the idiosyncratic shocks in the cash market washes out in the following credit market, as long as  $h$  remains in the interior. The following result is immediate:

**Result 1.** *The choice of  $c, \tilde{m}, k', b'$  only depends on the aggregate states  $S$ .*

Further, it can be shown that for low enough realizations of the shock  $\vartheta$ , cash balances are not spent in full, and that the resulting  $q_\vartheta$  is efficient, equalizing the marginal utilities of credit- and cash-good consumption. In contrast, if a shock  $\bar{\vartheta}$  results in a binding cash constraint, then for any  $\vartheta > \bar{\vartheta}$ , the constraint will also bind. This leads to underconsumption of the cash good relative to social optimum.

What does this say about consumer payment behavior? In the data, it is reasonable to expect that different consumers make different choices with respect to how much to consume with cash versus credit. In the model, the cash market is meant to capture transactions which cannot be paid using credit (see calibration section). However, the “credit” market in the first subperiod is more flexible: the model is silent on whether households pay using credit or cash, as these methods can be costlessly transferred one into the other. Thus the model implicitly can accommodate the kind of heterogeneity in portfolios and payment methods seen in the data. In order to discipline the model, we will, in calibration, match the total volume of consumer transactions done by

liquid payment methods (cash, check, debit) in aggregate data. Since the dynamics of interest are of aggregate variables, individual heterogeneity in payment methods will not affect them beyond getting the transaction shares right.

### 3 Idiosyncratic Uncertainty and Nominal Dynamics

In this section, we demonstrate analytically that there are at least three ways in which idiosyncratic shocks to cash-good preferences can improve the quantitative performance of the model. First, the dynamic behavior of the value of money and prices varies significantly with the probability that the marginal dollar is spent, i.e. that the cash constraint binds. As a result, the model with idiosyncratic shocks can accommodate values of the relative risk aversion (RRA) parameter in the standard RBC calibration range ( $\sigma \in [1, 4]$ ), whereas the model without shocks would require  $\sigma < 1$  to produce realistic dynamics of prices. Second, part of velocity fluctuation is now generated in the cash market, thus increasing the overall magnitude of velocity volatility, and velocity now depends in an intuitive way on nominal interest rates. Third, the standard general-equilibrium substitution channel in cash-credit good models between cash and credit good consumption is dampened, because cash consumption will now only adjust for the binding realization of the shock. Most proofs are in appendix B.

#### 3.1 Dynamic Behavior of Real Balances

The dynamic behavior of money will be an essential input for relating velocity to interest rates. It is also, however, empirically relevant in itself: one uncontroversial empirical regularity is the degree of persistence of interest rates, prices, and real balances, before and after detrending, over the business cycle. Nominal interest rates have an autocorrelation at quarterly frequency of 0.932; for real balances, it is 0.951.<sup>10</sup> It seems a minimal requirement that a monetary business cycle model can replicate the sign of these autocorrelations. This requirement turns out to have important implications for the range of the RRA parameter admissible in calibration.

For the sake of exposition, assume two preference shock realizations  $\vartheta_i$ , where  $\vartheta_h$  leads to a binding cash constraint, and  $\vartheta_l$  to a nonbinding constraint.<sup>11</sup> Write  $p$  for the probability of the high shock  $\vartheta_h$ . Note that if  $p = 1$ , there are no idiosyncratic shocks. We temporarily simplify the utility function in the credit market to be

<sup>10</sup>BP-filtered, nominal interest rate from three-month treasury bonds, real money balances from M2 and the GDP deflator (source: FRED2).

<sup>11</sup>In general, as  $i$  increases, more of the shock realizations may lead to a binding cash constraint; here, for exposition, we assume that only the high shock binds throughout. Generically, for small enough fluctuations in  $i$ , this assumption will hold. It will be relaxed in computation.

fully linear,  $U(c) = c$ . For the dynamics of real balances, the equilibrium conditions of the problem imply the relationship between real balances today  $\phi$  and expected real balances tomorrow  $\widetilde{\mathbb{E}\phi'} \equiv \mathbb{E} \left[ \frac{\phi'(S')}{1+\varpi} \frac{U'(c')}{U'(c)} \right]$ :

$$\phi = p \vartheta_h u'(\beta \widetilde{\mathbb{E}\phi'}) \beta \widetilde{\mathbb{E}\phi'} + (1-p) \beta \widetilde{\mathbb{E}\phi'}, \quad (12)$$

Recall that real balances  $\phi$  equal normalized money supply over the price level, and since normalized money stock is 1,  $\phi$  is also the real value of one unit of money. (12) gives that the real value of one unit of money is a weighted average of the real value of a unit of money in use on the margin, needed only with probability  $p$ , and the value when this unit is not used and is carried over to the next period.

**Lemma 1.** *The elasticity of real balances today with respect to real balances tomorrow evaluated at an equilibrium  $\phi, \beta \widetilde{\mathbb{E}\phi'}$ , is given by*

$$\varepsilon_{\phi, \beta \widetilde{\mathbb{E}\phi'}} = \left( 1 - \frac{1-p}{1+i} \right) (1-\sigma) + \frac{1-p}{1+i}. \quad (13)$$

Consider the case without idiosyncratic shocks, where the cash constraint always binds ( $p = 1$ ). In this case,  $\varepsilon_{\phi, \beta \widetilde{\mathbb{E}\phi'}} = 1 - \sigma$ . The value of the marginal unit of money when it needs to be spent depends on the strength of the households' motive to smooth consumption, captured by the intertemporal elasticity of substitution (IES)  $1/\sigma$ . Lower real value of money tomorrow implies that the value of a unit of money is also lower in today's cash market; fewer goods can be bought, raising the marginal utility. With  $\sigma > 1$ , households are averse enough to variation in consumption that, when real balances tomorrow are expected to decrease, the marginal utility rises so much that the demand for real balances in today's credit market goes up. This leads to a negative autocorrelation of real balances (equivalently,  $\varepsilon_{\phi, \beta \widetilde{\mathbb{E}\phi'}} < 0$ ). To avoid such counterfactual behavior of monetary variables in the setting without idiosyncratic shocks, one has to make households less averse to consumption fluctuations ( $0 < \sigma < 1$ ). This, however, will imply an IES that is nonstandard in business cycle modeling, leading to counterfactually strong responses of real variables to real shocks. In sum, with no idiosyncratic uncertainty, one must face a tradeoff between counterfactual monetary outcomes and counterfactual real outcomes.

With idiosyncratic preference shocks, one does not have to make this unattractive choice. With  $p < 1$ , the second term on the RHS of (13) is positive, and real balances tomorrow in part determine directly real balances today, because with probability  $(1-p)$  the marginal unit of money is not used. Thus the elasticity of real balances today with respect to real balances tomorrow is brought closer to 1, and the demand for money is smoothed relative to the no-shock model. This means that households can have a lower IES, without implying a

negative autocorrelation of real balances.<sup>12</sup>

### 3.2 Dynamic Behavior of Velocity

Denote by  $C$  aggregate consumption within a period. The consumption and output velocities of money in the above example with two idiosyncratic shocks can be written as

$$V_c = \frac{PC}{M} = \frac{c}{\phi} + (1-p) \frac{q_l(1+i)}{\phi} + p \frac{q_h(1+i)}{\phi}$$

$$V_y = \frac{PY}{M} = \frac{(y - (1-p)q_l - pq_h)}{\phi} + (1-p) \frac{q_l(1+i)}{\phi} + p \frac{q_h(1+i)}{\phi}$$

Looking at consumption velocity, observe that as in standard cash-in-advance and cash-credit-good models, the constrained part of the cash market always contributes 1 to the level of consumption velocity, and nothing to velocity fluctuations, because  $\frac{q_h(1+i)}{\phi} = 1$ . Thus, if  $p = 1$ , then all velocity movement has to come from the credit market - i.e. from  $c$  or  $\phi$ . Instead, in our model, velocity fluctuations are also created in the cash market, thanks to the low shocks where the cash constraint does not bind.<sup>13</sup>

One can also see this by looking at marginal rates of substitution between cash and credit market consumption. The MRS for the binding and non-binding cases is

$$\frac{\partial_h u'(q_h)}{U'(c)} = 1 + \frac{i}{p} \quad (14)$$

$$\frac{\partial_h u'(q_h)}{U'(c)} = 1, \quad (15)$$

Without preference shocks ( $p = 1$ ), cash market consumption thus always depends on nominal interest rates, as in (14). Preference shocks add agents who are not constrained ( $p < 1$ ), and whose cash market consumption does not depend on  $i$ , as in (15). Having arrived in the cash market, money holdings are predetermined, and households trade off spending a dollar in the cash market versus spending it next credit market. Given competitive pricing by retail firms', this results in an undistorted consumption allocation of unconstrained agents. Unconstrained agents do not adjust their consumption in response to changes in  $i$ , but in response to price changes, they adjust their money spending, and hence they contribute to fluctuations in velocity. Constrained agents respond to price

<sup>12</sup>In calibration, we will have a nominal interest rate rule with persistence. In a setting with  $p = 1$  and  $\sigma > 1$ , persistence in the nominal interest rate could be achieved by alternating expansions and contractions of the money supply. Again, this would be counterfactual.

<sup>13</sup>Models with variable search intensity also create velocity fluctuations in the cash market (Wang and Shi 2006). Standard search models with fixed match probabilities do not.

changes through consumption, but the total amount of money spent does not move.

This analysis also sheds light on the nature of the inflation tax, relative to the standard cash-credit good model. One can distinguish between the ex-ante inflation tax, borne in the credit market, and the ex-post tax, borne after the preference shocks are realized. The ex-ante inflation tax affects all agents equally, as long as the nominal interest rate is positive, because money is costly to hold. In a model without preference shocks, this tax is reflected in the higher cash market price than is dictated by the technology. With preference shocks, there is an additional aspect: the nominal interest rate is the cost of self-insurance against these shocks, and thus households economize on carrying real balances into the cash market. Furthermore, at higher prices in the cash market, less is sold in total, and hence less total output is produced in the credit market, which leads agents to work less in the credit market.

For the ex-post inflation tax, without preference shocks, the wedge in the MRS (14) affects everyone equally, distorting allocations relative to the optimum. With preference shocks, constrained and unconstrained agents are impacted differentially by this tax. The MRS of the unconstrained agents implies that their allocation is optimal; on their unused cash, the unconstrained agents bear the tax in the form of foregone interest. Instead, the constrained agents suffer a distortion of consumption in the cash market. Note in addition that, for a given level of  $q$  supplied by retailers, the MRS of constrained households is higher than that of the unconstrained households, and higher than the opportunity cost of holding money,  $1 + i$ . This is contrary to the solution of a planner constrained only by  $q$ , who would equate these marginal utilities across preference shock realizations, like in the no-shock case. For the constrained agents, the cost of inflation is higher than the average in the no-shock model. We will evaluate the quantitative implications of this in the Welfare section.

Returning to velocity dynamics, the elasticity of consumption velocity with respect to  $i$  can be divided into credit-market and cash-market components  $\varepsilon_{V_c, 1+i} = s_c(\varepsilon_{c, 1+i} - \varepsilon_{\phi, 1+i}) + s_{\text{cash, nb}} \cdot \varepsilon_{\psi, 1+i}$ , where  $s_c$  is the share of the credit good in total consumption expenditure, and  $s_{\text{cash, nb}}$  is the share of cash consumption under non-binding preference shocks.

**Lemma 2.** *Elasticity of the cash market price with respect to the interest rate is always positive, and is given by*

$$\varepsilon_{\psi, 1+i} = \varepsilon_{\frac{1+i}{\phi}, 1+i} = \frac{1+i}{\sigma(p+i)} > 0. \quad (16)$$

Thus, the less risk-averse the household is, or the smaller the probability of a binding constraint is, the more of

the velocity fluctuations originate in the cash market, *ceteris paribus*.

