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# Progress Calculating Decay Constants with NRQCD and AsqTad Actions

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We combine a light AsqTad antiquark with a nonrelativistic heavy quark to compute the decay constants of heavy-light pseudoscalar mesons using the ensemble of 3-flavor gauge field configurations generated by the MILC collaboration. Preliminary results for  $f_{B_s}$  and  $f_{D_s}$  are given and status of the chiral extrapolation to  $f_B$  is reported. We also touch upon results of the perturbative calculation which matches matrix elements in the effective theory to the full theory at 1-loop order.

## 1. INTRODUCTION

The ‘‘AsqTad’’ improved staggered quark action has made feasible the simulation of QCD with 3 flavors of sea quarks, with 2 of the flavors varying in mass from  $m_s$  to below  $m_s/5$ . Unquenched simulation with this action at light sea quark masses produces agreement with experiment for several quantities: the  $\Upsilon$  spectrum,  $B$  masses, and  $\pi$  and  $K$  decay constants. Using a single set of input parameters, these quantities could not previously reproduce experiment [1].

Having removed this discrepancy and constructed actions with good scaling properties, the uncertainties arising from chiral extrapolations can be studied more accurately.

In recent work [2] we proposed and tested the use of improved staggered quarks as the light quark in heavy-light mesons. Since the heavy quark does not have the doubling problem, taste-breaking effects present in light staggered hadrons are suppressed in heavy-light mesons. Consequently the same operators used in Wilson fermion simulations can be used here, employing the identity between naive and staggered quark propagators.

This work utilizes a subset of the public MILC configurations, the parameters of which we sum-

marize in Table 1; further details appear in [4]. The correct experimental kaon mass is obtained by tuning the valence strange quark mass to  $au_0m_q = 0.040(1)$  on the  $au_0m_{ud}^{\text{sea}} = 0.01$  lattice. For clarity, we will quote quark masses in units of the mass,  $m_s$ , corresponding to the physical strange mass.

The NRQCD (improved through  $1/M_0^2$ ) and improved staggered actions are exactly as in [2] (see references therein). The operator matching has been carried out through  $\mathcal{O}(1/M_0)$  at 1-loop order in perturbation theory [5,6].

## 2. $f_{B_s}$ AND $f_{D_s}$

The  $B_s$  decay constant is the simplest for us to compute since no extrapolations in valence quark mass are necessary and dynamical quark mass dependence is found to be small (see discussion below). The strange and bottom quark masses are fixed by tuning the bare masses to the physical  $K$  and  $\Upsilon$  masses. On the lattice with  $m_{ud}^{\text{sea}} \approx m_s/4$  we find

$$f_{B_s} = 260 \pm 7 \pm 26 \pm 8 \pm 5 \text{ MeV}. \quad (1)$$

The first uncertainty is the statistical errors in the matrix elements and lattice spacing. The dominant uncertainty is the estimate of  $\mathcal{O}(\alpha_s^2)$  effects neglected in the 1-loop matching calculation. The last two uncertainties estimate the errors due to

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Table 1

Simulation parameters and inverse lattice spacing from two  $\Upsilon$  splittings [3]. The lattice volume is  $20^3 \times 64$ .

$au_0 m_{ud}^{\text{sea}}$	$au_0 m_s^{\text{sea}}$	$N_{\text{conf}}$	$a^{-1}(2S - 1S)$	$a^{-1}(1P - 1S)$
0.01	0.05	568	1.59(2) GeV	1.58(3) GeV
0.02	0.05	468	1.61(2) GeV	1.64(2) GeV
0.03	0.05	564	1.60(3) GeV	1.68(4) GeV

