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On the Co-existence of Conventions

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Abstract

Recent stochastic evolutionary models have shown that the most likely convention when the probability of a mutation is su±ciently small is co-ordination on the risk-dominant strategy rather than the payo®-dominant one. This paper looks at the consequences of player movement between locations when there are constraints which limit the number of agents who can reside at each location. If the constraints are strong then the risk-dominance result continues to hold. However, we show that when su±cient movement is possible, the most likely outcome involves a mixed state in which agents at di®erent locations coordinate on di®erent strategies. In the asymmetric case, it is the location with the stronger constraint, limiting movement, that coordinates on the payo®-dominant strategy.

JEL classi¯cation numbers: C72, C73, D83.

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1 Introduction

How do players know which equilibrium to play when a game has multiple equilibria? This question has been at the heart of recent research in game theory. The focus of attention has been the 2x2 Coordination Game such as the one given in ¯gure 1 that has two Nash equilibria in pure strategies, one of which is Pareto-e±cient but is riskier to play than the other. Harsanyi and Selten (1988) call the former equilibrium payo®-dominant and the latter risk-dominant. Schelling (1960) appeals to the prominence of e±ciency to suggest that agents will play for the payo®-dominant equilibrium in the expectation that other agents will be similarly attracted by its focal status. But Harsanyi and Selten have emphasised that such an expectation may not be well-founded. If each player optimises on the assumption that the opponent is equally likely to play either strategy, the outcome will be the risk-dominant equilibrium of the game, which therefore also has a focal status that may outweigh that of the payo®-dominant equilibrium.

Evolutionary game theory has given the argument another perspective. By modelling the process by which agents adjust their strategies out of equilibrium we can analyse how it is that one equilibrium strategy rather than another may be selected. The principle underlying the dynamic systems studied in evolutionary game theory is that successful strategies will be used by a greater proportion of the population in future periods.

	s_1	s_2
s_1	5,5	0,3
s_2	3,0	4,4

Figure 1

2x2 Coordination Game

To illustrate the idea consider the Coordination Game of ¯gure 1. The game has two pure-strategy equilibria, $e_1 = (s_1; s_1)$ and $e_2 = (s_2; s_2)$. Notice that e_1 is payo®-dominant while e_2 is risk-dominant. There is also a mixed-strategy equilibrium where s_1 is played with probability 2=3. When expressed in terms of the fraction q of the population using strategy s_1, these Nash equilibria correspond respectively to $q = 1$, $q = 0$ and $q = 2$=3. Assume that members of the population are randomly matched each period to play this game. They adjust their choice by playing the strategy that yielded the highest expected payo® in the previous period when they are given the chance to do so. This simple adjustment process leads to a dynamic system for which the population states $q = 1$ and $q = 0$ are stable stationary points. Denote these stationary states by E_1 and E_2 respectively. Consider the case where $q > 2$=3. If a revision opportunity arises, then the optimal response against the current state is to play s_1. The proportion playing s_1 will therefore grow over time until the state where everyone plays s_1 is reached. The basin of attraction of E_1 is therefore (2=3; 1], since it will be selected from any state where $q > 2$=3. Similarly the basin of attraction of E_2, where everyone plays s_2, is [0; 2=3). A third possible stationary state is $q = 2$=3.

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At this point, no agent has an incentive to change his strategy. However, only E_1 and E_2 are locally stable.

Kandori et al. (1993) and Young (1993) added to this analysis by assuming that agents sometimes mutate by changing their strategies at random. Each agent has a positive probability of mutating each period. There is therefore a small but positive probability that there will be a large number of simultaneous mutations. Once in an equilibrium, it is therefore no longer the case that the system will stay there forever because enough simultaneous mutations will eventually occur to move the system into the other basin of attraction. The system therefore needs to be described in terms of a probability distribution over the states.

Kandori et al. show that, when the probability of mutation goes to zero, the distribution becomes concentrated entirely on the risk-dominant equilibrium, E_2. The reason for this is that more mutations are required to move from E_2 to E_1 than from E_1 to E_2. As the mutation rate goes to zero the probability of the ¯rst transition becomes negligible compared with the second. The time-limit of the distribution over population states therefore puts all its mass on E_2 when the mutation rate becomes vanishingly small. We can therefore say that E_2 is the most likely equilibrium when the mutation rate is very small. (Following Kandori et al., equilibria that have a positive probability as the mutation rate goes to zero will be called long-run equilibria). Ellison (1993) extends this result to a model of local interaction where players are located around a circle and interact only with a subset of the population who are close to them.

The stable states can be thought of as conventions. If a society is following a particular convention then it is in the interest of all participants to conform. The convergence to risk-dominance suggests that the dynamic process will not necessarily lead to the best convention for society. This negative result has been countered by introducing movement between locations. In Kandori et al., the location structure does not matter since each agent is equally likely to be matched with every other agent in the population. In models of local interaction, agents are more likely to be matched with neighbouring players. An agent's choice of location is therefore important, since this will determine his or her expected payoff. Thus, if agents are given the chance, they will move to the location where they get the highest expected payoff. In Ellison's model, however, this phenomenon is absent, since agents are located at fixed positions around a circle and remain there. If this assumption is relaxed, it may be more difficult to upset the payoff-dominant equilibrium because agents may move away from a locality in which deviant mutations have occurred in search of a higher payoff. Similarly, the risk-dominant equilibrium may now be easier to upset since a few localised mutations may entice movement towards this locality. Ely (1995) presents a model based on this idea in which such movement makes the long-run equilibrium E_1 rather than E_2.

A third possibility when local interaction is modelled is for conventions to co-exist. We often observe different societies coordinating on different strategies. For example, in some cultures it is conventional to greet people with a handshake

whereas in others a peck on the cheek (or on each cheek) is the conventional thing to do[1]. We demonstrate how the co-existence of conventions can arise as a result of constraints on the number of people that can reside at each location. Capacity constraints may arise from physical aspects of the locations or may be imposed by local governance. For example, there may be a certain number of slots at each location, limiting the number that can play there, or one can imagine neighbouring societies who impose restrictions on the maximum number of inhabitants. Once this limit is reached, further immigration is prohibited. Such constraints may be imposed when there is a negative payoff to congestion. There may also be a minimum number of agents at each location. Such a constraint may arise when some agents are immobile or simply prefer to stay put. We will call these agents patriots.