To conclude the analytical discussion, we now incorporate the response of credit-good consumption  $c$  to changes in prices and interest rates. To do this, drop the linearity assumption on  $U(c)$ ; this adds a general-equilibrium feedback effect linking nominal interest rates and velocity, through substitution between cash and credit goods. The only assumption needed for analytical tractability is that capital is constant; while this shuts down one equilibrium effect, it does not alter the other effects of interest.<sup>14</sup>

**Proposition 1.** *The implicit elasticity of consumption velocity with respect to the nominal interest rate, caused by a one-time fully anticipated money injection (in addition to the constant rate of money growth consistent with a given steady state level of  $1 + i$ ), is*

$$\varepsilon_{V_c, 1+i} = s_c \left( \frac{1}{\sigma} \frac{1+i}{p+i} - 1 \right) + s_{cash, nb} \left( \frac{1+i}{p+i} \right) \left( \frac{1}{\varepsilon_{U'(c), q_h} + \sigma} \right) \quad (17)$$

*Proof.* For this proof, redefine next period's real balances as  $\widetilde{\mathbb{E}\phi'} \equiv \mathbb{E}U'(c')\phi'/(1+\varpi)$ . From the free entry condition  $U'(c)q = \beta\widetilde{\mathbb{E}\phi'}$ , the elasticity of cash consumption to the money injection for a constrained agent is  $-\varepsilon_{q_h, 1+\varpi} = (\varepsilon_{U'(c), q_h} + 1)^{-1}$ . Then, from (12),

$$\varepsilon_{U'(c)\phi, \beta\widetilde{\mathbb{E}\phi'}} = \frac{d \ln U'(c)\phi}{d \ln \beta\widetilde{\mathbb{E}\phi'}} = \left( \frac{p+i}{1+i}(1-\sigma) \right) \frac{1}{\varepsilon_{U'(c), q_h} + 1} + \frac{1-p}{1+i}. \quad (18)$$

This gives elasticity of consumption velocity with respect to future value of money as

$$\varepsilon_{V_c, \beta\widetilde{\mathbb{E}\phi'}} = s_c \left\{ \left[ \left( -\frac{1}{\sigma} + 1 \right) \varepsilon_{U'(c), q_h} - \frac{p+i}{1+i}(1-\sigma) \right] \frac{1}{\varepsilon_{U'(c), q_h} + 1} - \frac{1-p}{1+i} \right\} - s_{cash, nb} \frac{1}{\varepsilon_{U'(c), q_h} + 1}.$$

This and (18) imply (17). A detailed proof is in appendix B.3. □

The only difference between (18) and (13) is the term  $(\varepsilon_{U'(c), q_h} + 1)^{-1}$ , which captures general equilibrium feedback between  $q_h$  and  $c$ , taking into account the optimal labor supply decision. As less is sold in the retail market, less has to be produced in the credit market. This improves marginal productivity of labor and raises credit consumption. As before, in (18), if  $p = 1$ ,  $\sigma < 1$  is needed to get positive autocorrelations of prices, real money stock and interest rates, and this constraint is relaxed if  $p < 1$ .

In (17), observe the channels through which idiosyncratic uncertainty works: (i) credit market effects through

---

<sup>14</sup>Capital fluctuations remain an important ingredient of the computed model below.

the leftmost term; (ii) cash market channel through the right-side  $(1 + i)/(p + i)$  term; and (iii) the general equilibrium substitution channel, through  $\varepsilon_{U'(c), qh}$ . We graph these components as a function of  $\sigma$  in figure 1, with  $p = 0.5$ . Idiosyncratic shocks raise the elasticity of velocity with respect to interest rates dramatically, as signified by the vertical difference between the grey dashed and black dashed lines in the graph, and allow for a positive elasticity for a much larger range of  $\sigma$ . Note also, in the difference between the top dotted and the top solid lines, that the general equilibrium effect is small, but raises the elasticity further. Keeping the size of the cash market fixed, and lowering  $p$ , it can be shown that the sensitivity of velocity to interest rates through the cash market channel is raised.

## 4 Calibration

The model period is a quarter. The utility function is CES:

$$U(c, q) = \frac{\left( (\alpha c^\nu + (1 - \alpha) \vartheta q^\nu)^{\frac{1}{\nu}} \right)^{1 - \sigma}}{1 - \sigma} \quad (19)$$

We will calibrate the model both with this function, and with the separable form used in most of the analytical work, which is a special case of the CES form. In that case,  $U(c) = \frac{c^{1 - \sigma}}{1 - \sigma}$  and  $u(q) = \frac{x_1 q^{1 - \sigma}}{1 - \sigma}$ . The production function is Cobb-Douglas:  $f(k, h) = k^\theta h^{1 - \theta}$ .

In the separable case, the parameters to be calibrated are  $\beta$ ,  $\sigma$ ,  $A$ ,  $\theta$  and  $\delta$ , as well as  $x_1$  and the process for the idiosyncratic shock  $\vartheta$ . For the CES utility, the parameters are  $\alpha$ , the share of credit goods in the consumption mix, and  $\nu$ , the parameter that guides elasticity of substitution between cash and credit goods. Finally, the parameters of the exogenous processes  $\{\xi\}$ ,  $\sigma_{\varepsilon_1}$  and  $\sigma_{\varepsilon_2}$  have to be calibrated.

We will use the M2 measure of money supply to compute the nominal moments in the data. This choice follows the literature, e.g. Hodrick et al (1991) and Wang and Shi (2006). Another reason is that M2 exhibits much more stationarity over time than M1. But the most important reason for this choice is that in the data, liquid payment methods include not only cash, but also checks and debit cards, which implies inclusion in the monetary aggregate of checking accounts. In addition, it is intuitive, in studying precautionary money demand, that the monetary aggregate should include savings accounts. A concern may arise that some money in M2 earns interest, which does not happen in the model. In response to this concern, quantitative results are also presented for a modified version of the model, in which we allow interest to be paid on money that is carried over from

period to period as savings by households. Appendix C presents this modification and the resulting equilibrium conditions.<sup>15</sup> Notice that in this modified model, there is an extreme assumption that the representative liquid savings earn interest; in the data, a significant portion of the precautionary balances is held in non-interest-paying forms like cash and checking accounts (see Telyukova (2012)), so this approach likely overstates the share of interest-bearing liquid balances. The calibration of all the model versions is in table 1.

#### 4.1 Preference and Production Parameters

The preference and production parameters of the model are calibrated as follows.  $\beta = 0.9901$  matches the annual capital-output ratio of 3.  $\sigma = 2$  is chosen within, and on the lower side of, the standard range of  $[1, 4]$  in the literature.  $A$  is chosen each time to match aggregate labor supply of 0.3. The capital share of output is measured in the data to give  $\theta = 0.36$ . Quarterly depreciation rate of 2%, consistent with estimates in the data, gives  $\delta = 0.02$ .

For the separable model, the constant  $x_1$  is calibrated to match the size of the retail (cash) market. The target is 75% of total consumption in the model, consistent with the aggregate fact, documented in Telyukova (2012), that roughly 75% of the total value of consumer transactions in 2001 took place using liquid payments methods - cash, checks, and debit cards. This number was quoted at 82% in 1986 in Wang and Shi (2006), based on a consumer survey.

In an alternative calibration,  $x_1$  is chosen to target the average level of M2 output velocity ( $V_y = 1.897$ ) in the data sample (1984-2007, as detailed below). This results in the size of the cash market of 50% of total consumption. The model calibrated this way produced the same dynamic results, so they are only shown in section 5.3 for appropriate comparison (table 7).

For the CES utility case, the parameter  $x_1$  is no longer used. The parameters  $\alpha$  and  $\nu$  have to be estimated jointly, together with  $\sigma_\theta$ , in an SMM procedure. For  $\alpha$ , we target the size of the cash-good market, 75%. For the parameter  $\nu$ , the target is the interest rate elasticity of the ratio of aggregate cash good consumption to aggregate credit good consumption. This measure is constructed using the Consumer Expenditure Survey over the period 1980-2004.<sup>16</sup> Over the entire period, household nondurable consumption items are separated into cash goods

<sup>15</sup>We also tried the version of the model where interest is paid on all money, both within period and across periods; the model results look very similar.

<sup>16</sup>Thanks to Dirk Krueger and Fabrizio Perri for providing us with the aggregated CEX data set for this period. All the expenditure components in their data set are deflated by the relevant component of the CPI.



and credit goods first, using the definitions discussed below for the calibration of preference shocks.<sup>17</sup> Then using household weights, individual cash and credit good consumption are aggregated. For the desired elasticity, the cash-to-credit log-filtered consumption ratio is then regressed on the T-Bill rate. The resulting interest rate elasticity is -1.24. The model is able to hit the targets exactly in each calibration. In the model without interest-paying money,  $\nu = 0.39$  implies the elasticity of substitution between the two types of goods of 1.6.

## 4.2 Idiosyncratic Preference Shock Process

Let the log of the preference shock be i.i.d.  $N(0, \sigma_{\vartheta})$ .<sup>18</sup> We interpret the preference shocks as causing fluctuations in household liquid consumption *beyond* expected (e.g. seasonal or planned) fluctuations in the data. The process for this shock is calibrated following Telyukova (2012), based on quarterly data from the 2000-2002 Consumer Expenditure Survey (CEX).

The key measure is the unpredictable component of volatility of cash-good consumption in the data. This component is assumed to reflect optimal responses by households to unexpected preference shocks.<sup>19</sup> The standard deviation of the shock process  $\sigma_{\vartheta}$  is estimated by SMM such that standard deviation of cash-good consumption in the model matches that in the data.

The process of this measurement is described in Telyukova (2012) in detail. The first step is to separate out cash goods in the CEX data. Based on the American Bankers Association’s 2004 survey of consumer payment methods, the following are defined as cash goods: food, alcohol, tobacco, rents, mortgages, utilities, household repairs, childcare expenses, other household operations, property taxes, insurance, public transportation, and health insurance. Even in 2004, consumers reported paying for these types of goods with liquid assets (primarily cash and check) in 90% or more of transactions. This proportion would clearly be higher over the longer period of inquiry, 1984-2007. This definition is also conservative along some other dimensions. First, volatility of expenses could be driven by seasonality (e.g. Christmas gift shopping); to control for that in part, any expenses reported to be gifts are removed, and the regression analysis below also controls for seasonality. Second, the cash-good category excludes many situations that may be reflections of emergencies that require liquid payment, such as an

<sup>17</sup>Clearly, in the 1980’s “cash goods” in the data would have been a much broader group than the 2004 definition used here. However, in the absence of survey data from that period detailing which goods were and were not cash goods, the decision of when and how to change the definition over time becomes arbitrary. The approach here is the most conservative: to keep the definition constant.

<sup>18</sup>Because of quasilinearity and credit markets, a shock process of the form  $\vartheta' = \vartheta^{\rho} \varepsilon'$  with  $\rho > 0$  would not change aggregate implications of the model, as can be shown from the first-order conditions of the problem.

<sup>19</sup>The preference shocks reflect any situation from being locked out of one’s house to a significant household repair that requires payment by cash or check, e.g. In these situations, not having the money to meet the expense is very costly, which is well captured by a parameter that shifts (marginal) utility.

emergency purchase of (or downpayment on) a durable to replace - rather than repair - a broken durable, such as a car or an appliance. Similarly, medical payments, which include co-pays or other out-of-pocket expenses, some of which are unpredictable and may require a liquid payment - are not included, because medical expenses may be payable by credit card today, even though historically this would not be the case. Thus, in measuring the volatility of cash-good consumption, using a lot of the “smooth” good categories while excluding many that may reflect emergencies other than household repairs, may understate the uncertainty that households face, against which they may hold liquid assets.

Using the above definition of cash goods, estimate the following model:

$$\begin{aligned}\log(c_{it}^{liq}) &= \beta \mathbf{X}_{it} + u_i + \varepsilon_{it} \\ \varepsilon_{it} &= \rho \varepsilon_{i,t-1} + \eta_{it}.\end{aligned}\tag{20}$$

The vector  $\mathbf{X}$  includes, depending on specification, household observables, such as age (a cubic), education, marital status, race, earnings, family size, homeownership status, as well as a set of month and year dummies.  $u_i$  is the household fixed effect. The residual  $\varepsilon_{it}$  is the idiosyncratic component of liquid consumption, consisting of a persistent and transitory components. Since the preference shock is assumed i.i.d., the autoregressive component above is taken as predictable, and the innovation  $\eta_{it}$  as reflecting household response to the preference shocks. Table 2 presents the standard deviation of  $\eta_{it}$  based on the benchmark cash-good measure, and two alternatives that exclude some more predictable expense groups. The most conservative benchmark standard deviation of 19.6% is used in estimation.

The estimate of the standard deviation of the log-preference shock that results from the SMM procedure is  $\sigma_{\eta} = 0.4045$  in the separable model, and 0.2042 in the CES model, where in the latter case, the parameter is estimated jointly with  $\alpha$  and  $\nu$ . The values adjust slightly if money savings are made interest-bearing in the model. The i.i.d. shock is discretized with 5 states using the Tauchen (1986) method, with shocks at maximum two standard deviations away from their mean.<sup>20</sup> To convince the reader that we do not overstate the amount of uncertainty in expenses through the shock calibration, figure 2 presents the steady-state distribution of the log of liquid consumption in the model, and compares it to the empirical distribution of the log-consumption residual ( $\eta_{it}$ ), with bins centered at the same states as in the model. It is key for the quantitative performance of the model that the probability of binding shocks is captured correctly; in this 5-state calibration, this is reflected in the

<sup>20</sup>The results are robust with an 11-state discretization.

top consumption state, as only the top shock binds in the calibrated equilibrium. From the figure, it is apparent that this probability is captured accurately. The discretized shock process implies that 7% of households have a binding cash constraint each period and spend all of their money. This is consistent with the findings of Telyukova (2012) that 5-7% of households have no liquid assets at a given point in time. The remaining 93% of households, in the model as in the data, are wealthy enough to hold liquid assets continually.

### 4.3 Aggregate Shock Processes

The technology and monetary policy shocks are modeled as two separate processes, as described above. For the model with no interest paid on money, these are estimated as:

$$\begin{aligned} z_t &= \xi_{zz} z_{t-1} + \varepsilon_1 \\ \ln \left( \frac{1+i_t}{1+\bar{i}} \right) &= \xi_{ii} \ln \left( \frac{1+i_{t-1}}{1+\bar{i}} \right) + \xi_{i\pi} \ln \left( \frac{1+\pi_{t-1}}{1+\bar{\pi}} \right) + \xi_{iy} \ln \left( \frac{y_{t-1}}{\bar{y}} \right) + \varepsilon_2 \end{aligned}$$

$z_t$  is the Solow residual measured in the standard way, with the linear trend extracted from the Solow residual and output. The variables with bars capture long-term averages of the respective variables in the sample period, 1984-2007, as is standard in estimating central banks' targets in policy rules. The choice of years captures the period when the Federal Reserve used (implicit) inflation targeting. Notice that the interest rate rule depends on endogenous variables. The interest rate variable is the Federal Funds rate.