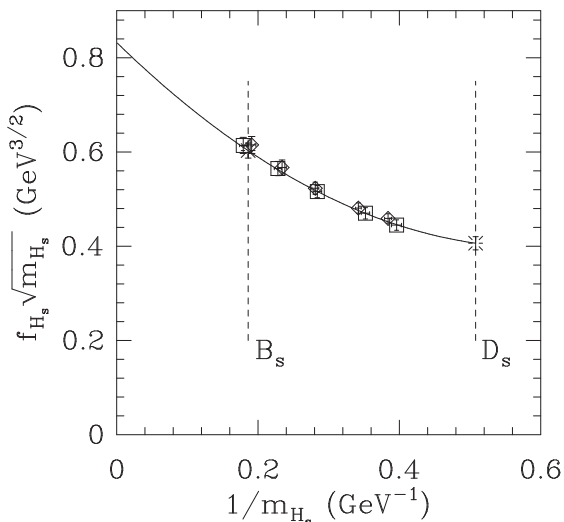


Figure 1.  $f_{H_s} \sqrt{m_{H_s}}$  vs. inverse meson mass. Squares correspond to the light sea quark mass  $m_{ud}^{\text{sea}} \approx m_s/4$  and diamonds to  $m_{ud}^{\text{sea}} \approx m_s/2$ . The curve is the fit (to the squares) described in the text and the bursts are the fit results at physical values of  $m_{H_s}$ .

higher orders in the heavy quark expansion and discretization errors.

Figure 1 shows the mass dependence of the combination  $f_{H_s} \sqrt{m_{H_s}}$ . We would like to extrapolate to the charm region, where we are unable to calculate using NRQCD at this lattice spacing. Fits to a power series

$$f_{H_s} \sqrt{m_{H_s}} = \Phi^{\text{stat}} \left( 1 + \sum_{n=1}^N \frac{C_n}{m_{H_s}^n} \right) \quad (2)$$

give acceptable  $\chi^2$ 's for  $N \geq 2$  – a linear fit gives an unacceptably large  $\chi^2/\text{DoF} = 3$ . The extrapolated value for  $f_{D_s}$  is 290 MeV. The largest uncertainty is again  $\mathcal{O}(\alpha_s^2) \approx 10\%$  perturbative corrections. Other uncertainties are still being esti-

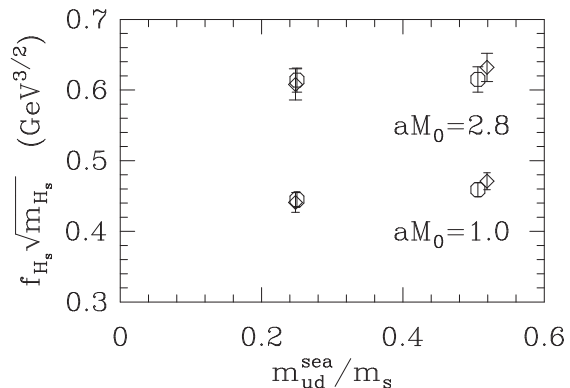


Figure 2. Light sea quark mass dependence on  $f_{H_s} \sqrt{m_{H_s}}$ . The heavy quark mass is  $\approx m_b$  for the top 4 points and  $\approx 1.5 m_c$  for the bottom 4 points. Octagons use  $a^{-1}$  from  $\Upsilon(2S - 1S)$  and diamonds use  $a^{-1}$  from  $\Upsilon(1P - 1S)$ .

mated.

Within statistical errors, there is no sea quark mass dependence apparent in Fig. 1. In Fig. 2 we plot the same quantity for 2 values of heavy quark mass and with 2 definitions of the lattice spacing. Note from Table 1 that with  $m_{ud}^{\text{sea}} \approx (3/4)m_s$  we see the reappearance of a scale ambiguity. With  $m_{ud}^{\text{sea}} \approx m_s/2$  the 2 lattice spacings differ by  $1.5\sigma$ . This effect apparently masks any sea quark mass dependence, as can be seen in Fig. 2, consequently we conclude that a chiral extrapolation in  $m_{ud}^{\text{sea}}$  will be an effect smaller than the other quoted errors. We emphasize that within statistical errors of 1.3% no scale ambiguity exists for  $m_{ud}^{\text{sea}} \approx m_s/4$  which is where our result (1) is taken.