Goyal and Janssen (1997) find that conventions can co-exist if there is some initial period of isolation when the societies evolve to different conventions. They present a deterministic model that restricts attention to non-exclusive conventions where agents can choose to be flexible at some cost allowing them to coordinate correctly with whoever they meet. Co-existence then arises if the cost is at some intermediate level. In contrast, there is no initial period of isolation in our model. We show that the situation where two societies have different conventions can be the most likely scenario when constraints are present. When no movement is possible, we know the long-run equilibrium will involve agents from both islands

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[1]I'll avoid completing the analogy by suggesting one of the conventions is efficient!

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coordinating on the risk-dominant strategy. This continues to be the case if a limited amount of movement is possible. The coexistence arises when su±cient movement is possible. That is, when the capacity constraints are not too strong and a su±cient proportion of the population is mobile. In equilibrium one location will play the e±cient equilibrium and the other will play the risk-dominant one with a binding constraint (either the location playing the risk-dominant equilibrium has only patriots left or the location playing the e±cient equilibrium is full to capacity). In the symmetric case each location is equally likely to be the one that plays the e±cient equilibrium. The most interesting result arises when we look at the asymmetric case where the locations have di®erent capacity constraints and di®erent numbers of patriots. The location that has the stronger capacity constraint (or fewer patriots) will be the one that plays the e±cient equilibrium.

In section 2 we look at a simple symmetric model where strategy revision is instantaneous, i.e. everybody revises their strategy each period, but the chance to move to the other location only arises with some positive probability. We start with this model to make clear the techniques that are being used. In section 3 we show that the results hold when there is inertia in strategy revision and section 4 looks at the asymmetric case.

2 Basic Model

We begin by presenting a basic model of local interaction with movement between locations. We assume that there are two islands and agents are randomly matched with someone on the same island to play the game of ¯gure 2 in which $A > C$, $D > B$, $A > D$ and $A + B < C + D$. Hence $e_1 = (s_1; s_1)$ is the payo®-dominant equilibrium while $e_2 = (s_2; s_2)$ is risk-dominant. The probability with which s_1 is played in the mixed-strategy equilibrium is $q^{\pi} = (D_{\text{i}} B)=(A_{\text{i}} C + D_{\text{i}} B) > 1=2$:

	s_1	s_2
s_1	A,A	B,C
s_2	C,B	D,D

Figure 2

Each island has pN patriots who never change location where $0 < p < 1$: The global population is $2N$ and the capacity of each island is cN where $c > 1$. The maximum number of agents who can reside at each location is therefore dN where $d = \min(c; 2_{\text{i}} p)$ and $1 < d < 2$: Strategy revision is instantaneous, that is everybody chooses a strategy that is a best response to the state in the previous period. The opportunity for an agent to change islands arises with a positive probability each period. When such an opportunity arises, a mobile agent will choose the island and strategy that would have maximised their expected payo® in the previous period. If the agent is indi®erent between two choices then we assume they choose either with a positive probability. However, an agent cannot move to an island that is full to capacity. If the number of agents who wish to

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move to island i is greater than $(Nd - n_i)$, where n_i is the current number on the

island, then only $(Nd - n_i)$ of them will be allowed to move.

The state space is

$$S = \{(\frac{n_1^1}{n_1}; \frac{n_2^1}{2N - n_1}) : n_1^1 \in (0, 1, ..., n_1); n_2^1 \in (0, 1, ..., 2N n_1); N(2 - d) \leq n_1 \leq Nd\}$$

where n_i^1 is the number playing strategy s_1 on island i and n_1 is the number of

agents on island 1. Denote a state of the system by $s = (q_1; q_2; n_1) \in S$, where q_i

is the proportion of the population playing s_1 on islands i.

The dynamics give rise to a Markov process, P, on state space S. From any

initial condition, the system will move to a state or set of states where it remains.

Following Young (1993), such a set will be called a recurrent communication class.

The recurrent communication classes are characterised later.

Without mutations, the system will move to one of these classes and remain

there. Now assume that each agent mutates independently, with probability ",

with the consequence that a strategy is re-selected at random on their current

island[2]. This allows the system to move between classes and gives rise to the

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[2]All the results go through if we assume a strategy and location is re-selected at random by a mobile agent with the restriction that the capacity constraint cannot be broken. If the number who re-select a location at random would take that location over its capacity then that island becomes full to capacity and some agents select a strategy at random on their current island instead.

perturbed transition matrix P^{ε} given by,

$$P_{ij}^{\varepsilon} = P_{ij}(1 - \varepsilon)^{2N} + \sum_{k=1}^{P} c_{ijk}\varepsilon^{k}(1 - \varepsilon)^{2N - k} \qquad (1)$$

where P_{ij} is the ijth element of P, the unperturbed transition matrix and c_{ijk} is some non-negative number that depends on the exact speci¯cation of the dynamics.

Proposition 1: P^{ε} has a unique stationary distribution $\mu(\varepsilon)$ and $\lim_{\varepsilon \to 0} \mu(\varepsilon)$ exists.

Proof. Young (1993) shows that this is true if P^{ε} is a regular perturbation of P which requires that the following conditions hold,

i) P^{ε} is aperiodic and irreducible.

ii) $\lim_{\varepsilon \to 0} P_{ij}^{\varepsilon} = P_{ij}$:

iii) $P_{ij}^{\varepsilon} > 0$ for some ε implies $9r > 0$ such that $0 < \lim_{\varepsilon \to 0} \varepsilon^{-r}P_{ij}^{\varepsilon} < 1$:

From (1) conditions (ii) and (iii) are clearly satis¯ed. When $P_{ij}^{\varepsilon} > 0$ then r is 0 if $P_{ij} > 0$ or equal to the lowest value of k such that $c_{ijk} > 0$. We now show that P^{ε} is aperiodic and irreducible. The diagonal elements of P^{ε} are all positive. This is because in any state, there is a positive probability that nobody moves and that there are mutations that keep the same numbers playing each strategy on both islands. Hence P^{ε} is aperiodic. There is a positive probability of going from any state to the class in which both islands coordinate on the same equilibrium.

