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# Efficient Allocations in Economies with Asymmetric Information when the Realized Frequency of Types is Common Knowledge

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## Abstract

We consider a general economy, where agents have private information about their types. Types can be multi-dimensional and potentially interdependent. We show that, if the realized frequency of types (the exact number of agents for each type) is common knowledge, then a mechanism exists, which is consistent with truthful revelation of private information and which implements first-best allocations of resources as the unique equilibrium. The result requires weak restrictions on preferences (Local Non-Common Indifference Property) and on the Pareto correspondence (Anonymity).

**Keywords:** adverse selection, first-best, full implementation, mechanism design, single-crossing property

**JEL Classification:** D71, D82, D86

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# 1 Introduction

As first shown by Akerlof (1970), Spence (1973) and Rothschild and Stiglitz (1976), hidden-types (adverse selection) problems can have significant consequences in terms of efficiency on economic outcomes. More specifically, incentive compatibility constraints limit the set of feasible allocations that can be attained. How are these restrictions relaxed as more information becomes common knowledge? And what is the minimum additional information required for achieving first-best efficiency? These are some of the questions that have emerged in the attempt to better understand the effects of information aggregation on efficiency. Indeed, some early papers by McAfee (1992), Armstrong (1999) and Casella (2002) already point towards this direction.

In this paper we claim that if the number of agents with the same type is known for all types in a population (in other words, the realized frequency of types is known), then it is possible, under general conditions, to implement first-best allocations as a unique equilibrium. More precisely, we consider an economy with asymmetric information, where each agent has private information about his type. We also assume that: (i) the realized frequency of types is common knowledge, (ii) preferences satisfy the Local Non-Common Indifference Property, and (iii) the social choice rule satisfies Anonymity. Given these conditions, we show that it is possible to construct a mechanism which has a unique equilibrium, where all agents reveal their type truthfully and they receive a first-best allocation. We obtain our equilibrium by using iterated elimination of strictly dominated strategies, and hence it is also a Bayes-Nash equilibrium.

The result is interesting because we examine an asymmetric information problem which is situated in-between the problem of Maskin (1999) (in which all agents know the state of world but the mechanism designer does not know it) and the classic adverse selection (in which each agent knows only his own type and the mechanism designer knows the ex-ante distribution of types). The intuition behind the result is that, if the realized frequency of types is known, then one can aggregate the messages that all agents are sending out and uncover any misreport(s), even if the identity of the liar is not known. That is appropriately designed punishments for lying can induce agents to reveal their information truthfully. We talk about appropriately designed punishments, because one of the features of our mechanism is that punishments must not be too harsh. If the punishment when a lie is detected is too severe, then some agents may deliberately lie about their type in order to force other agents to also do so. The lies cancel out at the aggregate level and the former agents “steal” the allocations of the latter, who are forced to lie under the fear of the extreme punishments. This can lead to coordination failures and multiplicity of equilibria. Therefore, uniqueness of the equilibrium requires a careful construction of the off-the-equilibrium path allocations when lies are detected. We show that such punishments exist when the indifference curves of different types are not locally identical, meaning that in the neighborhood of any allocation one can find other allocations such that each type prefers one of these over the rest.

This result is also interesting for two more reasons. First, one may consider economic

applications with a finite number of agents, where, in addition to the private information that each individual has, there is knowledge about how many agents have each type. This additional information could come from a positive or negative informational shock. For example, a retail store has received pre-paid orders from its customers, has already the goods in stock and is ready to make the deliveries. However, the records on the orders get destroyed due to an accident and the store’s manager does not know who made each order. What is he to do? Can he induce the customers to truthfully reveal the orders they have made without them making unreasonable claims or receiving orders that were meant for other customers? We claim that this is possible, as long as the manager posts a list with all the orders made and gives to each customer a basket of goods, which depends on how many other agents have claimed to have ordered it.

Second, there are some well-known models of adverse selection (for example Akerloff (1970) and Spence (1973)) which assume that the proportion of each type in the population is common knowledge. For these models, the mechanism presented in this paper can be used in order to provide first-best allocations. To the best of our knowledge, this efficiency result has not been provided in the literature so far.