### 3. CHIRAL BEHAVIOR

The main benefit of using staggered fermions is being able to simulate closer to the chiral limit. In order to make maximal use out of the gauge field

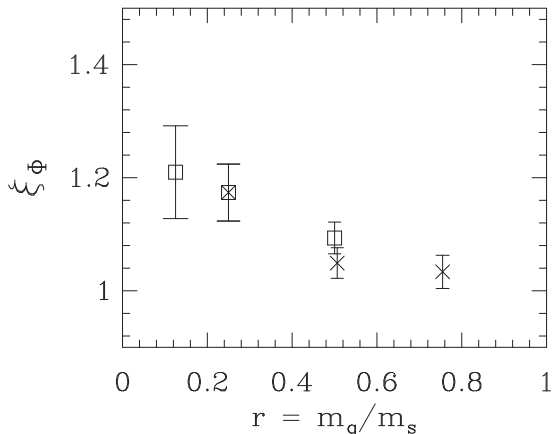


Figure 3.  $\xi_\Phi \equiv f_{B_s} \sqrt{m_{B_s}} / f_{B_q} \sqrt{m_{B_q}}$  plotted as a function of valence light quark mass in units of  $m_s$ . The crosses have  $m_q = m_{ud}^{\text{sea}}$  and are uncorrelated, and the squares have fixed  $m_{ud}^{\text{sea}} = m_s/4$  so are correlated.

configurations, several values of valence quark mass  $m_q$  are used. So far we have accumulated data with  $m_{ud}^{\text{sea}} \geq m_s/4$  and  $m_q \geq m_s/8$ , compared to  $m_q \geq 0.7 m_s$  which is the state-of-the-art using Wilson-like fermions [7].

In Fig. 3 we plot  $f_{B_s}/f_{B_q}$  (times  $\sqrt{m_{B_s}/m_{B_q}}$  or 1.01) against  $m_q/m_s$ . Crosses are unquenched, except that the dynamical strange quark mass is slightly heavier ( $m^{\text{sea}} = (5/4)m_s$ ) than the valence strange quark mass ( $m^{\text{val}} = m_s$ ), and squares have light sea quark mass fixed to  $m_s/4$ . A partially quenched analysis will utilize the correlations between data points computed on the same configurations.

Bećirević *et al.* recently noticed that the coefficient of the chiral logs in the double ratio

$$R \equiv \frac{f_{B_s} \sqrt{m_{B_s}}}{f_B \sqrt{m_B}} \bigg/ \frac{f_K}{f_\pi} \quad (3)$$

is numerically smaller than in either ratio individually [8]. Combining our results with the MILC collaboration results [9] for  $f_K/f_\pi$  (which is 1.22 experimentally) yields Fig. 4. Extrapolating  $R$  to  $r = 0$ , a range of 1.0 – 1.1 would correspond to  $\xi$  between 1.22 and 1.34. ( $\sqrt{B_{B_s}/B_{B_d}} = 1.01(3)$  [7].) Much work remains to be done before we have a trustworthy and precise final result, but

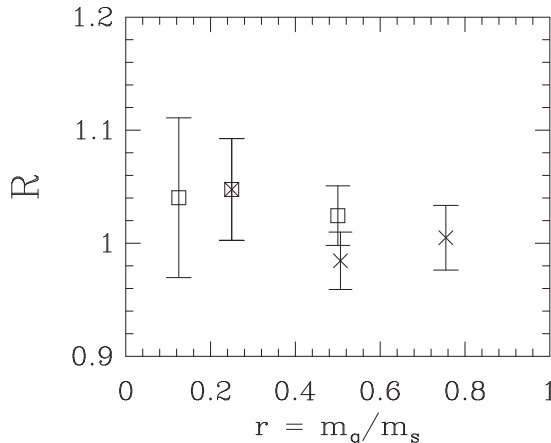


Figure 4. The double ratio  $R$  (see Eq. (3)) plotted as a function of valence light quark mass in units of  $m_s$ . Symbols are as in the previous figure.

these simulations with masses below  $m_s/2$  should cast new light on the chiral extrapolation of this important quantity.

## ACKNOWLEDGMENTS

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