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This simply requires a certain number of mutations on each island. We can then have any number of agents on each island up to Nd and for a given number of agents on each island, we can have any number playing each strategy, as there is a positive probability that nobody moves while a certain number mutate. It is therefore possible to go from any state to any other and the process is irreducible.

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De¯nition 1: The set of states in the support of $\lim_{\mu \to 0} \mu^1(\varepsilon)$ will be called the long-run equilibria.

De¯nition 2: A k-tree, h, de¯ned on state space R (the set of recurrent communication classes), is a set of ordered pairs,$(i \to j)$ $i, j \in R$, such that each state $x \neq k$ is the initial point of one arrow and from every state there is a path which leads to k.

Let $M(i, j)$ be the minimum number of mutations required to go from class i to j. We know that such a number exists because P^{ε} is irreducible. The cost of a k-tree is $\sum_{(i \to j) \in h} M(i, j)$:

Proposition 2: The long-run equilibria are the set of states in the recurrent communication class which has the lowest cost k-tree.

For the proof the reader is referred to Young (1993). The intuition is clear. The long-run equilibria are the set of states in the recurrent communication class that is easiest to °ow into from all other recurrent communication classes. Hence to ¯nd the long-run equilibria we need to characterise the recurrent communication classes and the costs $M(i, j)$ of moving between them and then ¯nd the class

that has the lowest cost k-tree.

Recurrent communication classes.

One recurrent communication class is the set of all states where $q_1 = q_2 = 0$. The basin of attraction of this class is $\{(q_1, q_2) : q_1 \leq q^\pi, q_2 \leq q^\pi\}$, since best replies will lead both islands to coordinate on the risk-dominant equilibrium. In this class the system will move between states where $q_1 = q_2 = 0$ and $n_1 \in (N(2 - d), Nd)$, since agents move with a positive probability when they are indifferent and n_1 must lie in this range due to the constraint[3].

Now consider any initial condition with $q_1 > q^\pi$ and $q_2 \leq q^\pi$. Best replies will move the system towards $q_1 = 1$ and $q_2 = 0$. This will result in movement into island 1, since the higher payoff equilibrium is being played there. The system will eventually move to the equilibrium state $(1, 0, Nd)$. Similarly the set of states with $q_1 \leq q^\pi$ and $q_2 > q^\pi$ form the basin of attraction of the equilibrium $(0, 1, N(2 - d))$. The final possibility is for both populations to coordinate on the payoff-dominant equilibrium. The basin of attraction for this class is $\{(q_1, q_2) : q_1 > q^\pi, q_2 > q^\pi\}$, and the recurrent communication class is the set of all states with $q_1 = q_2 = 0$ and $n_1 \in (N(2 - d), Nd)$. The four recurrent communication

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[3]If agents are randomly paired then a mobile agent who knows that her current location has an even number of agents will strictly prefer not to move when agents on both locations are coordinating on the same strategy to avoid the small chance of remaining unmatched. However, it is not necessary to assume that agents know the exact number at their current location. The only crucial restriction is that the dynamics must reinforce successful strategies. A boundedly rational agent may adopt a successful strategy simply through imitation. If we want to assume agents do know the exact numbers then we need the additional assumption that agents move around in pairs or that in the event of an odd one out, mobile agents are always matched.
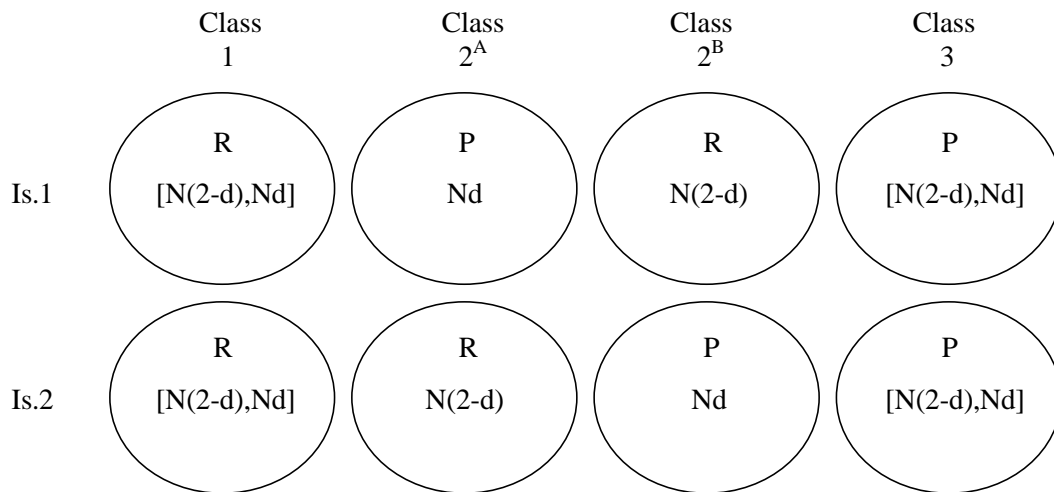
classes are illustrated in figure 3.

	Class 1	Class 2^A	Class 2^B	Class 3
Is.1	R [N(2-d),Nd]	P Nd	R N(2-d)	P [N(2-d),Nd]
Is.2	R [N(2-d),Nd]	R N(2-d)	P Nd	P [N(2-d),Nd]

Figure 3: Recurrent communication classes.

Row i of circles illustrates the equilibrium played on island i

in each of the classes (risk dominant, R or payoff dominant, P),

plus the values n_i can take in that class.

Lemma 1: To find the class which has the minimum cost k-tree it is sufficient to find the minimum cost trees between just three classes, ruling out either 2^A or 2^B.