For the model with interest-paying money, denote by  $R$  the ratio  $\frac{1+i^m}{1+i^b}$ , where  $i^m$  is the interest rate paid on money, and  $i^b$  is the interest rate on bonds. The second equation above becomes the following system, estimated by VAR using M2 own interest rate for  $i^m$ :

$$\begin{aligned} \ln \left( \frac{1+i_t^b}{1+i^b} \right) &= \xi_{ii} \ln \left( \frac{1+i_{t-1}^b}{1+i^b} \right) + \xi_{i\pi} \ln \left( \frac{1+\pi_{t-1}}{1+\bar{\pi}} \right) + \xi_{iy} \ln \left( \frac{y_{t-1}}{\bar{y}} \right) + \varepsilon_2 \\ \ln \left( \frac{R_t}{\bar{R}} \right) &= \xi_{rr} \ln \left( \frac{R_{t-1}}{\bar{R}} \right) + \xi_{ri} \ln \left( \frac{1+i_{t-1}^b}{1+i^b} \right) + \xi_{r\pi} \ln \left( \frac{1+\pi_{t-1}}{1+\bar{\pi}} \right) + \xi_{ry} \ln \left( \frac{y_{t-1}}{\bar{y}} \right) + \varepsilon_3. \end{aligned}$$

## 5 Results

To highlight the quantitative role of precautionary demand for money, we compute, in addition to the four versions of the model above, a version of the separable benchmark with the preference shocks shut down. In this version,

all agents receive the highest preference shock with probability 1; thus everyone's cash constraint always binds. This is the "no-shock model"; it closely replicates standard cash-credit good models with only aggregate risk. The computational method is in Appendix D.

## 5.1 The Role of Precautionary Money Demand

Table 3 summarizes the dynamic properties of some key nominal variables. The first column of the table presents the data moments. The second column presents the results for the no-shock model. The last four columns show the results in the model with separable and CES utility, with and without interest-paying money. Notice that none of the result moments are targeted in calibration.

For any model version, precautionary demand for money makes a dramatic difference for the performance of the model: without it, the model is not able to capture almost any of the moments in the data, while introducing precautionary demand makes the model align successfully on nearly all of the dimensions. When mean output velocity is not a target, the model underpredicts the level of both velocities. In this model, as in other monetary business cycle models, money turns over only once a quarter, so it is not surprising that the level is not high enough. It is also not surprising that in the no-shock model, mean velocity is higher than in the model with shocks; when the cash constraint always binds, the cash market contributes exactly 1 to velocity level every period. In the model with shocks, that contribution is less than 1 for all the non-binding shock realizations; all but 7% of the households do not spend all of the money, and hence contribute less than 1 to velocity. If the level of velocity is targeted instead, the model matches the levels well without affecting the rest of the moments. (See table 7).

Focusing first on the models without interest-bearing money, as the analytical results suggested, the models with preference shocks do much better, in terms of volatility of velocity, than those without. This is true even with the relatively low risk aversion parameter, a parameter that needed to be high in both Hodrick et al (1991) and Wang and Shi (2006) to begin to get significant velocity volatility. For output velocity, the separable model with the preference shocks produces 40% higher volatility than the no-shock model; for consumption velocity, the benchmark produces 60 *times* the volatility of the no-shock model. As discussed in the theory section, the reason is that with the introduction of preference shocks, the cash market contributes significantly to volatility of velocity, whereas it would contribute nothing if the cash constraint were always binding. Notice also that the model with precautionary demand gets the proportion of consumption velocity to output velocity volatility right,

while it is very far off target in the no-shock model. Since consumption velocity is a major component of output velocity, consumption expenditure being 75% of GDP, the no-shock model is an unsatisfying theory of velocity dynamics because these dynamics come from the wrong source in that model.

Due to the properties of the exogenous driving process, output volatility is slightly and equally overpredicted in both the benchmark and the no-shock model. The latter, however, underpredicts volatility of velocity dramatically; thus, excess output volatility is not creating excess velocity volatility.

The model with preference shocks replicates most other moments very well too, and much better than the model without shocks. From the model analysis, we know that for  $\sigma > 1$ , as it is here, the correlation of velocity with nominal interest rates, and its elasticity, will be negative in the no-shock model, counter to the data. The relevant rows in the table confirm this. Instead, with the preference shocks, the relationship between velocity and nominal interest rates has the correct sign and magnitude. The correlations of output with growth of output and consumption also flip signs relative to data if the model has no idiosyncratic risk; with the shocks, the signs are correct, and the magnitudes mostly close.

The CES and separable models produce almost the same results; in the CES case, volatility of velocity is even slightly high. If the separable model is changed to have interest-bearing money, the role of precautionary demand for money is robust. The presence of interest payment on money reduces model volatility of velocity, and elasticity of velocity with respect to the bond interest rate, particularly in the separable model. This is not surprising: for unconstrained households, consumption in the cash market now responds to the movement not of the bond interest rate, but to the ratio of bond-to-money interest rates, which dampens the response of cash consumption. In addition, due to the extreme assumption that all precautionary balances earn interest, our model likely presents the lower bound on volatility of velocity. Notice however, that with CES and interest-bearing money, we recover most of the volatility properties, as well as other dynamic properties, of the benchmark without interest-bearing money; in some dimensions, the model's performance even improves relative to data. This is a strong endorsement for the role of precautionary demand for money: not only does the model do well along the standard dimensions in the literature, where typically money is not modeled as interest-bearing and is calibrated to M2, but it passes an even higher bar of introducing interest-bearing money.

Table 4 presents the dynamic behavior of real balances ( $M/P$ ), as well as autocorrelations of velocity, inflation and real balances. These moments, all close to the data in the benchmark model, show that velocity volatility does not come from excessive volatility in the value of the money stock or from volatility at the wrong frequency.

Also, in the no-shock model, volatility of real balances is extremely low, again implying that it is unable to reproduce dynamics of the sources of velocity fluctuations.

## 5.2 Some Other Aggregate Facts

We now assess the performance of the model according to an additional set of facts, listed by Cooley and Hansen (CH, 1995) as some of the significant monetary features of business cycles. A set of these facts is presented in table 5. The first five columns list the performance of the model against the data, while the last two show the comparable moments from the CH data sample and model. The facts are that velocity is procyclical, prices are countercyclical, and that correlation of output and inflation with the growth of money supply is negative, with the latter being small. The model matches these facts and gets the magnitudes about right. The correlation of money growth and inflation is the only fact that is affected by introduction of interest-bearing money: the sign reverses, although in the CES case, the magnitude approaches zero. Even so, the model performs better than the CH model on all dimensions. See appendix E or cross-correlations of some real and nominal variables with output in the model.

Finally, table 6 highlights some aspects of the data that the model is not so successful in capturing. As the previous models in its class, the model misses the liquidity effect, i.e. the negative correlation of nominal interest rates with money growth, and prices and inflation are too flexible relative to data. The CH cash-in-advance model similarly misses these moments. The absence of the liquidity effect is not surprising, since by design, the model lacks rigidity in price adjustment.

## 5.3 Pitfalls of Calibration without Precautionary Demand

If precautionary demand for money is omitted from the model, the standard practice of calibrating monetary models to aggregate money demand, which may have a significant precautionary component in the data, may produce misleading parameter values and thus affect the quantitative performance of the model. To demonstrate this, we consider a version of the separable no-shock model where the calibration now uses two standard targets in the monetary literature: the expected value of velocity,  $\mathbb{E}(V_y)$ , and the elasticity of inverse velocity with respect to the nominal interest rate,  $-\varepsilon_{V,1+i}$ . This exercise produces different values for parameters  $x_1$  (0.51),  $A$  (3.1) and, most importantly, the RRA parameter  $\sigma$ . The model without preference shocks can only reproduce the

monetary targets with the value of  $\sigma = 0.15$ .<sup>21</sup> This is not surprising: the analytical result was that the no-shock model cannot get the sign of the elasticity of velocity with respect to nominal interest rates right unless  $\sigma < 1$ .

The dynamic properties of this model are presented in table 7, which compares our separable model, now calibrated to match  $\mathbb{E}(V_y)$ , with the no-shock model with the same target. Even when targeting  $\mathbb{E}(V)$  and  $\varepsilon_{V,1+i}$ , the no-shock model does badly along other nominal dimensions, even for the same moments of *consumption* velocity, and now the quantitative implications on the real side of the model suffer noticeably as well. For instance, here the no-shock model doubles volatility of output, consumption and investment relative to data, while the benchmark gets these standard deviation measures fairly close to the data. In other words, even if one were willing to accept the calibrated parameters that this exercise requires, the results that the no-shock model produces are far inferior to the performance of the benchmark with preference shocks. This is true along both nominal and real dimensions, even though the no-shock model is rigged in its parameters to do well quantitatively on the nominal side.

With CES utility, one can derive the value of the parameter  $\nu$  needed in order to match the target  $\varepsilon_{V_c,1+i} = 4.15$ , as described above. First, the elasticity of consumption velocity with respect to the nominal interest rate is given by  $\varepsilon_{V_c,1+i} \approx s_{cr} \frac{c}{c+q} \frac{\nu}{1-\nu} \approx 4.15$ . With credit consumption  $c/(c+q)$  at 47% of total consumption, as was the case to target the expected level of consumption velocity, this expression implies the parameter  $\nu = 0.915$ . If we target the credit consumption share of 25%, as in the SMM procedure described before, then we derive  $\nu = 0.94$ . These values of parameters imply elasticity of substitution between cash and credit goods of 11.8 and 20, respectively, for the CES model with no preference shocks to match the elasticity target.

But the model with no preference shocks also gives a closed-form implication for the parameter  $\nu$  that can be estimated in the data directly. The model with CES utility implies the MRS between cash and credit goods to be  $\frac{1-\alpha}{\alpha} \left(\frac{q}{c}\right)^{\nu-1} = 1 + i$ . Taking logs and rearranging yields

$$\log\left(\frac{q}{c}\right) = -\log\left(\frac{1-\alpha}{\alpha}\right) \frac{1}{\nu-1} + \frac{1}{\nu-1} \log(1+i).$$

Based on logged and detrended data used before, the result is  $\nu = 0.85$ , which implies the elasticity of substitution between the two goods of 6.8. Thus the data targets once again prove impossible to match using the model with no precautionary demand. To sum up, omitting precautionary money demand from monetary models may produce

---

<sup>21</sup>Values of  $\sigma$  very close to this commonly arise in monetary models without precautionary money demand, from sticky-price models (Rotemberg and Woodford, 1997) to monetary search models (Lagos and Wright, 2005).

inaccurate results in not only matching data facts, but also conducting policy experiments and drawing normative conclusions.

## 6 Welfare Costs of Inflation

The model is now used to evaluate the welfare cost of both anticipated and unanticipated inflation.

### 6.1 Inflation Level

As discussed above, in a model without precautionary demand for money, all agents bear the inflation tax equally: it comes from the wedge in the marginal rate of substitution between the cash and credit good created by the nominal interest rate. In the model with idiosyncratic risk, changes in steady state inflation distort cash-good consumption only for the binding shock realizations, but the constrained households value this consumption more than average, and are therefore more sensitive to these distortions.<sup>22</sup> Relative to a model without precautionary demand, the first channel would diminish the welfare cost of inflation, because not all agents bear it, while the second would increase it for the constrained agents. Comparing steady states, the welfare cost of inflation is the percentage reduction in consumption under the Friedman Rule that would make a household indifferent between the Friedman Rule and higher inflation. This measure is  $1 - \Delta$ , with  $\pi_{FR}$  denoting inflation at the Friedman Rule:

$$U(\Delta c(\pi_{FR})) + \mathbb{E}[\vartheta u(\Delta q_{\vartheta}(\pi_{FR}))] - Ah(\pi_{FR}) = U(c(\pi)) + \mathbb{E}[\vartheta u(q_{\vartheta}(\pi))] - Ah(\pi),$$

We solve for  $\Delta$ , and derive the steady state quantities in closed form (see appendix F). The dashed (red) line in panel (a) of figure 3 is the welfare cost of inflation in the no-shock model; the solid (blue) line is the cost of welfare in the model with idiosyncratic shocks. First, the welfare cost of 10% yearly inflation, relative to the Friedman rule, is 0.2% of the efficient level of consumption in the no-shock model, but is more than twice that, 0.5%, in the benchmark model with idiosyncratic risk.