Admittedly, the assumption that the realized distribution is common knowledge is stronger than the standard assumption of only the ex-ante distribution being common knowledge, which is more commonly used. However, in a closely related paper (Boukouras and Koufopoulos, 2013) we show that this limitation can be overcome. In particular, we show that if the ex-ante distribution is common knowledge, then there exists a mechanism which implements allocations arbitrarily close to the first-best allocations as the number of agents becomes large. Even though the mechanism used there is not the same as the one in this paper, many of the results and insights come from the work presented here.

The most closely related paper to ours is Jackson and Sonnenschein (2007), who consider an economy where agents play multiple copies of the same game at the same time and their types are independently distributed across games. They allow for mechanisms, which “budget” the number of times that an agent claims to be of a certain type. If the number of parallel games becomes very large, then all the Bayes-Nash equilibria of these mechanisms converge to first-best allocations. Our model is different from theirs, because we do not require multiple games to be played at the same time but we impose a stronger assumption on what is common knowledge. Moreover, we allow for interdependent values, while they consider an independent values setting, and in our model asymmetric information may include other individual characteristics apart from preferences (productivity parameters, proneness to accidents, etc.).

McLean and Postlewaite (2002, 2004) consider efficient mechanisms in economies with interdependent values. The state of the world is unknown to all agents, but each individual receives a noisy private signal about the state. They show that when signals are sufficiently correlated with the state of the world and each agent has small informational size (in the sense that his signal does not contain additional information about the state of the world when the signals of all the other agents are taken into account), then their mechanism implements allocations arbitrarily close to first-best

allocations. However, in the model of McLean and Postlewaite, when private signals are perfectly correlated, all agents learn not only their own type but also the type of all other agents. That is, in the limit, the framework of McLean and Postlewaite is one of complete information. In contrast, in our setting agents know, at most, the realized frequency of types<sup>1</sup>.

VCG-mechanisms (Vikrey, 1961, Clarke, 1971, Groves, 1973) are often reference points in terms of results on efficiency. With respect to these mechanisms, our paper has the following differences: (i) they assume quasi-linear preferences while we allow for general preferences, (ii) VCG mechanisms may violate budget balance, while we provide conditions, which ensure that this never happens on or off the equilibrium path.

Our paper is also related to the auctions literature with interdependent types. In this context, Crémer and McLean (1985) and Perry and Reny (2002, 2005), show the existence of efficient auctions when types are interdependent. Crémer and McLean, however, require quasi-linear preferences and large transfers which may violate ex-post feasibility. Also, Perry and Reny require the single crossing property on preferences which is a stronger restriction than ours. Our general framework can encompass auction design problems as well. Furthermore, our main focus is the uniqueness of the equilibrium, an issue which is not studied in these papers.

In the framework of auction design the papers by Maskin (1992), Dasgupta and Maskin (2000) and Jehiel and Moldovanu (2001) show, in increasing generality, that efficiency and incentive compatibility can not be simultaneously satisfied if the single crossing condition is violated or if signals are multidimensional. In that respect, the additional information of our environment allows us to overcome this impossibility and implement efficient outcomes, even if conditions, which are necessary in the standard mechanism design literature for implementation, are violated.

Rustichini, Satterthwaite and Williams (1994) show that the inefficiency of trade between buyers and sellers of a good, who are privately informed about their preferences, rapidly decreases with the number of agents involved in the two sides of the market and in the limit it reaches zero. Effectively, the paper examines the issue of convergence to the competitive equilibrium as the number of agents increases. However, their model is limited to private values problems and hence it can be seen as a special case of our formulation.

More recently, the papers by Mezzetti (2004) and Ausubel (2004),(2006) examine the issues of efficient implementation under interdependent valuations and independently distributed types. However, they also assume that agents' preferences are quasi-linear with respect to the transfers they receive, whereas in our model utility may not be transferable. Moreover, the mechanisms proposed in these papers may generate multiple equilibria (in most of which truth-telling is violated), while we are interested in a mechanism which has a unique truth-telling equilibrium.

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<sup>1</sup>In a sense, in our model agents receive private signals as well, but one can think of them as perfect signals about the frequency of types.