Proof. Let h be a minimum cost k-tree. Adjust the tree so that at least one of 2^A or 2^B have no predecessors without changing the cost. This is easy to do since $i \to 2^A$ can be transferred to $i \to 2^B$ (or vice versa) leaving 2^A with no predecessors (and this will not change the cost as $M(i; 2^A) = M(i; 2^B)$ and

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$M(2^A; i) = M(2^B; i)$ for $i \in (1,3)$ from symmetry). We can split the adjusted k-tree into two parts, a minimum cost k^0-tree defined on the vertices (1,2,3) and 2^A added at minimum cost. It must be a minimum cost k^0-tree because any adjustments which reduce the cost would also reduce the cost of the k-tree but we started with a minimum cost k-tree. Hence we can find the minimum cost k-tree by first finding the minimum cost k^0-tree and then adding a 2-state at minimum cost. This cost will be common to all k-trees and so does not need to be considered. ∎

1-trees	2-trees	3-trees
$A^1 : 3 \to 2 \to 1$	$A^2 : 1 \to 2 \tilde{A} 3$	$A^3 : 1 \to 2 \to 3$
$B^1 : 2 \to 3 \to 1$	$B^2 : 3 \to 1 \to 2$	$B^3 : 2 \to 1 \to 3$
$C^1 : 2 \to 1 \tilde{A} 3$	$C^2 : 1 \to 3 \to 2$	$C^3 : 2 \to 3 \tilde{A} 1$

Table 1

k-trees

This leaves nine trees that we need to compare (3 for each communication class). These trees are illustrated in table 1. To compare the costs of the k-trees we need to find the minimum cost of moving between the 3 classes. Consider the transition from class 1 to 2^A. We want the minimum number of mutations required to get into the basin of attraction of class 2^A, $f(q_1; q_2) : q_1 > q^*; q_2 \leqslant q^* g$,

from a state in class 1, $(0; 0; n_1)$. Hence we require a proportion q^* of island 1 to mutate. Now the less populated island 1 is, the smaller the number of mutations required to achieve this. The minimum value of n_1 is $N(2 - d)$ so $M(1; 2^A) = N(2 - d)q^*$. The dynamics will then move the system to the state $(1; 0; Nd)$. The cost of moving back is $Nd(1 - q^*)$ since we require the system to move back to a state where $q_1 \ge q^*$ and island 1 is full to capacity. A direct jump will not necessarily yield the minimum number of mutations. For example consider the transition from class 1 to 3. A direct jump from class 1 to 3 requires $2Nq^*$ simultaneous mutations. However, it is easier to go from class 1 to 2 and then from 2 to 3 since this only requires $2(2 - d)Nq^*$ mutations. Hence $M(1; 3) = 2(2 - d)Nq^*$. All the minimum costs are given in figure 4.
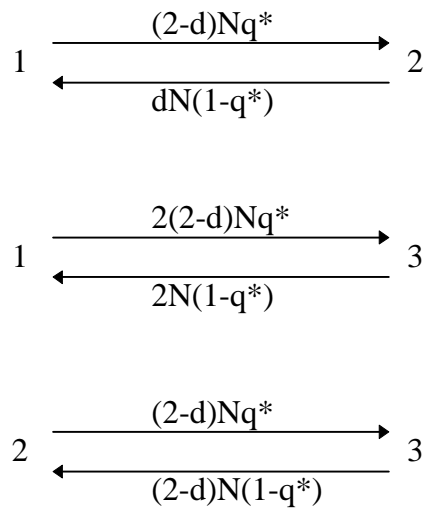

Figure 4

Minimum costs $(M(i; j)$'s) of moving between recurrent

communication classes.

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Proposition 3: The long-run equilibria are: the set of states in class 1 if $d < 2q^{\pi}$; states 2^A and 2^B if $d > 2q^{\pi}$.

Proof. From proposition 2, the long-run equilibria are the set of states in the recurrent communication class which has the lowest cost k-tree. It is a simple exercise to see that the lowest cost 1-tree is A^1. The other two 1-trees include the transition $3 \to 1$, which has the same cost as A^1 but also include a transition from class 2 at some cost. Similarly, the lowest cost 3-tree is A^3 as the other two 3-trees include the transition $1 \to 3$, which has the same cost as A^3. Finally, the lowest cost 2-tree is A^2. The other two 2-trees include the transitions $3 \to 1$ and $1 \to 3$. In each case the cost is reduced by going directly to class 2.

The only difference between the cost of A^2 and A^3 is in the transition between classes 2 and 3. Since $M(2;3) > M(3;2)$ (as $q^{\pi} > \frac{1}{2}$), A^2 always has a lower cost. This leaves two candidates for minimum cost k-tree, A^1 and A^2. The cost of A^1 is less than the cost of A^2 if $M(2;1) < M(1;2)$. Hence class 1 has the lowest cost k-tree if

$$d(1 - q^{\pi}) < (2 - d)q^{\pi} => d < 2q^{\pi}$$

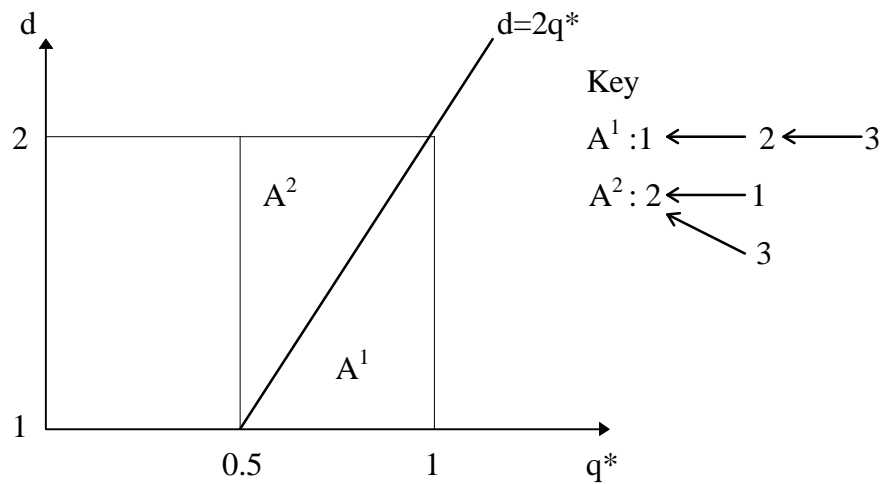If the inequality is reversed then class 2 has the minimum cost k-tree. ∎