Second, the difference between the two lines is increasing with the level of annual inflation. This is because not only the cost of inflation increases for a given constrained agent as inflation rises, but in addition, the proportion of constrained agents increases with inflation. At 100% inflation, where in the benchmark model close

<sup>22</sup>In the model, the timing is such that it is the firms who take cash from one period to the next, after selling the cash goods. Given free entry of firms, the full incidence of the inflation tax is still borne by the households, because they pay a higher price for cash goods than for credit goods, adjusted for marginal utility.



to 30% of households are constrained, compared to only 7% at inflation rates below 9%, the welfare cost in the no-shock model is 1.6%, while in the model with shocks, it is 2.8%. Thus, a model with a full distribution of idiosyncratic shocks allows a better approximation of welfare costs at high levels of inflation than models in which the cash constraints always binds, and in which the share of consumption subject to this constraint is calibrated using velocity in low-inflation data. The underestimate of welfare costs in the no-shock model is increasing over a large range of inflation (at least up to an annual inflation of 1200%), and is relatively stable afterwards.

## 6.2 Inflation Uncertainty

In the benchmark model, there is no uncertainty about inflation within a period: the current aggregate state is revealed at the beginning of the period, households make their decisions on how much money to hold, and firms set supply of the cash good, knowing the value of money, and hence the cash-good price, in advance. One can modify the timing of the model, however, to introduce inflation “surprises”, so that households and firms make their credit-market decisions before they know the second-subperiod price level. We modify the benchmark to allow households to find out the aggregate state of the economy for period  $t + 1$  at the start of subperiod 2 of  $t$ . The information structure in this model is meant to be comparable to Svensson (1985); the details are in appendix G.

This timing adjustment adds a distortion from increases in inflation, because the cash-good price  $\psi_t$  will adjust mid-period to the information regarding next period’s aggregate state. Now retail firms decide on how much good  $q_t$  to take out of the credit market, and households choose money holdings, before they know  $\psi_t$ . Suppose that households observe that inflation will be higher than expected, implying lower value of money  $\phi_{t+1}$ . The constrained households cannot increase their cash-good consumption, and can buy less in the high-shock state than they expected before. However, the supply of the cash good is fixed from the credit market, and the shortfall of demand from constrained households has to be made up by demand from unconstrained households. This means that relative to a case without unexpected inflation, the MRS of a constrained household will increase as its consumption will decline, while that of an unconstrained household will be lower than 1, so the distortion between the two types of households is exacerbated by unexpected inflation changes. Thus, an *unexpected* increase in inflation would decrease welfare by a larger amount than an *expected* increase, because  $q$  cannot adjust. As a result, a mean-preserving increase in the variance of inflation shocks can lower ex-ante welfare in this version of the model, whereas it would have no impact in the model with no inflation surprises.

As panel (b) of figure 3 shows, welfare cost of inflation uncertainty accelerates as the standard deviation of log-inflation increases; the welfare cost of 10% standard deviation of inflation is just below 2.5% of consumption.

## 7 Conclusion

Aggregate implications of precautionary demand for money are significant. This study demonstrates, theoretically and quantitatively, the importance of modeling the precautionary motive for holding liquidity. By incorporating idiosyncratic expenditure risk into a standard monetary model with aggregate risk, and by carefully calibrating the idiosyncratic shocks to data, we find that the model matches many dynamic moments of nominal variables well, and greatly improves on the performance of existing monetary models without such shocks. In addition, omitting precautionary demand while targeting, in calibration, data properties of money demand – a standard calibration practice – produces inferior performance in terms of matching the data, potentially misleading implications for parameters of the model, and an understatement of welfare costs of inflation, and may therefore adversely affect the model’s policy implications as well.

## References

- [1] Alvarez, F., Lippi, F., 2009. Financial Innovation and the Transactions Demand for Cash. *Econometrica* 77(2), 363-402.
- [2] Cooley, T., Hansen, G., 1995. Money and the Business Cycle. In: Cooley T. (Ed.), *Frontiers of Business Cycle Research*. Princeton University Press, pp. xxx-xxx.
- [3] Cooley, T., Hansen, G., 1989. The Inflation Tax in a Real Business Cycle Model. *American Economic Review* 79(4), 733-748.
- [4] Diamond, D. and Dybvig, P., 1983. Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy* 91(3), 401-419.
- [5] Faig, M., Jerez, B., 2006. Precautionary Balances and Velocity of Circulation of Money. *Journal of Money, Credit and Banking* 39(4), 843-873.
- [6] Hansen, G., 1985. Indivisible Labor and the Business Cycle. *Journal of Monetary Economics* 16(3), 309-327.
- [7] Hodrick, R., Kocherlakota, N., Lucas D., 1991. The Variability of Velocity in Cash-in-Advance Models. *Journal of Political Economy* 99(2), 358-384.
- [8] Hagedorn, M. 2008. A Monetary Model with Strong Liquidity Effects. Mimeo, U. of Zuerich.
- [9] Keynes, J. M. 1936. *The General Theory of Employment, Interest and Money*. Prometheus Books.

- [10] Lagos, R., Rocheteau, G., 2005. Inflation, Output and Welfare. *International Economic Review* 46(2), 495-522.
- [11] Lagos, R., Wright, R., 2005. A Unified Framework for Monetary Theory and Policy Analysis. *Journal of Political Economy* 113(3), 463-484.
- [12] Levine, D. K., 1991. Asset Trading Mechanisms and Expansionary Policy. *Journal of Economic Theory* 54(1), 148-164.
- [13] Lucas, R., 1980. Equilibrium in a Pure Currency Economy. *Economic Inquiry* 18, 203-220.
- [14] Lucas, R., 2000. Inflation and welfare. *Econometrica* 68, 247-274.
- [15] Lucas, R., Stokey, N., 1987. Money and Interest in a Cash-in-Advance Economy. *Econometrica* 55(3), 491-513.
- [16] Rocheteau, G., Wright, R., 2005. Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium. *Econometrica* 73(1), 175-202.
- [17] Rogerson, R., 1988. Indivisible Labor, Lotteries and Equilibrium. *Journal of Monetary Economics* 21(1), 3-16.
- [18] Rotemberg, J., Woodford, M., 1997. An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy. *NBER Macroeconomics Annual* 12: 297-346.
- [19] Shi, S., 1998. Search for a Monetary Propagation Mechanism. *Journal of Economic Theory* 81, 314-352.
- [20] Svensson, L., 1985. Money and Asset Prices in a Cash in Advance Economy. *Journal of Political Economy* 93, 919-944.
- [21] Tauchen, G., 1986. Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions. *Economics Letters* 20, 177-181.
- [22] Telyukova, I.A., 2012. Household Need for Liquidity and the Credit Card Debt Puzzle. *Forthcoming, Review of Economic Studies*.
- [23] Telyukova, I.A., Wright, R., 2008. A Model of Money and Credit, with Application to the Credit Card Debt Puzzle. *Review of Economic Studies* 75(2), 629 - 647.
- [24] Temzelides, T., Yu, J., 2004. Lack-Of-Recall And Centralized Monetary Trade. *International Economic Review* 45(4), 1221-1227.
- [25] Wallace, N., 2001. Whither Monetary Economics? *International Economic Review* 42(4), 847-870.
- [26] Wang, W., Shi, S., 2006. The Variability of Velocity of Money in a Search Model. *Journal of Monetary Economics* 53, 537-571.
- [27] Whalen, E. L., 1966. A Rationalization of the Precautionary Demand for Cash. *Quarterly Journal of Economics* 80, 314-325.

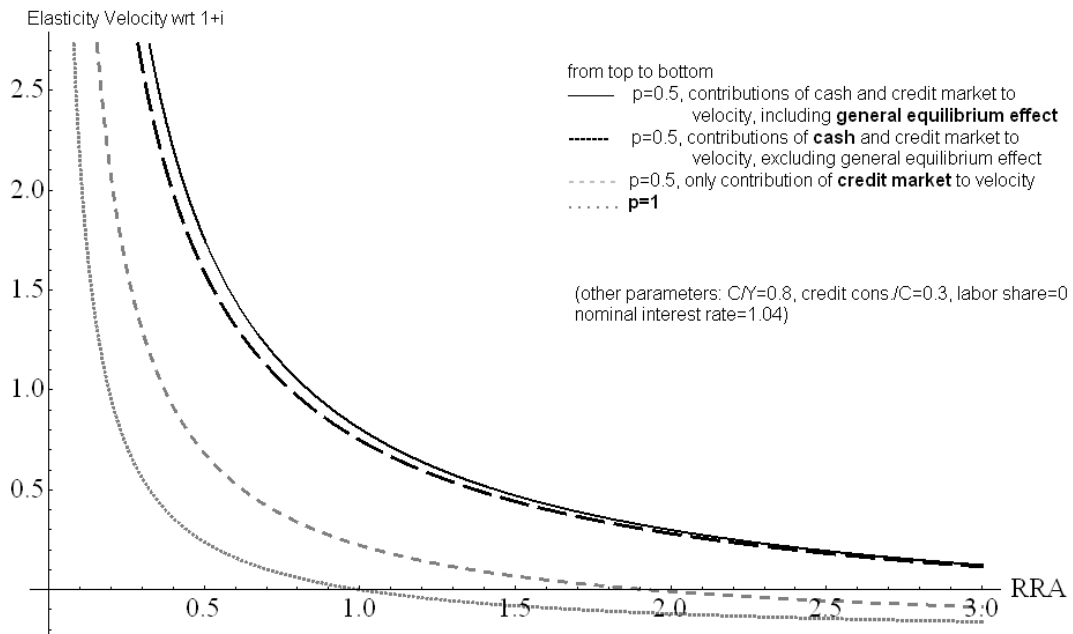


Figure 1: Contributions of Idiosyncratic Uncertainty to Interest Rate Elasticity of Velocity

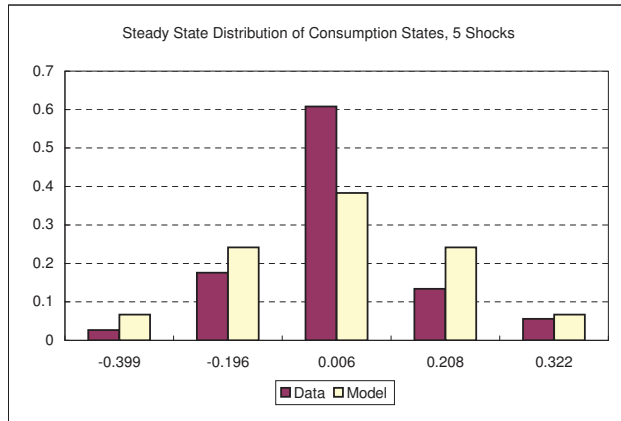
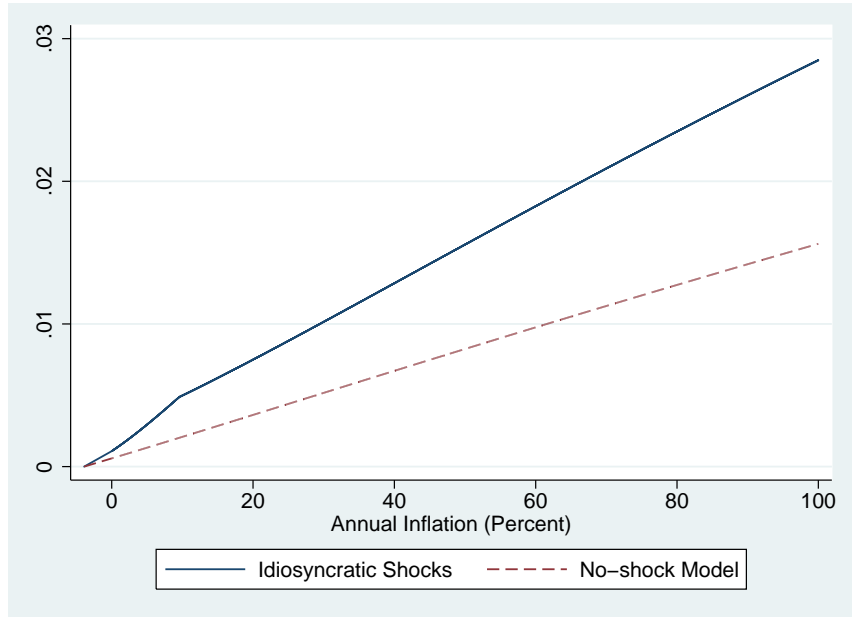
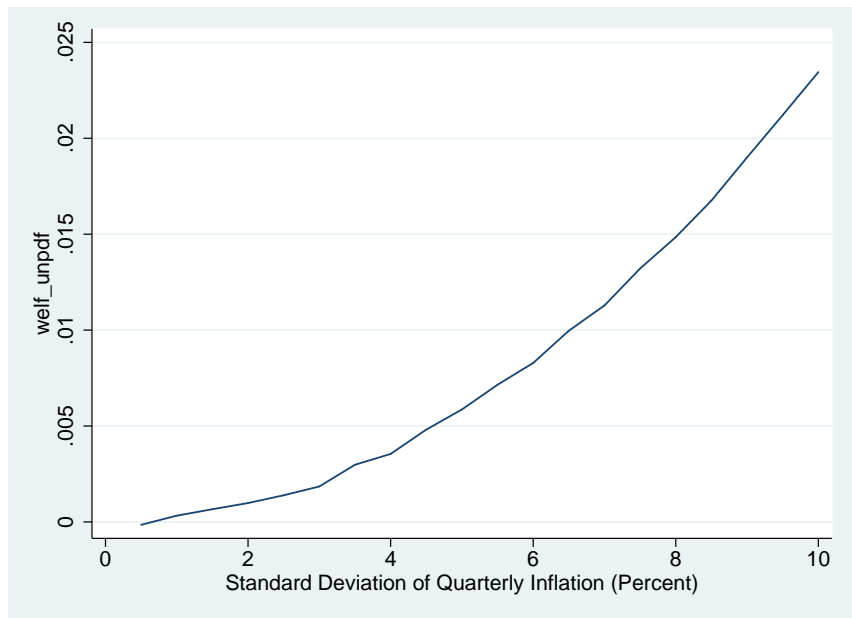


