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Citation for published version:
Assanto, G & Smyth, N 2013, 'Comment on ``Solitons in highly nonlocal nematic liquid crystals: Variational approach'' by N.B. Aleksi\'c, M.S. Petrovi\'c, A.I. Strini\'c and M.R. Beli\'c' Physical Review A, vol 87, 047801. DOI: 10.1103/PhysRevA.87.047801

Digital Object Identifier (DOI):
10.1103/PhysRevA.87.047801

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Publisher's PDF, also known as Version of record

Published In:
Physical Review A

Comment on "Solitons in highly nonlocal nematic liquid crystals: Variational approach"

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(Received 2 November 2012; published 17 April 2013)

In their recent paper [N. B. Aleksić, M. S. Petrović, A. I. Strinić, and M. R. Belić, Phys. Rev. A **85**, 033826 (2012)], Aleksić *et al.* numerically study the propagation of spatial solitary waves in nematic liquid crystals in the presence of noise. As expected, and reported earlier in their previous work on the same topic, the authors find that optical solitary waves in the presence of perturbations are no longer stationary, oscillate in amplitude and width as they propagate, and eventually decay to linear waves. Surprisingly, they conclude that spatial solitary waves are difficult to observe in nematic liquid crystals, in contrast to numerous experimental reports and the vast literature on the topic. We argue with such a conclusion in light of the behavior of wave-packet solutions of nonlinear Schrödinger-type equations.

DOI: 10.1103/PhysRevA.87.047801 PACS number(s): 42.65.Tg, 42.65.Jx, 42.70.Df

The recent paper [1] derives numerical steady-state solitary wave solutions, known also as nematicons [2,3], of the coupled system of equations governing nonlinear optical beam propagation in nematic liquid crystals in the presence of an external bias [4]. The simplified model governing a nonlinear optical beam in reorientational nematic liquid crystals consists of a coupled system of a nonlinear-Schrödinger (NLS)-type equation for the light beam envelope and an elliptic equation for the medium response, that is the field-induced rotation of the molecular director [4,5]. In addition, a variational approach, based on a chirped Gaussian trial function for the beam [6], is used to derive analytical approximations to the exact nematicon solution. As known from previous reports on the theoretical model, as well as numerics and experiments in nematic liquid crystals [5,7], they find that when a beam which is not an exact solitary wave is launched, it oscillates in width and intensity [5]. The main conclusion the authors of Ref. [1] draw from this is that nematicons, i.e., optical solitary waves in space, are difficult to observe in nematic liquid crystals, a claim which is in sharp contrast to the existing literature [8]. This conclusion has a number of problems in light of the behavior of wave-packet solutions of NLS-type equations.

The standard (1 + 1)-D NLS equation is

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + |u|^2 u = 0, \tag{1}$$

which has the soliton solution

$$u = a \operatorname{sech} ax\, e^{\frac{1}{2}a^2 z}. \tag{2}$$

If the input wave packet is not an exact soliton, then the beam oscillates in amplitude and width, shedding diffractive radiation, in order to reach the exact soliton solution [9]. This behavior is guaranteed by the inverse scattering solution of the NLS equation, as any initial beam must evolve to a fixed number of solitons plus diffractive radiation [10–12]. This oscillatory evolution is in contrast to the exponential evolution to the steady state for the Korteweg–de Vries (KdV) model, which is another equation with an inverse scattering solution [10–12]. This oscillatory behavior of initial nonsolitary wave beams holds for any NLS-type equation, as has been shown for perturbed NLS equations [12] and coupled systems of NLS equations [13,14]. So, general results for NLS-type equations show that the oscillations seen in [1] for non-nematicon input beams are due to an excitation adjusting to become an exact solitary wave. It is not evidence that exact nematicons do not exist or that they are difficult to observe. In experiments, for large nonlocality, the evolution to the steady state is slow [15] and so the typical Gaussian input does not have enough propagation distance to evolve to a steady state over the usual millimeter lengths [4]. In addition, the width oscillations are not evidence that a steady breather has formed [1], as over longer length scales, the oscillations decay in amplitude [15]. These oscillations are not the same as those shown by exact breathers of the sine-Gordon equation [12], which are solitary waves oscillating in a periodic fashion on any length scale and shedding no radiation: they are steady, apart from the breathing amplitude. The same comments apply to the evolution of a beam in a cell in which noise is introduced as a perturbation [1]. The beam oscillations are the usual behavior for an NLS-type solitary wave adjusting to local changes. Although the linear, highly nonlocal (in fact, infinitely nonlocal) model of Snyder and Mitchell [16] has steadily oscillating solitary-wave solutions, this is an artifact of the approximation. The infinite nonlocality approximation reduces the nematic response to an infinite parabolic potential which traps all waves, so that the solitary wave cannot shed radiation to evolve to a steady state. Without this linear approximation, the "nematic potential" is finite, so that the solitary wave can shed radiation and evolve to a steady state.

Reference [1] concludes that steady nematicons are difficult to observe experimentally based on numerical solutions of the standard scalar system of equations governing such beams. Such a conclusion could simply be recast as follows: Exact solitary waves (i.e., mathematical solutions) do not exist in any medium whose response is real, i.e., whose response cannot be reduced to ideal equations neglecting losses, noise, perturbations, etc. The main point about any such equations governing a real phenomenon is that they are always an approximation to a complicated physical process: assumptions and approximations are made as to which physical effects can be ignored to lead to a tractable system of governing equations. Experimental measurements and observations reflect the physical reality. Hence, making strong predictions from the results of simplified mathematical models should be treated with caution. Solitary waves have been observed and reported in many physical systems, e.g., internal waves in the ocean and atmosphere, surface waves in fluids, plasmas, and optics [10,17–21]. Indeed, this work dates back to the pioneering solitary-wave studies of Russell [22]. In all of these cases, these waves are neither steady nor exact solutions of some simplified governing equation, but the existence of solitary waves in such systems is accepted, as the observed solitary waves are, to a good approximation, governed by the simplified equations when effects not included in them are acknowledged.

In conclusion, the oscillatory beam behavior reported in [1] is expected due to the general form of the governing equations. This behavior is not evidence that solitary waves cannot be observed in nematic liquid crystals or other nonlinear media.

This work was supported by the Royal Society of London under Grant No. IE111560.

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