Figure 5

Long-run equilibria

The long-run equilibria are illustrated in ¯gure 5. Hence the long-run equi-
libria are the set of states where everyone plays s_2, the risk-dominant strategy if
$d < 2q^{\pi}$. The critical value of d where class 1 becomes the long-run equilibrium
increases with the degree of risk-dominance. If d is above this critical value then
class 2 has the minimum cost k-tree. The long-run equilibria are the two states
where agents on the two islands coordinate on di®erent strategies. In fact, from
the symmetry of the cost structure, states 2^A and 2^B will each have a probability
of one half in the limit-distribution.

The intuition follows from the analysis. In class 2, the island playing the
payo®-dominant equilibrium becomes more populated as d increases because in
equilibrium it is full to capacity. The transition to class 1 therefore becomes more
di±cult. Eventually class 1 becomes more costly to °ow into. Hence with su±-

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ciently large capacities/su±ciently few patriots, the payo®-dominant equilibrium will be played on one of the islands. We never observe class 3 as the long-run equilibrium where both islands coordinate on the payo®-dominant equilibrium. The reason for this is that in class 2 all mobile agents want to move out of the island playing the risk-dominant equilibrium. However, no movement out is possible because the other island is full to capacity (or because it contains only patriots). Hence it is as if the other island does not exist. As it is more di±cult to get out of a risk-dominant equilibrium than to get in, the long-run equilibrium involves the island coordinating on the risk-dominant strategy[45].

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[4]We have only considered the case where there is a constraint (d < 2): If d = 2 (p = 0 and c > 2) then the island playing the risk-dominant equilibrium becomes empty. As in Ely (1995), the long-run equilibrium would then involve all agents coordinating on the payo®-dominant strategy.

[5]One of the interpretations for the capacity constraints given in the introduction is that they are imposed by local governance to limit congestion. A congestion e®ect can be introduced by imposing a tie-breaking rule where agents move to an island that is less congested when they are otherwise indi®erent. With this additional e®ect, class 1 reduces to the state $(0; 0; N)$ and class 3 reduces to the state, $(1; 1; N)$. The four recurrent communication classes are then 4 single states. This will not signi¯cantly a®ect the long-run equilibria as only one mutation is required to get agents moving between islands. (The conditions in proposition 3 become, $d < 2q^{\mathtt{n}} + 1{=}N$ and $d > 2q^{\mathtt{n}} + 1{=}N$).

3 Inertia in strategy revision.

The previous results rely on the assumption that strategy revision is instanta-
neous. We now extend the model to the case where there is a positive probability
that agents simply continue to use the strategy they used in the previous pe-
riod. However, if they do revise their strategy they do so by playing a best
reply. As before, there is a positive probability that they are given the chance
to move islands and agents will then choose the island and strategy that would
have maximised their expected payo® in the previous period, as long as this does
not involve moving to an island that is full to capacity. The state space S, is the
same as in the case with no inertia. The above dynamics, however, give rise to a
di®erent transition matrix, P^0. All other aspects of the model are the same. The
perturbed transition matrix is given by

$$P_{ij}^{0"} = P_{ij}^0 (1 \text{ ¡ } ")^{2N} + \sum_{k=1}^{} c_{ijk} "^k (1 \text{ ¡ } ")^{2N \text{ ¡ } k}$$

The same reasoning as before can be used to show that $P^{0"}$ is aperiodic and
irreducible. Hence we can apply propositions 1 and 2 and ¯nd the long-run
equilibria by ¯nding the recurrent communication class that has the lowest cost
k-tree. The recurrent communication classes are the same as in the model with
no inertia and are illustrated in ¯gure 3. However, the basins of attraction of
the recurrent communication classes are now di®erent. This changes the cost

of moving between classes. In the model without inertia, the minimum cost of

the transition $2 \to 1$ is $Nd(1 - q^*)$. We can now achieve this transition with

fewer mutations because after a certain number of mutations on the efficient

island, it will be optimal for agents to move and get a payoff of D. If there

are $(2 - d)(1 - q^*)N$ mutations followed by movement, then there is a positive

probability that $2N(d - 1)$ agents move and that all the agents that move were

playing s_1, while nobody revises their strategy on the efficient island. Hence the

proportion playing s_1 will be

$$(Nd - 2N(d - 1) - (2 - d)(1 - q^*)N) = (Nd - 2N(d - 1)) = q^*$$

However, we must ensure that it is optimal to move and this requires a proportion

$(1 - q')$ of the efficient island to mutate, where q' satisfies $Aq' + B(1 - q') = D$ or

$q' = (D - B) / (A - B)$. Hence $(1 - q')dN$ mutations are required before anyone will

move. Since the number of mutations must satisfy both of the above conditions,

the minimum number of mutations required will be $\max[(1 - q')dN; (2 - d)(1 - q^*)]$.