Figure 2: Distribution of Log-Consumption of Cash Goods, Data vs. Model Discretization



(a) Inflation Level



(b) Inflation Uncertainty, 2% Annual Average Inflation

Figure 3: Welfare Cost of Inflation

Table 1: Calibrated Parameters

|   | <b>Separable</b> | <b>CES</b> | <b>Separable<br/>with <math>i^m</math></b> | <b>CES<br/>with <math>i^m</math></b> |
|---|------------------|------------|--|--------------------------------------|
| $\beta$                                 | 0.99             | 0.99       | 0.99                                       | 0.99                                 |
| $\sigma$                                | 2                | 2          | 2  | 2                                    |
| $A$                                     | 34               | 5          | 34   | 5                                    |
| $x_1$                                   | 6                | –          | 6.33                                       | –                                    |
| $\alpha$                                | –                | 0.33       | –  | 0.33                                 |
| $\nu$                                   | –                | 0.39       | –  | 0.43                                 |
| $\theta$                                | 0.36             | 0.36       | 0.36                                       | 0.36                                 |
| $\delta$                                | 0.02             | 0.02       | 0.02                                       | 0.02                                 |
| $\sigma_\vartheta$                      | 0.41             | 0.20       | 0.39                                       | 0.21                                 |
| $\xi_{zz}$                              | 0.94             | 0.94       | 0.94                                       | 0.94                                 |
| $\xi_{ii}$                              | 0.78             | 0.78       | 0.78                                       | 0.78                                 |
| $\xi_{i\pi}$                            | 0.13             | 0.13       | 0.12                                       | 0.12                                 |
| $\xi_{iy}$                              | 0.01             | 0.01       | 0.01                                       | 0.01                                 |
| $\xi_{rr}$                              | –                | –          | 0.89                                       | 0.89                                 |
| $\xi_{rr}$                              | –                | –          | 0.01                                       | 0.01                                 |
| $\xi_{ry}$                              | –                | –          | -0.006                                     | -0.006                               |
| $\xi_{r\pi}$                            | –                | –          | -0.05                                      | -0.05                                |
| $\sigma_{\varepsilon_1}$                | 0.006            | 0.006      | 0.006                                      | 0.006                                |
| $\sigma_{\varepsilon_2}$                | 0.001            | 0.001      | 0.001                                      | 0.001                                |
| $\sigma_{\varepsilon_3}$                | –                | –          | 0.0007                                     | 0.0007                               |
| $\sigma_{\varepsilon_2, \varepsilon_3}$ | –                | –          | $-7 \times 10^{-7}$                        | $-7 \times 10^{-7}$                  |

“Separable” – benchmark model with separable utility; “CES” - benchmark with CES utility. “With  $i^m$ ” is the version of the model with interest-bearing money.

Table 2: Unpredictable Volatility of Liquid Consumption, Quarterly CEX Data

|                                   | Standard Deviation of $\eta_{it}$ (%) |
|-----------------------------------|---------------------------------------|
| Benchmark                         | 19.6                                  |
| Excluding food                    | 27.5                                  |
| Excluding food and property taxes | 29.4                                  |

Standard deviation, converted into percent, of the transitory component of the residual of the regression of log-cash good consumption on household characteristics, CEX 2000-2002. See equation (20).



Table 3: Dynamic Properties of the Model

| Moment                     | Data   | No-Shock Model | Separable | CES    | Separable with $i^m$ | CES with $i^m$ |
|----------------------------|--------|----------------|-----------|--------|----------------------|----------------|
| $\mathbb{E}(V_y)$          | 1.897  | 1.812          | 1.357     | 1.339  | 1.293                | 1.248          |
| $\mathbb{E}(V_c)$          | 1.120  | 1.380          | 1.033     | 1.020  | 0.984                | 0.950          |
| $\sigma(V_y)$              | 0.017  | 0.010          | 0.014     | 0.021  | 0.011                | 0.013          |
| $\sigma(V_c)$              | 0.014  | 0.0002         | 0.012     | 0.019  | 0.006                | 0.009          |
| $\sigma(y)$                | 0.009  | 0.012          | 0.012     | 0.012  | 0.012                | 0.012          |
| $\sigma(1 + i^b)$          | 0.0026 | 0.001          | 0.002     | 0.0025 | 0.002                | 0.002          |
| $corr(V_y, y)$             | 0.638  | 0.993          | 0.585     | 0.386  | 0.817                | 0.684          |
| $corr(V_y, g_y)$           | 0.059  | 0.289          | 0.142     | 0.078  | 0.217                | 0.159          |
| $corr(V_c, g_y)$           | -0.094 | 0.110          | -0.071    | -0.065 | -0.074               | -0.084         |
| $corr(V_y, g_c)$           | 0.127  | 0.539          | 0.233     | 0.109  | 0.242                | 0.464          |
| $corr(V_c, g_c)$           | -0.027 | 0.176          | -0.155    | -0.148 | -0.128               | -0.134         |
| $corr(V_y, 1 + i^b)$       | 0.714  | -0.210         | 0.645     | 0.558  | 0.390                | 0.585          |
| $corr(V_c, 1 + i^b)$       | 0.690  | -0.896         | 0.897     | 0.648  | 0.912                | 0.915          |
| $\varepsilon_{V_y, 1+i^b}$ | 5.072  | -1.747         | 4.546     | 4.744  | 2.542                | 4.115          |
| $\varepsilon_{V_c, 1+i^b}$ | 4.158  | -0.123         | 5.072     | 5.046  | 3.363                | 4.653          |
| $corr(1 + \pi, 1 + i^b)$   | 0.529  | 0.768          | 0.361     | -0.032 | 0.572                | 0.465          |

All data logged and BP filtered. Data period: 1984-2007. Velocity moments calculated based on M2. Bond interest rate is the Fed Funds rate. Inflation measured based on GDP deflator.  $g_y$  refers to output growth. “Separable” – benchmark model with separable utility; “CES” - benchmark with CES utility. “With  $i^m$ ” is the version of the model with interest-bearing money. “No-Shock” model is the version of the separable model with idiosyncratic preference shocks shut down.

Table 4: Properties of Velocity Volatility

| Moment                       | Data  | No-Shock Model | Separable | CES   | Separable with $i^m$ | CES with $i^m$ |
|------------------------------|-------|----------------|-----------|-------|----------------------|----------------|
| $\sigma(M/P)$                | 0.013 | 0.003          | 0.012     | 0.018 | 0.007                | 0.010          |
| $\text{corr}(V_y, V_{y,-1})$ | 0.941 | 0.898          | 0.898     | 0.902 | 0.901                | 0.903          |
| $\text{corr}(V_c, V_{c,-1})$ | 0.937 | 0.896          | 0.898     | 0.902 | 0.911                | 0.911          |
| $\text{corr}(\pi, \pi_{-1})$ | 0.901 | 0.870          | 0.844     | 0.840 | 0.864                | 0.851          |
| $\text{corr}(M/P, M/P_{-1})$ | 0.944 | 0.921          | 0.898     | 0.901 | 0.908                | 0.908          |

All data logged and BP filtered. Data period: 1984-2007. Velocity and money supply moments calculated based on M2. Inflation measured based on GDP deflator. “Separable” – benchmark model with separable utility; “CES” - benchmark with CES utility. “With  $i^m$ ” is the version of the model with interest-bearing money. “No-Shock” model is the version of the separable model with idiosyncratic preference shocks shut down.

Table 5: More Monetary Business Cycle Facts

| Moment           | Data  | Sep.  | CES   | Sep. + $i^m$ | CES + $i^m$ | CH data | CH    |
|------------------|-------|-------|-------|--------------|-------------|---------|-------|
| $corr(V, y)$     | 0.64  | 0.58  | 0.39  | 0.82         | 0.68        | 0.37    | 0.948 |
| $corr(p, y)$     | -0.13 | -0.28 | -0.10 | -0.26        | -0.28       | -0.57   | -0.22 |
| $corr(g_m, y)$   | -0.11 | -0.15 | -0.10 | -0.27        | -0.25       | -0.12   | -0.01 |
| $corr(g_m, \pi)$ | -0.06 | -0.13 | -0.40 | 0.38         | 0.07        | -0.29   | 0.92  |

All data logged and BP filtered. Data period: 1984-2007. Velocity moments calculated based on M2. Inflation and prices measured based on GDP deflator.  $g_m$  refers to money supply growth. “Separable” – benchmark model with separable utility; “CES” - benchmark with CES utility. “With  $i^m$ ” is the version of the model with interest-bearing money.

Table 6: Liquidity Effect

| Moment         | Data              | Separable Model | CH data | CH Model |
|----------------|-------------------|-----------------|---------|----------|
| $corr(g_m, i)$ | -0.7(M1)/0.07(M2) | 0.79            | -0.27   | 0.72     |
| $corr(y, i)$   | 0.54              | -0.13           | 0.40    | -0.01    |
| $corr(y, \pi)$ | 0.37              | -0.25           | 0.34    | -0.14    |
| $corr(g_m, p)$ | 0.03              | 0.61            | -0.16   | 0.43     |

All data logged and BP filtered. Data period: 1984-2007. Money supply moments calculated based on M2. Bond interest rate is the Fed Funds rate. Inflation measured based on GDP deflator.  $g_m$  refers to money supply growth. “Separable” – benchmark model with separable utility.

Table 7: No-Shock Model Targeting Money Demand in the Data, Separable Utility

| Moment                   | Data         | Separable<br>(Target $\mathbb{E}(V_y)$ ) | No-Shock Model<br>(Target Money Demand) |
|--------------------------|--------------|--|---|
| $\mathbb{E}(V_y)$        | <b>1.897</b> | <b>1.898</b>                             | <b>1.895</b>                            |
| $\mathbb{E}(V_c)$        | 1.120        | 1.445                                    | 1.442                                   |
| $\sigma(V_y)$            | 0.017        | 0.014                                    | 0.028                                   |
| $\sigma(V_c)$            | 0.014        | 0.011                                    | 0.003                                   |
| $\sigma(y)$              | 0.009        | 0.012                                    | 0.022                                   |
| $\sigma(c)$              | 0.005        | 0.003                                    | 0.012                                   |
| $\sigma(inv)$            | 0.050        | 0.044                                    | 0.114                                   |
| $\sigma(1+i)$            | 0.0026       | 0.002                                    | 0.002                                   |
| $corr(V_y, y)$           | 0.638        | 0.602                                    | 0.905                                   |
| $corr(V_y, g_y)$         | 0.059        | 0.145                                    | 0.411                                   |
| $corr(V_c, g_y)$         | -0.094       | -0.070                                   | -0.185                                  |
| $corr(V_y, g_c)$         | 0.127        | 0.262                                    | 0.323                                   |
| $corr(V_c, g_c)$         | -0.027       | -0.139                                   | 0.256                                   |
| $corr(V_y, 1+i)$         | 0.714        | 0.638                                    | 0.333                                   |
| $corr(V_c, 1+i)$         | 0.690        | 0.897                                    | 0.999                                   |
| $\varepsilon_{V_y, 1+i}$ | <b>5.072</b> | 4.469                                    | <b>5.030</b>                            |
| $\varepsilon_{V_c, 1+i}$ | 4.158        | 4.994                                    | 1.763                                   |
| $corr(1+\pi, 1+i)$       | 0.529        | 0.358                                    | 0.657                                   |
| RRA $\sigma$             |              | 2.0                                      | 0.15                                    |

All data logged and BP filtered. Data period: 1984-2007. Velocity moments calculated based on M2. Bond interest rate is the Fed Funds rate. Inflation measured based on GDP deflator.  $g_y$  refers to output growth. Benchmark: separable model, calibrated to target  $\mathbb{E}(V_y)$ . “No-Shock” model is the version of the separable model with idiosyncratic preference shocks shut down. Bolded quantities represent calibration targets. RRA  $\sigma$  is the value of the relative risk aversion parameter needed in calibration in order to match the targets.

## A APPENDIX: Analysis of Household Problem

### A.1 Walrasian Market creates Homogeneity

For general utility functions, different realizations of the idiosyncratic preference shock would lead to a nontrivial distribution of wealth (with, for example, those who have recently experienced a sequence of high  $\vartheta$ s now being poorer on average). In turn, households with different wealth could make different portfolio decisions, and hence the distribution across individual state variables would be relevant for equilibrium prices.

However, the quasi-linear specification of the problem allows equilibria in which all heterogeneity created in the second subperiod washes out in the credit market.<sup>23</sup> This occurs if the boundary conditions of  $h$  are never hit, which we assume to be the relevant case below. Our quantitative strategy later is to solve the problem assuming that the optimal choice of  $h$  is interior, and check in our calibrated equilibrium whether this is indeed the case.