## 2 An Example: Spence (1973)

First we demonstrate how the knowledge of the realized frequency of types can be used to implement first best allocations as a unique equilibrium by applying the main idea to the classic paper by Spence (1973). The economy consists of two types of workers. Type 1 has low productivity  $\underline{a}$  and its proportion of the population is  $q_1$ . Type 2 has high productivity  $\bar{a}$ , ( $\bar{a} > \underline{a}$ ) and its proportion of the population is  $1 - q_1$ <sup>2</sup>. Acquiring  $y$  units of education costs  $y/\underline{a}$  for type 1 and  $y/\bar{a}$  for type 2. Productivity parameters are private information and firms hire workers according to a wage schedule, based on verifiable educational attainment. The payoff for an individual is the value of his wage minus the educational cost and for a firm the productivity parameter minus the wage.

Spence argues that agents will acquire education (which does not increase productivity in his model) in order to signal their productivity to firms. In equilibrium, the wage schedules are such that high productivity workers acquire some education and credibly signal their type, while low productivity workers acquire no education, and firms correctly infer that they are of low productivity. The education acquired by type 2 is a deadweight loss, but necessary for credible signaling.

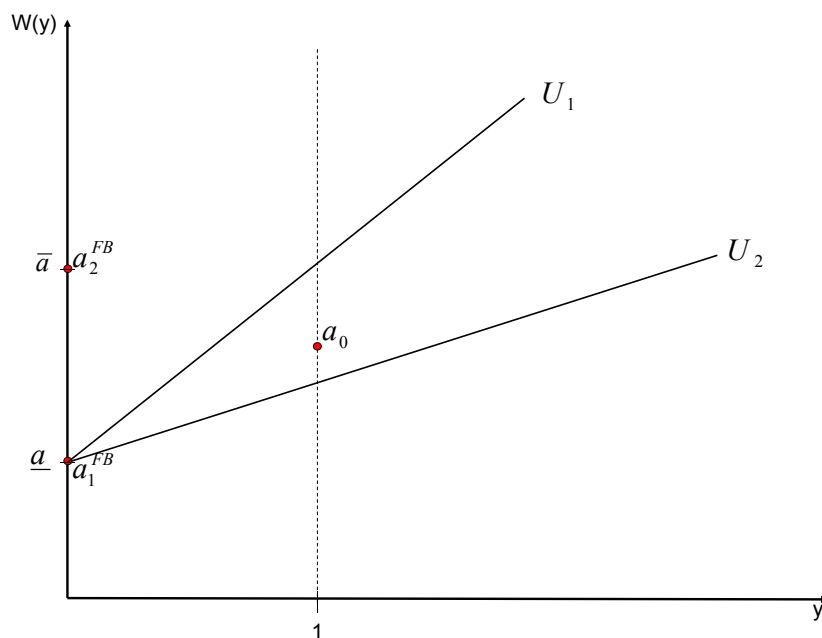


Figure 1: Spence, 1973

<sup>2</sup>Note that in the original paper, Spence made the assumption that a known proportion of the population belongs to one type and the remainder proportion belongs to the other type. Hence, he implicitly made the assumption that the realized frequency of types is common knowledge and, hence, we can apply our mechanism directly into his economy.

Assume that the total population is  $N$ . Then  $Nq_1$  is the total number of agents of type 1 and  $N(1 - q_1)$  is the total number of agents of type 2. Given that, the following mechanism can separate types without any agent incurring educational costs in equilibrium.

Let all workers report their type. If the number of agents who report type 1 and 2 is  $Nq_1$  and  $N(1 - q_1)$  respectively, then agents who report type 1 receive wage  $w_1 = \underline{a}$  and those who report type 2, receive wage  $w_2 = \bar{a}$ . In any other case, where the reported number of types do not match their population size, those who report type 1 receive  $w_1 = \underline{a}$  and those who report type 2, are asked to undertake one unit of education and receive  $w_2 = \underline{a} + \epsilon$ , with  $\frac{1}{\bar{a}} < \epsilon < \frac{1}{\underline{a}}$  (recall that a unit of education costs  $\frac{1}{\bar{a}}$  for high productivity workers and  $\frac{1}{\underline{a}}$  for low productivity workers).