The minimum costs of moving between recurrent communication classes are given

in figure 6.

$$1 \xrightarrow{\;(2\text{-}d)Nq^*\;} 2$$
$$\overleftarrow{\max[(1\text{-}d)N(1\text{-}q^*),(1\text{-}q')Nd]}$$

1 $\xrightarrow{\;2(2\text{-}d)Nq^*\;}$ 2

$$\max[(1\text{-}d)N(1\text{-}q^*),(1\text{-}q')Nd]+(2\text{-}d)N(1\text{-}q^*)$$

$$2 \xrightarrow{\;(2\text{-}d)Nq^*\;} 3$$
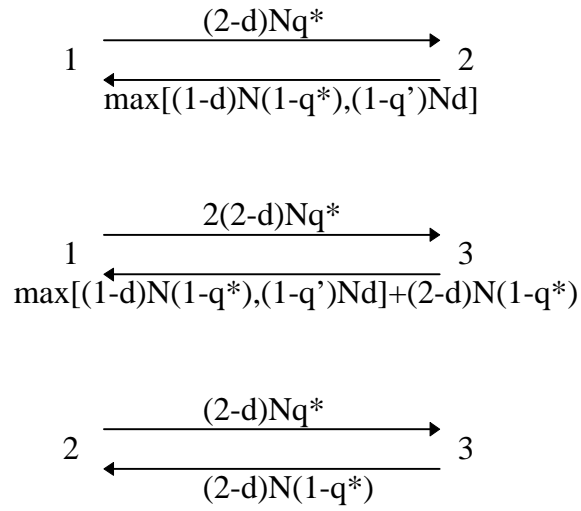$$\overleftarrow{(2\text{-}d)N(1\text{-}q^*)}$$

Figure 6

Minimum costs of moving between recurrent

communication classes with inertia.

Proposition 4: The long-run equilibria with inertia are: all states in class 1 if $d < \frac{2q^\alpha}{q^\alpha+1-q^0}$; states 2^A and 2^B if $d > \frac{2q^\alpha}{q^\alpha+1-q^0}$:

Proof. From proposition 2 we know that we need to find the class with the minimum cost k-tree. Also to find the class which has the minimum cost k-tree it is sufficient to find the minimum cost trees between just three classes, ruling out either 2^A or 2^B (lemma 1). Of the nine trees (table 1), it is a simple exercise to see that the minimum cost tree is either $3 \to 2 \tilde{\to} 1$ or $3 \to 2 \to 1$[6]. Hence the set of states in class 1 will be the long-run equilibria when $\max[(2-d)(1-q^\alpha),(1-q^0)d] < (2-d)q^\alpha$. This condition reduces to $(1-q^0)d < (2-d)q^\alpha$ as $(2-d)(1-q^\alpha)$ is

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[6]The reasoning is exactly the same as in the proof of proposition 3.

always less than $(2 - d)q^{\ast}$: Solving for d gives $d < \frac{2q^{\ast}}{q^{\ast}+1 - q^{0}}$: If the inequality is reversed then class 2 has the minimum cost k-tree. ∎

Apart from the transitions $2 \to 1$ and $3 \to 1$ the minimum costs of moving between recurrent communication classes are the same as the model with no inertia. The only significant difference is that it is easier to move from class 2 to class 1 as fewer mutations are required. Class 1 therefore has a slightly larger range over which it is the long-run equilibrium. Otherwise the long-run equilibria are similar - class 1 if d is below some critical value and class 2 if it is above this value, where the critical value is increasing with $q^{\ast}$:

4 Asymmetric constraints.

We now turn to the case where the limit on the number of agents who can reside at each location is di®erent. This may be because the locations do not have the same number of patriots or because they have di®erent capacity constraints. We extend the basic model to allow for these asymmetries: Assume the capacity of island i is c_iN where $c_i > 1$ and the number of patriots on island i is p_iN where $0 < p_i < 1$. The maximum number of agents who can reside on island i is therefore d_iN where $d_i = \min(c_i, 2 - p_j)$ and $1 < d_i < 2$: We can follow the reasoning of section 2 to characterise the recurrent communication classes and the minimum costs of moving between them. These are given in ¯gures 7 and 8.
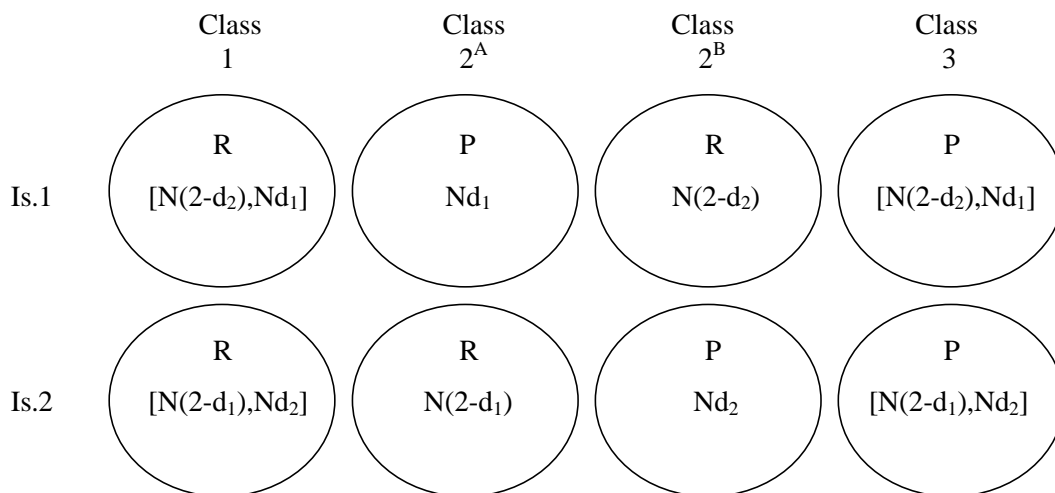
	Class 1	Class 2^A	Class 2^B	Class 3
Is.1	R $[N(2-d_2),Nd_1]$	P Nd_1	R $N(2-d_2)$	P $[N(2-d_2),Nd_1]$
Is.2	R $[N(2-d_1),Nd_2]$	R $N(2-d_1)$	P Nd_2	P $[N(2-d_1),Nd_2]$

Figure 7

$$1 \xrightarrow{(2-d_2)Nq^*} 2^A \qquad \text{with} \qquad \xleftarrow{d_1N(1-q^*)}$$

1 $\xrightarrow{(2-d_2)Nq^*}$ $\xleftarrow{d_1N(1-q^*)}$ 2^A 1 $\xrightarrow{(2-d_1)Nq^*}$ $\xleftarrow{d_2N(1-q^*)}$ 2^B

1 $\xrightarrow{(2-d_1)Nq^*+(2-d_2)Nq^*}$ $\xleftarrow{2N(1-q^*)}$ 3 2^A $\xrightarrow{(2-d_2)N(1-q^*)+(2-d_1)Nq^*}$ $\xleftarrow{(2-d_1)N(1-q^*)+(2-d_2)Nq^*}$ 2^B