After substituting the budget constraint for  $h$  into the household's value function, we can split the value function in two parts:

$$V(s, S) = A \left( \frac{\phi m + (1 + r - \delta)k + \phi b(1 + i_{-1})}{w} \right) + \max_{\dots} \left\{ U(c) - A \left( \frac{c + \phi \tilde{m} + k' + \phi b'}{w} \right) + \mathbb{E}_{\vartheta} [\vartheta u(q_{\vartheta}) + \beta \mathbb{E}_{z', i'} [V(s', S')]] \right\}. \quad (21)$$

From this, Result 1 is immediate, under the assumption of interiority on  $h$ .

Household wealth differs at the beginning of the Walrasian market, due to heterogeneous trading histories in the previous cash market, but households adjust their hours worked to be able to get the same amount of  $c, \tilde{m}, k', b'$ . The value function  $V(\cdot)$  is differentiable in  $k, m, b$ , and the envelope conditions are independent of the individual state variables. Hence, the expectation over  $\vartheta$  does not matter for intertemporal choice variables, for example:

$$\mathbb{E}_{\vartheta} [\mathbb{E}_{z', i'} V_m(s, S)] = \mathbb{E}_{z', i'} V_m(s, S) = \mathbb{E}_{z', i'} \left[ \frac{A\phi(S)}{w(S)} \right].$$

The problem is weakly concave in capital, labor and bond holdings, and the solution is interior, as long as  $h$  is interior. The first-order condition with respect to credit-good consumption, and the Euler equations with respect to capital and bonds, thus look as follows:

<sup>23</sup>This result has been used extensively in models that combine Walrasian markets with bilateral trade and idiosyncratic *matching* risk, such as Lagos and Wright (2005) and Rocheteau and Wright (2005). Here we use it to combine Walrasian markets with cash markets and idiosyncratic *preference* risk.

$$U'(c(S)) = \frac{A}{w(S)} \quad (22)$$

$$U'(c(S)) = \beta \mathbb{E}_{z', i'} [U'(c(S'))(1 + r(S') - \delta)] \quad (23)$$

$$\phi U'(c(S)) = \beta \mathbb{E}_{z', i'} \left[ \frac{\phi'}{1 + \varpi} U'(c(S')) \right] (1 + i) \quad (24)$$

For future reference, we introduce the following notation, using marginal utilities defined in terms of the marginal productivity of labor (22):

$$\mathbb{E} \left[ \frac{U'(c')}{U'(c)} \right] \equiv \widetilde{\mathbb{E}}, \quad \mathbb{E} \left[ \frac{\phi'(S') U'(c')}{1 + \varpi U'(c)} \right] \equiv \widetilde{\mathbb{E}\phi'}.$$

## A.2 The Choice of Money Holdings and Cash Market Consumption

Denote by  $\mathbb{P}(\vartheta)$  the probability of a particular shock  $\vartheta$  occurring. Taking as given (23)-(24), the first-order conditions with respect to  $\tilde{m}$  and  $q_\vartheta$  give

$$\mathbb{P}(\vartheta) \left( \frac{\vartheta u'(q_\vartheta)}{U'(c)} - 1 \right) - \psi \frac{\mu_\vartheta}{U'(c)} = 0 \quad (25)$$

$$-\phi + \sum_{\vartheta} \frac{\mu_\vartheta}{U'(c)} + \beta \widetilde{\mathbb{E}\phi'} = 0, \quad (26)$$

with the appropriate complementary slackness conditions (see equilibrium conditions below). It is immediate that in this model cash balances are not spent in full for realizations of  $\vartheta$  that are low enough. Since a social planner would equate  $U'(c)$  to  $\vartheta u'(q_\vartheta)$ , the following conclusions can be drawn:

**Result 2.** *If a shock  $\vartheta$  results in a nonbinding constraint, then  $q_\vartheta$  is the efficient quantity. Moreover, as long as the cash constraint does not bind, the quantity  $q_\vartheta$  does not respond to the interest rate.*

Moreover, also observe that if some  $\bar{\vartheta}$  leads to a binding constraint, then for every  $\vartheta > \bar{\vartheta}$ , the cash constraint will bind. If  $\bar{\vartheta}$  leads to a slack cash constraint, any  $\vartheta < \bar{\vartheta}$  will lead to a nonbinding constraint. A binding cash constraint leads to underconsumption of the cash good relative to the social optimum.

## A.3 Equilibrium Conditions

Let  $\mu_\vartheta$  be the Lagrange multiplier on the cash constraint in the second subperiod. We summarize the above discussion in the system of first-order conditions and Euler equations that characterize the equilibrium of the problem:

$$\begin{aligned}
U'(c) &= \beta \mathbb{E}[U'(c')(1+r' - \delta)] \\
U'(c) &= \frac{A}{w} \\
\psi &= \frac{1+i}{\phi} \\
\frac{\mu_{\vartheta}}{U'(c)} &= \mathbb{P}(\vartheta) \left( \frac{\vartheta u'(q_{\vartheta})}{U'(c)\psi} - \frac{\phi}{1+i} \right); \mu_{\vartheta}(\tilde{m} - \psi q_{\vartheta}) = 0 \forall \vartheta \\
\phi &= \frac{\sum_{\vartheta} \mu_{\vartheta}}{U'(c)} + \frac{\phi}{1+i} \\
\frac{\phi}{1+i} &= \beta \widetilde{\mathbb{E}}\phi' \\
y + (1-\delta)k &= c + k' + \sum_{\vartheta} \mathbb{P}(\vartheta)q_{\vartheta} \\
z' &= \xi_{zz}z + \varepsilon'_1 \\
(\widehat{1+i}') &= \xi_{ii}(\widehat{1+i}) + \xi_{i\pi}\hat{\pi} + \xi_{iy}\hat{y} + \varepsilon'_2
\end{aligned} \tag{27}$$

## B Proofs of Analytical Results: Idiosyncratic Uncertainty and Nominal Dynamics

### B.1 Lemma 1

*Proof.* The derivative of  $\phi$  with respect to  $\beta \widetilde{\mathbb{E}}\phi'$ , using (12), (24), the cash-market money constraint and  $\tilde{m} = 1$ , is

$$\frac{d\phi}{d(\beta \widetilde{\mathbb{E}}\phi')} = p \vartheta_h(u''(\beta \widetilde{\mathbb{E}}\phi')\beta \widetilde{\mathbb{E}}\phi' + u'(\beta \widetilde{\mathbb{E}}\phi')) + (1-p)$$

Divide both sides by  $\phi/(\beta \widetilde{\mathbb{E}}\phi')$ , and using (12), we find

$$\varepsilon_{\phi, \beta \widetilde{\mathbb{E}}\phi'} = p \frac{\vartheta_h(u''(\beta \widetilde{\mathbb{E}}\phi')\beta \widetilde{\mathbb{E}}\phi' + u'(\beta \widetilde{\mathbb{E}}\phi'))}{p \vartheta_h u'(\beta \widetilde{\mathbb{E}}\phi') + (1-p)} + (1-p) \frac{\beta \widetilde{\mathbb{E}}\phi'}{\phi}.$$

Rewriting this as a function of the interest rate  $(\phi/\beta \widetilde{\mathbb{E}}\phi')$ , this elasticity then becomes equation (13).  $\square$



## B.2 Lemma 2

*Proof.* One can derive that  $\varepsilon_{\frac{1+i}{\phi}, 1+i} = 1 - \varepsilon_{\phi, 1+i}$ . Substituting in

$$\varepsilon_{\phi, 1+i} = \frac{1}{\varepsilon_{1+i, \phi}} = \frac{1}{1 - \varepsilon_{\beta \widetilde{\mathbb{E}} \phi', \phi}} = \frac{\varepsilon_{\phi, \beta \widetilde{\mathbb{E}} \phi'}}{\varepsilon_{\phi, \beta \widetilde{\mathbb{E}} \phi'} - 1}, \quad (28)$$

we find

$$\varepsilon_{\psi, 1+i} = \varepsilon_{\frac{1+i}{\phi}, 1+i} = 1 - \varepsilon_{\phi, 1+i} = \frac{-1}{\varepsilon_{\phi, \beta \widetilde{\mathbb{E}} \phi'} - 1}$$

Putting the last equation together with (13) yields (16).  $\square$

## B.3 Proposition 1.

**Proposition 1.** *The implicit elasticity of consumption velocity with respect to the nominal interest rate, caused by a one-time fully anticipated money injection (in addition to the constant rate of money injection consistent with a given steady state level of  $1+i$ ), is*

$$\varepsilon_{V_c, 1+i} = s_c \left( \frac{1}{\sigma} \frac{1+i}{p+i} - 1 \right) + s_{cash, nb} \left( \frac{1+i}{p+i} \right) \left( \frac{1}{\varepsilon_{U'(c), q_h} + \sigma} \right) \quad (29)$$

*Proof.* The elasticity of velocity with respect to a change in the interest rate (caused by a one-time anticipated additional injection of money  $1+\varpi$ ) is

$$\varepsilon_{V_c, 1+i} = \frac{d \ln V_c}{d \ln 1 + \varpi} \Big/ \frac{d \ln 1 + i}{d \ln 1 + \varpi}$$

Velocity is given by

$$V_c = \frac{PC}{M} = \frac{c}{\phi} + (1-p) \frac{q_l(1+i)}{\phi} + p \frac{q_h(1+i)}{\phi},$$

hence,

$$\varepsilon_{V_c, 1+\varpi} = s_c(\varepsilon_{c, 1+\varpi} - \varepsilon_{\phi, 1+\varpi}) - s_{cash, nb} \varepsilon_{q_h, 1+\varpi},$$

using  $\varepsilon_{\psi, 1+\varpi} = -\varepsilon_{q_h, 1+\varpi}$ .

Since a one-time fully anticipated money injection does not affect tomorrow's  $\phi$  or  $U'(c')$ , we formulate the

elasticities in terms of  $\widetilde{\mathbb{E}\phi'}$ , which for the duration of the proof we define as  $\widetilde{\mathbb{E}\phi'} \stackrel{def}{=} U'(c')\phi'/(1 + \varpi)$ ; then  $\varepsilon_{1+\varpi, \beta\widetilde{\mathbb{E}\phi'}} = -1$ , and

$$\varepsilon_{V_c, 1+i} = \varepsilon_{V_c, \widetilde{\mathbb{E}\phi'}} / \varepsilon_{1+i, \widetilde{\mathbb{E}\phi'}}.$$

To derive  $\varepsilon_{c, \widetilde{\mathbb{E}\phi'}}$ , let us derive how  $h$  varies with  $c$  in the equilibrium. From equation (22), the elasticity is

$$\varepsilon_{c, h} = -\frac{\alpha}{\sigma}, \quad \varepsilon_{U'(c), h} = \alpha \quad (30)$$

Now, from the household budget constraint, we use (30) to derive

$$\varepsilon_{c, q_h} = -\frac{s_{q_h}}{s_c + \frac{1-\alpha}{\alpha}\sigma}, \quad \varepsilon_{U'(c), q_h} = \frac{s_{q_h}\sigma}{s_c + \frac{1-\alpha}{\alpha}\sigma} > 0, \quad (31)$$

where  $s_{q_h}$  is the share of total output going to  $q_h$  consumption,  $s_{q_h} = (pq_h)/Y$ ; likewise  $s_c = c/Y$ . Moreover,  $\varepsilon_{c, q_h} = -\frac{1}{\sigma}\varepsilon_{U'(c), q_h}$ .<sup>24</sup>

The free entry condition now allows us to link tomorrow's value of money  $\widetilde{\mathbb{E}\phi'}$  to today's movements in  $q_h$ . Free entry gives  $U'(c)q_h = \beta\widetilde{\mathbb{E}\phi'}$  (which is consistent with  $q_h = \beta\widetilde{\mathbb{E}\phi'}$  in the old definition), which means that

$$\varepsilon_{q_h, \beta\widetilde{\mathbb{E}\phi'}} = \frac{1}{\varepsilon_{U'(c), q_h} + 1}, \quad (32)$$

leading to

$$\varepsilon_{c, \beta\widetilde{\mathbb{E}\phi'}} = -\frac{\varepsilon_{U'(c), q_h}}{\sigma(\varepsilon_{U'(c), q_h} + 1)}. \quad (33)$$

To calculate  $\varepsilon_{\phi, \beta\widetilde{\mathbb{E}\phi'}}$ , use

$$U'(c)\phi = p\vartheta u'(q_h) \frac{\beta\widetilde{\mathbb{E}\phi'}}{U'(c)} + (1-p)\beta\widetilde{\mathbb{E}\phi'} = p\vartheta u'(q_h)q_h + (1-p)\beta\widetilde{\mathbb{E}\phi'}, \quad (34)$$

<sup>24</sup>Equation (31) captures the general equilibrium effect from changes in  $q_h$ : a shift away from cash consumption will lead to an increase in credit market consumption. This effect is proportional to the share of cash consumption under the binding shock in total consumption. In case of idiosyncratic uncertainty,  $s_{q_h}$  is smaller (by a factor smaller than  $p$ ) than total cash market consumption, and hence the elasticity in equation (31) is smaller.