2^A $\xrightarrow{(2-d_1)Nq^*}$ $\xleftarrow{(2-d_1)N(1-q^*)}$ 3 2^B $\xrightarrow{(2-d_2)Nq^*}$ $\xleftarrow{(2-d_2)N(1-q^*)}$ 3
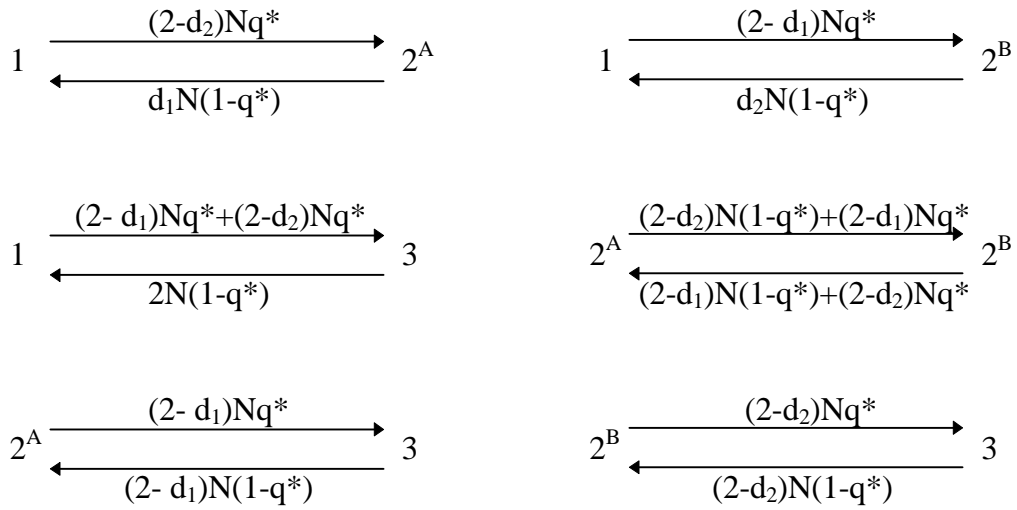
Figure 8

As before, the recurrent communication classes either involve agents on both islands coordinating on the same strategy or the agents coordinating on di®erent strategies with the island playing the e±cient equilibrium full to capacity.

Proposition 5: The long-run equilibria with asymmetric constraints are:

the set of states in class 1 if $d_1 < \min[2 - \frac{(1-q^*)}{q^*}d_2, \frac{2}{1-q^*} - \frac{q^*}{1-q^*}d_2]$;

state 2^A if $d_1 > \frac{2}{1-q^*} - \frac{q^*}{1-q^*}d_2$ and $d_1 < d_2$;

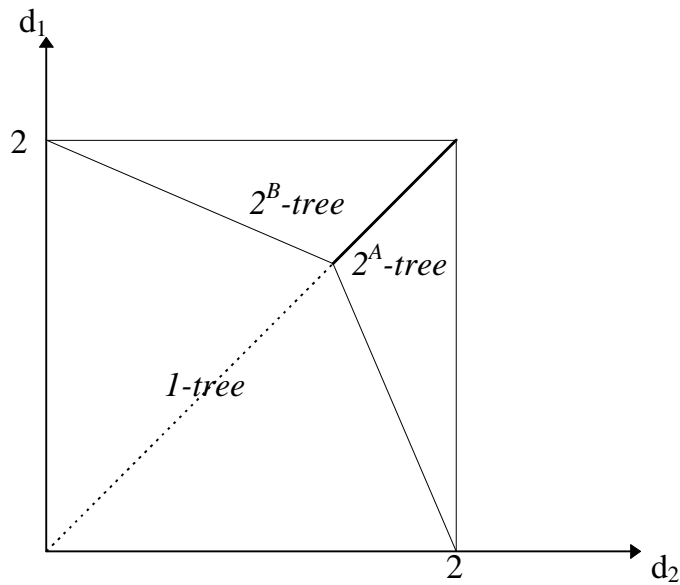state 2^B if $d_1 > 2 - \frac{(1-q^*)}{q^*}d_2$ and $d_1 > d_2$.

Figure 9

The equilibria are illustrated in ¯gure 9 for the case where $q^\alpha = \frac{2}{3}$. The proof is given in the appendix. As before, we move away from the equilibrium where both locations coordinate on the risk-dominant strategy when su±cient movement is possible but not to the equilibrium where both locations coordinate on the payo®-dominant strategy. The intuition is the same as in the basic model. However, with asymmetric constraints we ¯nd that the most likely state when su±cient movement is possible involves the location with the stronger constraint coordinating on the payo®-dominant strategy and the other location coordinating on the risk-dominant strategy.

To see why this is consider the cost of moving between states 2^A and 2^B. Assume $d_1 < d_2$ so that island 1 has the stronger constraint. To move between

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2^A and 2^B either via class 3 or class 1 requires that one island is converted from coordinating on the risk-dominant strategy and the other island is converted from coordinating on the payoff-dominant strategy. Converting an island from coordinating on the risk-dominant strategy requires a larger proportion (q^π) of the population to mutate and it is therefore this part of the transition that dominates the relative costs of the transitions. Now since the minimum possible number on island 2, $(2 - d_1)$ is greater than the minimum possible number on island 1 $(2 - d_2)$; the transition $2^A \to 2^B$ is more difficult than the transition $2^B \to 2^A$: Intuitively, when island 2 is coordinating on the risk-dominant strategy, it has a larger population than when island 1 is coordinating on the risk-dominant strategy and is therefore more difficult to escape from.

Now, consider the case where there are no capacity constraints. The long-run equilibria will be determined by the number of patriots on each island. If there are many patriots so that only limited movement is possible, the equilibrium will involve all agents coordinating on the risk-dominant strategy. However, if the number of mobile agents is sufficiently large, then the most likely state when the mutation rate is small will be agents on the island with fewer patriots coordinating on the payoff-dominant strategy, and agents on the other island coordinating on the risk-dominant strategy. Again, the intuition for this result is that the island with more patriots is more difficult to convert when it is playing the risk-dominant equilibrium. The islands can impose capacity constraints to further limit mobility (by setting $c_i < 2 - p_j$). An island that has more patriots can then counter

having more patriots by setting a su±ciently strong capacity constraint. The minimum possible number that can reside at the other island will then increase making it more di±cult for that island to escape from playing the risk-dominant equilibrium. However, if both islands compete by increasing constraints we will end up with little mobility and all agents coordinating on the risk-dominant strategy.

5   Conclusions

Game theorist have turned to evolutionary models in recent years to address the question of equilibrium selection. A particularly useful technique has been used to characterise the limit of the stationary distribution of a stochastic Markov process when the mutation rate goes to zero. This allows us to address the question of equilibrium selection in the long-run when the mutation rate is very small. Kandori et al. and Young show that the equilibrium selected in this way is the risk-dominant one rather than the payo®-dominant one.

We show that the introduction of movement between locations will upset the long-run equilibrium where everyone plays the risk-dominant strategy if su±cient movement is possible. However, if there are constraints which limit the number of agents who can reside on each island then we do not move to an equilibrium where everyone coordinates on the payo®-dominant strategy. Instead, when su±cient movement is possible, the most likely scenario will be that the islands have different conventions. Agents on one island will coordinate on the payo®-dominant strategy while agents on the other island will coordinate on the risk-dominant one.

On a more practical level, the results show that imposing restrictions on movement hinders e±ciency. If both islands impose a su±cient capacity constraint the equilibrium will involve all agents playing the risk-dominant strategy. However, if no constraints are imposed and one island has more patriots than the other, it

is in the interest of that island to impose a su±cient capacity constraint. Hence, without a binding agreement, capacity constraints will result in the risk-dominant convention on both islands.

A   Appendix

Proof of propostion 5.   Consider the minimum cost 1-trees. A 1-tree that includes the transition $2^i \to 2^j$ cannot be a minimum cost 1-tree as the 1-tree where the transition $2^i \to 2^j$ is replaced by $2^i \to 1$ has a lower cost. We can also eliminate any 1-tree that includes the transition $3 \to 1$ but not $2^i \to 2^j$ as $M(3;1) = M(3;2^i) + M(2^i;1)$ and a 1-tree with the transition $3 \to 1$ replaced by $3 \to 2^i \to 1$ will reduce the cost as the transition from 2^i is removed elsewhere. Similarly, a 3-tree that includes the transition $2^i \to 2^j$ or $1 \to 3$ cannot be a minimum cost 3-tree. A 2^i-tree that includes either $1 \to 3$ or $3 \to 1$ can be eliminated as we can replace the transitions with $1 \to 2^i$ or $3 \to 2^i$: Finally, a 2^i-tree that includes the transition $2^j \to 2^i$ but not $1 \to 3$ or $3 \to 1$ can be eliminated as there will be a lower cost transition, $2^j \to 1$ or $2^j \to 3$: In each case this leaves us with four possible candidates for minimum cost trees which are illustrated in the table below.

1-trees	2^A-trees	2^B-trees	3-trees
$3 \to 2^B \to 1 \tilde{A} 2^A$	$3 \to 2^B \to 1 \to 2^A$	$2^A \to 1 \to 2^B \tilde{A} 3$	$2^A \to 1 \to 2^B \to 3$
$2^B \to 3 \to 2^A \to 1$	$2^B \to 3 \to 2^A \tilde{A} 1$	$1 \to 2^A \to 3 \to 2^B$	$2^B \to 1 \to 2^A \to 3$
$3 \to 2^A \to 1 \tilde{A} 2^B$	$2^B \to 1 \to 2^A \tilde{A} 3$	$3 \to 2^A \to 1 \to 2^B$	$1 \to 2^A \to 3 \tilde{A} 2^B$
$2^A \to 3 \to 2^B \to 1$	$1 \to 2^B \to 3 \to 2^A$	$2^A \to 3 \to 2^B \tilde{A} 1$	$1 \to 2^B \to 3 \tilde{A} 2^A$

Since $M(3;2^i) < M(2^i;3)$ we can eliminate all 3-trees as possible candidates. Now assume $d_1 < d_2$: Since $M(3;2^A) > M(3;2^B)$ all 1 and 2-trees in row 3 can be eliminated by the trees in row 1. The 2-trees in row 4 can be eliminated by trees in row 2 (as $M(1;2^A) < M(1;2^B)$) and it is easy to check that the 1-tree in row 2 always has a lower cost than the 1-tree in row 4. The 2^A-tree in row 1 has a lower cost than the 2^B-tree in row 1 as

$$M(2^B;1) + M(1;2^A) < M(2^A;1) + M(1;2^B):$$

The 2^A-tree in row 2 has a lower cost than the 2^B-tree in row 2 as

$$M(2^B;3) + M(3;2^A) < M(2^A;3) + M(3;2^B):$$

This leaves us with a simple comparison between 1-trees and 2^A-trees which amounts to comparing $M(1;2^A)$ and $M(2^A;1)$: A 1-tree will have the lowest cost if $M(1;2^A) > M(2^A;1)$ or,

$$N(2 \text{ i } d_2)q^{\alpha} > N(1 \text{ i } q^{\alpha})d_1$$

$$d_1 < \frac{2}{1 \text{ i } q^{\alpha}} \text{ i } \frac{q^{\alpha}}{1 \text{ i } q^{\alpha}}d_2$$

If the inequality is reversed, a 2^A i tree will have the lowest cost. Repeating the analysis for the case where $d_1 > d_2$, we come down to a comparison between

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$M(1; 2^B)$ and $M(2^B; 1)$: A 1-tree will have the lowest cost if

$$N(2 - d_1)q^{\alpha} > N(1 - q^{\alpha})d_2$$

$$d_1 < 2 - \frac{1 - q^{\alpha}}{q^{\alpha}}d_2$$

∎

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