derived from the FOCs of  $m, q_h$ , to find

$$\begin{aligned} \frac{d \ln \phi}{d \ln \beta \mathbb{E} \phi'} &= \frac{p \vartheta_h u'(q_h) q_h}{p \vartheta_h u'(q_h) q_h + (1-p) \beta \mathbb{E} \phi'} \frac{d \ln(u'(q_h) q_h)}{d \ln q_h} \frac{d \ln q_h}{d \beta \mathbb{E} \phi'} \\ &+ \frac{(1-p) \beta \mathbb{E} \phi'}{p \vartheta_h u'(q_h) q_h + (1-p) \beta \mathbb{E} \phi'} - \frac{d \ln U'(c)}{d \ln q_h} \frac{d \ln q_h}{d \ln \mathbb{E} \phi'}, \end{aligned} \quad (35)$$

Combining (35) and (34), we find

$$\varepsilon_{\phi, \mathbb{E} \phi'} = \left( \frac{p+i}{1+i} (1-\sigma) - \varepsilon_{U'(c), q_h} \right) \frac{1}{\varepsilon_{U'(c), q_h} + 1} + \frac{1-p}{1+i}. \quad (36)$$

Likewise, we can calculate

$$\varepsilon_{U'(c) \phi, \mathbb{E} \phi'} = \frac{d \ln U'(c) \phi}{d \ln \beta U'(c) \phi'} = \left( \frac{p+i}{1+i} (1-\sigma) \right) \frac{1}{\varepsilon_{U'(c), q_h} + 1} + \frac{1-p}{1+i}. \quad (37)$$

With  $\varepsilon_{\beta \mathbb{E} \phi', 1+i} = (\varepsilon_{U'(c) \phi, \mathbb{E} \phi'} - 1)^{-1}$  and  $\varepsilon_{U'(c) \phi, \beta \mathbb{E} \phi'}$  from (37), we find

$$\varepsilon_{1+i, \mathbb{E} \phi'} = - \left( \frac{1+i}{p+i} \cdot \frac{\varepsilon_{U'(c), q_h} + 1}{\varepsilon_{U'(c), q_h} + \sigma} \right). \quad (38)$$

Now we are able to calculate the elasticity of velocity with respect to  $\mathbb{E} \phi'$ .

$$\begin{aligned} \varepsilon_{V_c, \beta \mathbb{E} \phi'} &= \frac{d \ln V_c}{d \ln \mathbb{E} \phi'} = - \frac{d \ln V}{d \ln 1 + \varpi} = -\varepsilon_{V_c, 1+\varpi} = s_c \left( \frac{d \ln c}{d \ln \mathbb{E} \phi'} - \frac{d \ln \phi}{d \ln \mathbb{E} \phi'} \right) + s_{cash, nb} \frac{d \ln \psi}{d \ln \mathbb{E} \phi'} \\ &= s_c \left( \left( \left( -\frac{1}{\sigma} + 1 \right) \varepsilon_{U'(c), q_h} - \frac{p+i}{1+i} (1-\sigma) \right) \frac{1}{\varepsilon_{U'(c), q_h} + 1} - \frac{1-p}{1+i} \right) - s_{cash, nb} \frac{1}{\varepsilon_{u'(c), q_h} + 1}. \end{aligned} \quad (39)$$

From (39) it follows that, for  $p = 1$ , this elasticity is negative if  $\sigma < 1$ ; for  $p < 1$ , a larger  $\sigma$  will also lead to a negative elasticity. Combining (38) and (39) yields

$$\begin{aligned} \varepsilon_{V_c, 1+i} &= \left( \frac{1+i}{p+i} \cdot \frac{\varepsilon_{U'(c), q_h} + 1}{\varepsilon_{U'(c), q_h} + \sigma} \right) \times \\ &\left( s_c \left\{ \left[ \left( \frac{1}{\sigma} - 1 \right) \varepsilon_{U'(c), q_h} + \frac{p+i}{1+i} (1-\sigma) \right] \frac{1}{\varepsilon_{U'(c), q_h} + 1} + \frac{1-p}{1+i} \right\} + s_{cash, nb} \frac{1}{\varepsilon_{u'(c), q_h} + 1} \right), \end{aligned} \quad (40)$$

which simplifies to

$$\varepsilon_{V_c, 1+i} = s_c \left( \frac{1+i}{\sigma p+i} - 1 \right) + s_{cash, nb} \left( \frac{1+i}{p+i} \right) \left( \frac{1}{\varepsilon_{U'(c), q_h} + \sigma} \right) \quad (41)$$

□

## C A Version of the Model with Interest-Paying Money

We modify our benchmark model by introducing interest payment on money balances carried over from period to period. Thus, we are introducing interest payment on liquid savings. (We have also tried a version where all money is interest-bearing, even within the period; the quantitative implications of that model are similar, but this version is a more natural counterpart of savings accounts in the data, since short-term liquid balances, carried in cash or checking accounts, are not typically interest-bearing.)

Denote by  $i^m$  the interest rate paid on money; the nominal interest rate on bonds becomes  $i^b$ . The aggregate state variables become  $S = (z, i_{-1}^m, i_{-1}^b, i^m, i^b, (1 + \varpi_{-1})\phi_{-1})$ . Firm problem remains the same. Given this, we have the household problem as follows, where only the budget constraint and the exogenous shock processes are affected:

$$V(k, m, b; S) = \max_{c, h, \tilde{m}, k', b', \{q_\vartheta\}} \left\{ U(c) - Ah + \mathbb{E}_\vartheta \left[ \vartheta u(q_\vartheta) + \beta \mathbb{E}_{z', i'} V(k', m', \frac{b'}{1 + \varpi}; S') \right] \right\} \quad (42)$$

$$\text{s.t. } c + \phi \tilde{m} + k' + \phi b' = \phi m(1 + i_{-1}^m) + \phi b(1 + i_{-1}^b) + (1 + r - \delta)k + wh \quad (43)$$

$$\psi q_\vartheta \leq \tilde{m} \quad (44)$$

$$\psi = \frac{1 + i^b}{\phi} \quad (45)$$

$$\pi = \frac{(1 + \varpi_{-1})\phi_{-1}}{\phi} \quad (46)$$

$$1 + \varpi = \Omega(S) \quad (47)$$

$$m' = \frac{\tilde{m}}{1 + \varpi} - \frac{\psi q_\vartheta}{1 + \varpi} + \frac{\varpi}{1 + \varpi} \quad (48)$$

$$z' = \xi_{zz}z + \varepsilon'_1 \quad (49)$$

$$\widehat{(1 + i^b)} = \xi_{ii}\widehat{(1 + i^b)} + \xi_{i\pi}\widehat{\pi} + \xi_{iy}\widehat{y} + \varepsilon'_2 \quad (50)$$

$$\left( \frac{\widehat{1 + i^m}}{\widehat{1 + i^b}} \right)' = \xi_{rr} \left( \frac{\widehat{1 + i^m}}{\widehat{1 + i^b}} \right) + \xi_{ri}\widehat{(1 + i^b)} + \xi_{r\pi}\widehat{\pi} + \xi_{ry}\widehat{y} + \varepsilon'_3 \quad (51)$$

The equilibrium conditions for this version of the model are:

$$\begin{aligned}
U'(c) &= \beta \mathbb{E}_{z', i'} [U'(c')(1 + r' - \delta)] \\
U'(c) &= \frac{A}{w} \\
\psi &= \frac{1 + i^b}{\phi} \\
\frac{\mu_\vartheta}{U'(c)} &= \mathbb{P}(\vartheta) \left( \frac{\vartheta u'(q_\vartheta)}{U'(c)\psi} - \frac{\phi(1 + i^m)}{1 + i^b} \right); \mu_\vartheta(\tilde{m} - \psi q_\vartheta) = 0 \quad \forall \vartheta \\
\phi &= \frac{\sum_\vartheta \mu_\vartheta}{U'(c)} + \frac{\phi(1 + i^m)}{1 + i^b} \\
\frac{\phi}{1 + i^b} &= \beta \widetilde{\mathbb{E}} \phi' \\
y + (1 - \delta)k &= c + k' + \sum_\vartheta \mathbb{P}(\vartheta) q_\vartheta
\end{aligned} \tag{52}$$

In terms of dynamics of the model, notice that now, the decisions of the household are often determined not by the dynamics of the nominal bond interest rate, but of the ratio of the bond-to-money interest rates. This is the channel which results in dampened volatilities in this version of the mode (with separable utility). As we show in the text, the CES version of the model, even with interest-paying money, brings the dynamics back to data levels.

## D Computation Procedure

To compute the model we employ the Parameterized Expectations Approach. The method approximates the expectations terms in our Euler equation system (27) - two in total - by polynomial functions of the state variables, and the coefficients of the approximation and solved for. We choose the following forms:

$$\begin{aligned}
\mathbb{E} \left[ (c')^{-\sigma} (1 + e^{z'} \theta (k')^{\theta-1} (h')^{1-\theta} - \delta) \right] &= \psi^1(\chi; \gamma^1) \\
\mathbb{E} \left[ \frac{1}{w'} \right] &= \frac{\widetilde{\mathbb{E}}}{w} = \psi^2(\chi; \gamma^2)
\end{aligned} \tag{53}$$

where

$$\psi^j(\chi; \gamma^j) = \gamma_1^j \exp(\gamma_2^j \log k + \gamma_3^j \log z + \gamma_4^j \log i_{-1} + \gamma_5^j \log i + \gamma_6^j \log[\phi_{-1}(1 + \varpi_{-1})]).$$

The accuracy of approximation can be increased by raising the degree of approximating polynomials above;

we have experimented with several forms and found the results robust to them. We find that the convergence properties of our model are good: convergence is monotone and very robust. In order to compute moments from the model, we re-run the model solution 100 times given parameters of the model, and the simulations within each run are for 10,000 periods, where we discard the first 1,000 in computing the moments.

## E Cross-Correlations of Aggregate Variables with Output in Benchmark Model

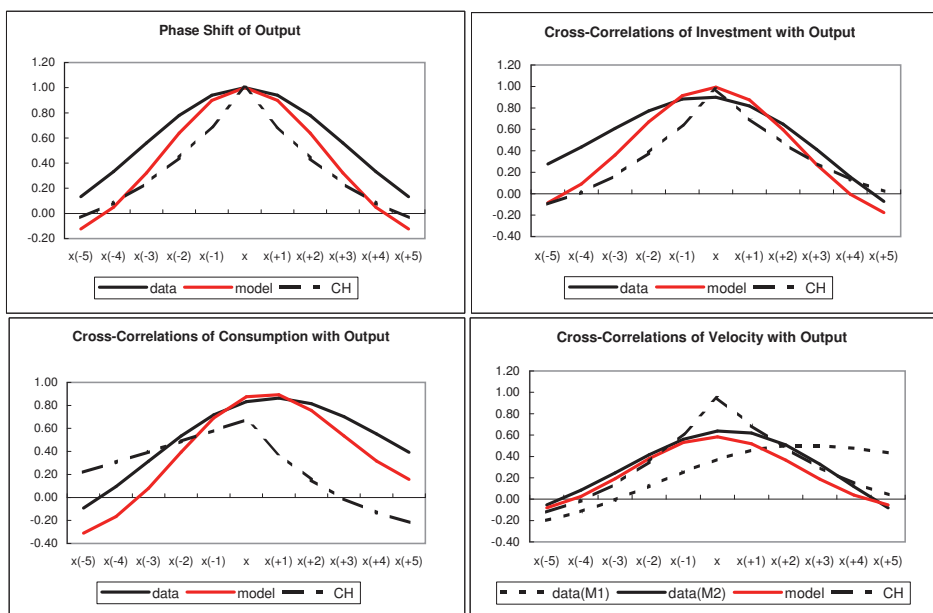


Figure 4: Cross-Correlations of Endogenous Variables with Output

To analyze our performance further with respect to facts highlighted by Cooley and Hansen, we present cross-correlations of several endogenous variables with output in graphical form, in figures 4 - 6. Where possible, we also graph the cross-correlations presented by Cooley and Hansen from their model.<sup>25</sup> With respect to the correlations of real variables with output, we do as well as the Cooley-Hansen model, or better. A notable improvement in our model relative to Cooley and Hansen concerns the dynamic pattern of output velocity (bottom right panel): we match the data for M2 velocity a lot more closely than they did. This last fact is the product of adding precautionary demand for money into the model; we further demonstrate this in figure 5 which shows the same cross-correlations, but comparing our benchmark to our no-shock model. In the bottom right panel, it is

<sup>25</sup>Obviously, this comparison is limited in that their model is calibrated to a different time period; we do not present their data for space reasons.

clear that the model with preference shocks does a lot better at matching the data than the model without. The other three panels of that figure also show that the improvement in dynamic patterns of real variables relative to Cooley-Hansen results in large part from our driving processes, rather than from preference shocks: we use an interest rate rule, while Cooley and Hansen used a money growth rule.

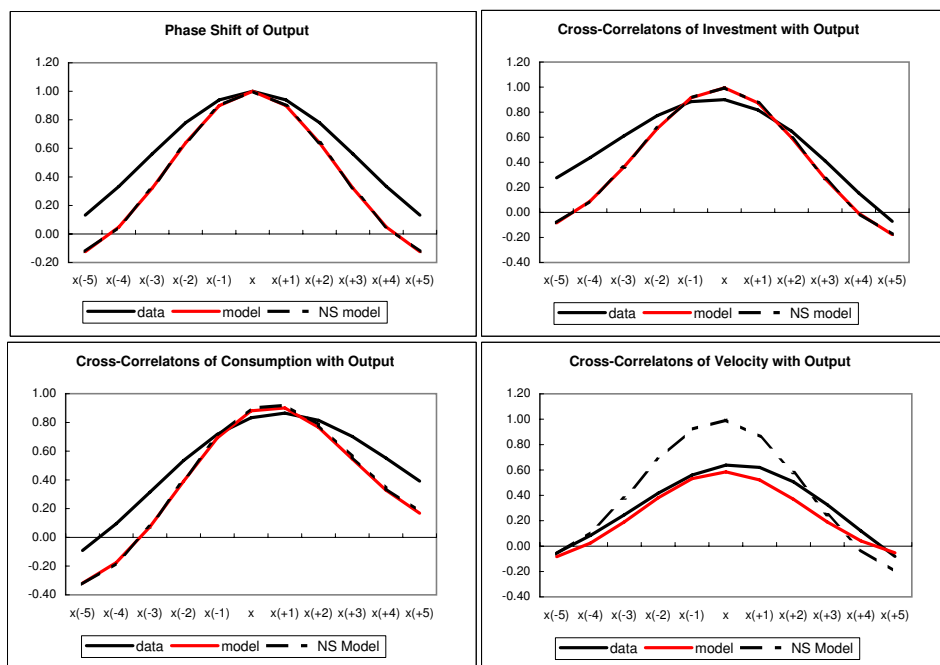


Figure 5: Cross-Correlations of Endogenous Variables with Output, Benchmark vs No-Shock Model

Finally, in figure 6 we present some further cross-correlations that we get less well. While we get the dynamic pattern of money supply half-right (although our cross-correlation bottoms out later than the data suggest), and we improve on Cooley and Hansen’s cross-correlation of nominal interest rates, we get neither these two, nor the dynamic patterns of prices and inflation. Our performance on the bottom two panels is fairly close to Cooley and Hansen’s. Again, we do not expect to get the patterns of prices to replicate the data with a price adjustment mechanism that is as flexible and frictionless as ours.

## F Steady State Consumption in Closed Form

We have to solve both for credit market and cash market consumption in order to conduct the welfare cost experiment. From the characterizing equation system (27), we get steady-state credit market consumption, after

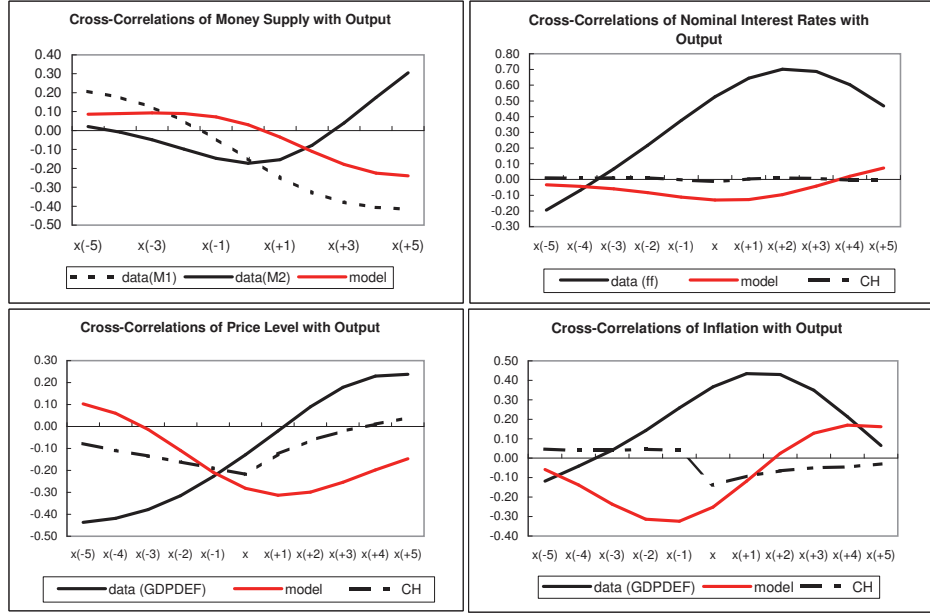


Figure 6: Cross-Correlations of Endogenous Variables with Output

substituting in the capital-labor ratio, from

$$\bar{c} = \left[ \frac{A}{1-\theta} \left( \frac{1}{\theta} \left( \frac{1}{\beta} - 1 + \delta \right) \right)^{\frac{\theta}{1-\theta}} \right]^{-\frac{1}{\sigma}}.$$

To get cash market consumption we again appeal to the system (27). The issue for the welfare-cost analysis is that as inflation rate increases, more of the discrete shocks cause the cash constraint to bind. Thus, we solve in closed form here for the general case: suppose that the total number of discrete shock states is  $n$  and  $k$  of these shocks, from  $\vartheta_{n-k+1}$  to  $\vartheta_n$ , bind. For any binding shock, the following system holds, given our functional forms:

$$\begin{aligned} \bar{\mu}_{\vartheta_i} &= \mathbb{P}(\vartheta_i) \left( \vartheta_i x_1 \bar{q}_{\vartheta_i}^{1-\sigma} \bar{c}^\sigma - \frac{\bar{\phi}}{1+\bar{i}} \right) \quad \forall i \in \{n-k+1, k\} \\ \bar{\phi} &= \left( \frac{1+\bar{i}}{\bar{i}} \right) \sum_{i=n-k+1}^k \bar{\mu}_{\vartheta_i} \\ \bar{q}_{\vartheta_i} &= \frac{\bar{\phi}}{1+\bar{i}} \quad \forall i \in \{n-k+1, k\}. \end{aligned}$$

From this, one can solve for the relevant  $\mu_{\vartheta_i}$ , which then determine  $\phi$ , and finally  $q$  in all the binding states, which is a function of the nominal interest rate but not of the binding shock level, as expected. Instead, in the



remaining (non-binding) states, consumption is given simply by

$$\bar{q}_{\vartheta_i} = (\vartheta_i x_i)^{\frac{1}{\sigma}} \bar{c} \quad \forall i \in \{1, n - k\},$$

and is not a function of the nominal interest rate, but does change with the level of the non-binding shock.

## G A Version of the Model with Inflation Surprises

In this version of the model, we want to change the timing of agents' information. In the benchmark model, there is no *within-period* aggregate uncertainty: agents find out the aggregate state  $S$  at the beginning of the period, and the only uncertainty they face within the period is the idiosyncratic preference uncertainty. In this setup, inflation impacts agents via its level, but its variability does not impact firms or households. Here, we change the model by introducing within-period inflation surprises: now, we will assume that at the beginning of the cash market, households find out not only their preference shocks, but all agents in addition find out the values of the next period's monetary and productivity shocks.

We rewrite the problem, making explicit the dependence on aggregate state where necessary to make the changed timing clear:

$$V(k, m, b, S) = \max_{c, h, \tilde{m}, k', b', \{q_{\vartheta}\}} \left\{ U(c) - Ah + \mathbb{E}_{\vartheta, S'} \left[ \vartheta u(q_{\vartheta}(S')) + \beta \mathbb{E}_{S'} V(k', m', \frac{b'}{1 + \varpi}, S') \right] \right\}$$

$$\text{s.t. } c(S) + \phi(S)\tilde{m}(S) + k'(S) + \phi b'(S) = \phi m + \phi b(1 + i_{-1}) + (1 + r - \delta)k + wh(S) \quad (54)$$

$$\mathbb{E}(\psi(S')) = \frac{1 + i}{\phi} \quad (55)$$

$$\psi(S')q_{\vartheta}(S') \leq \tilde{m}(S) \quad (56)$$

$$\pi = \frac{(1 + \varpi_{-1})\phi_{-1}}{\phi} \quad (57)$$

$$1 + \varpi = \Omega(S) \quad (58)$$

$$m' = \frac{\tilde{m}}{1 + \varpi} - \frac{\psi q_{\vartheta}}{1 + \varpi} + \frac{\varpi}{1 + \varpi} \quad (59)$$

$$z' = \xi_{zz}z + \varepsilon'_1 \quad (60)$$

$$(\widehat{1 + i'}) = \xi_{ii}(\widehat{1 + i}) + \xi_{i\pi}\hat{\pi} + \xi_{iy}\hat{y} + \varepsilon'_2 \quad (61)$$

The equilibrium conditions for this model are:

$$\begin{aligned}
U'(c) &= \beta \mathbb{E}[U'(c')(1 + r' - \delta)] & (62) \\
U'(c) &= \frac{A}{w} \\
\mathbb{E}[\psi(S')] &= \frac{1 + i}{\phi} \\
\psi(S') \frac{\mu_{\vartheta}(S')}{U'(c)} &= \mathbb{P}(\vartheta_i, S'|S) \left( \frac{\vartheta u'(q_{\vartheta}(S'))}{U'(c)} - \beta \mathbb{E} \left[ \frac{\phi(S') \psi(S') U'(c')}{1 + \varpi} \frac{U'(c')}{U'(c)} \right] \right); \\
\frac{\mu_{\vartheta}(S')}{U'(c)} (\tilde{m} - \psi(S') q_{\vartheta}(S')) &= 0, \quad \forall \vartheta, S' \\
\phi &= \sum_{\vartheta, S'} \frac{\mu_{\vartheta}(S')}{U'(c)} + \frac{\phi}{1 + i} \\
\frac{\phi}{1 + i} &= \beta \widetilde{\mathbb{E}} \phi' \\
q(S') &= \sum_{\vartheta} \mathbb{P}(\vartheta) q_{\vartheta}(S') \quad \forall S' \\
y + (1 - \delta)k &= c + k' + q(S')
\end{aligned}$$

The key change here is that now, firms decide on the amount of good  $q$  to take into the retail market, and households on their money holdings, before they know next period's state, but consumption in the cash market occurs after the next period's aggregate state is revealed. Suppose that agents find out that next period's inflation will be higher. This means that, holding price of the cash good constant, while constrained households today cannot adjust their consumption, the unconstrained households will want to consume strictly more. But supply of the cash good is fixed, because retailers cannot produce additional goods in the cash market. Thus, in order to clear the market, the cash-good price  $\psi$  will rise in response to the inflation surprise.

What does this imply for agents' welfare? With a surprise increase in inflation, the constrained agents will now have even higher marginal utility, since the price of the cash good higher, while their money holdings are fixed. The constrained agents will have lower marginal utility. Thus, the distortion in relative marginal utilities of constrained versus unconstrained agents in this version of the model will be even higher.

From this, we conclude that an expected change in inflation will be less detrimental for welfare than an unexpected one, because in the former case, the supply of the cash good also adjusts, while in the latter case, this cannot happen.

In terms of the dynamics of nominal and real moments, the performance of this model is quantitatively similar

Table 8: Dynamic Properties of the Model with Changed Timing

| Moment                     | Data   | Separable<br>Benchmark | Separable<br>Changed Timing |
|----------------------------|--------|------------------------|-----------------------------|
| $\mathbb{E}(V_y)$          | 1.897  | 1.357                  | 1.428                       |
| $\mathbb{E}(V_c)$          | 1.120  | 1.033                  | 1.087                       |
| $\sigma(V_y)$              | 0.017  | 0.014                  | 0.023                       |
| $\sigma(V_c)$              | 0.014  | 0.012                  | 0.025                       |
| $\sigma(y)$                | 0.009  | 0.012                  | 0.013                       |
| $\sigma(1 + i^b)$          | 0.0026 | 0.002                  | 0.002                       |
| $corr(V_y, y)$             | 0.638  | 0.585                  | 0.733                       |
| $corr(V_y, g_y)$           | 0.059  | 0.142                  | 0.110                       |
| $corr(V_c, g_y)$           | -0.094 | -0.071                 | -0.025                      |
| $corr(V_y, g_c)$           | 0.127  | 0.233                  | 0.741                       |
| $corr(V_c, g_c)$           | -0.027 | -0.155                 | -0.470                      |
| $corr(V_y, 1 + i^b)$       | 0.714  | 0.645                  | 0.781                       |
| $corr(V_c, 1 + i^b)$       | 0.690  | 0.897                  | 0.885                       |
| $\varepsilon_{V_y, 1+i^b}$ | 5.072  | 4.546                  | 7.606                       |
| $\varepsilon_{V_c, 1+i^b}$ | 4.158  | 5.072                  | 9.347                       |
| $corr(1 + \pi, 1 + i^b)$   | 0.529  | 0.361                  | 0.306                       |

All data logged and BP filtered. Data period: 1984-2007. Velocity moments calculated based on M2. Bond interest rate is the Fed Funds rate. Inflation measured based on GDP deflator.  $g_y$  refers to output growth.

to the benchmark; see table 8. In this model, velocity of money becomes more volatile (standard deviation 2.3% versus benchmark's 1.4%), because the unconstrained agents' money demand responds not only to the idiosyncratic shock, but also to the aggregate shock that is realized mid-period, when the cash-good price changes but supply  $q$  cannot. This reflects also in higher elasticity of velocity with respect to the interest rate as well. Other than this, however, other moments remain close to the benchmark dynamics. This comparison between the benchmark model and the model with changed timing gives a quantitative bound on the role that uncertainty in monetary policy can play in generating fluctuations in money demand, and hence in the velocity of money.