The above mechanism fully implements the first-best allocations in this economy. First, consider the strategies of type 2. It is clear that, irrespectively of the reports of the other agents, it is a strictly dominant strategy for him to report his type truthfully. This is because, when everybody else reports truthfully, he prefers to report truthfully as well (then his payoff is  $\bar{a}$ ) than to misreport his type (then his payoff is  $\underline{a}$ ), given that  $\bar{a} > \underline{a}$ . Similarly, if someone else lies, he prefers to report truthfully and receive a payoff of  $\underline{a} + \epsilon - \frac{1}{\bar{a}}$  than to cover the lie by misreporting and receive  $\underline{a}$ , given that  $\underline{a} + \epsilon - \frac{1}{\bar{a}} > \underline{a}$ . Given the dominant strategy of type 2 and  $\underline{a} > \underline{a} + \epsilon - \frac{1}{\bar{a}}$ , it is a best-response for type 1 to report truthfully as well. Hence, all agents report truthfully in equilibrium and acquire zero education. In Figure 1, contract  $a_0$  denotes the offer to the workers, who report high productivity, when lies are detected.

### 3 The Economy

The previous example was used in order to show that it is possible to eliminate asymmetric information problems if the realized frequency is common knowledge. We now proceed to show that this result is general and does not depend on the specifics of the example. First, we introduce the economy and the notation.

The economy consists of a finite set  $I$  of agents, with  $I$  standing for the aggregate number of agents as well.  $\Theta$  is the finite set of potential types with elements  $\vartheta$ . Each agent has private information about his own type, but does not know the types of the other agents.  $\beta$  is the vector of realized frequencies of types in the population. That is  $\beta$  denotes the *ex post* distribution of types in the population, i.e. the relative frequency of each type, which materializes after types are drawn. Therefore,  $\beta(\vartheta)$  is the proportion of agents who have type  $\vartheta$  in the population and  $N(\vartheta)$  is the total number of agents of type  $\vartheta$ :  $N(\vartheta) = \beta(\vartheta)I$ .

Let  $A$  be the set of all feasible allocations, with elements  $a \in A \subseteq \mathbf{R}_+^{I \times L}$ , with  $L \geq 2$ .  $L$  can be interpreted as the number of commodities in the economy. Also, for any subset  $J$  of the set  $I$ , let  $A^J$  be the set of of feasible allocations for the agents in  $J$  ( $A^J \subseteq \mathbf{R}_+^{J \times L}$ ). For the analysis that follows it is also useful to define allocations on an individual basis. That is, given an allocation  $a \in A$ , the *individual allocation*

$a_i \in \mathbf{R}_+^L$  is the bundle that agent  $i$  consumes. Moreover, since later on we will require that agents of the same type consume the same bundle, it is useful to denote individual allocations with respect to types. That is,  $a_\vartheta$  denotes the individual allocation that an agent of type  $\vartheta$  consumes within allocation  $a$ .

$u : \mathbf{R}^L \times \Theta \times \Theta^{\wedge i} \rightarrow \mathbf{R}$  is the *ex post* utility function for agent  $i$ , which we assume to be strictly quasi-concave. Since the implementation takes place through some mechanism, we denote by  $\mathbf{m}$  the message profile sent by agents:  $\mathbf{m} = \{m_1, \dots, m_i, \dots, m_I\}$ , by  $\mathbf{m}_{-i}$  the message profile of all other agents apart from agent  $i$  and by  $a(\vartheta, \mathbf{m}_{-i})$  the allocation which one gets if he reports type  $\vartheta$ , conditional on all other messages.  $\mu(\vartheta)$  is the proportion of the population who have reported type  $\vartheta$ .

The following definitions are also useful.  $L_\vartheta(a_\vartheta)$  is the **lower-contour set** of an agent with type  $\vartheta$  associated with individual allocation  $a_\vartheta$ :  $L_\vartheta(a_\vartheta) = \{c \in \mathbf{R}_+^L : u_\vartheta(c) < u_\vartheta(a_\vartheta)\}$ .  $V_\vartheta(a_\vartheta)$  is the **upper-contour set** of type  $\vartheta$  associated with  $a_\vartheta$ :  $V_\vartheta(a_\vartheta) = \{c \in \mathbf{R}_+^L : u_\vartheta(c) > u_\vartheta(a_\vartheta)\}$ .  $C_{\vartheta\epsilon}(a_\vartheta) = \{c \in \mathbf{R}_+^L : u_\vartheta(c) = u_\vartheta(a_\vartheta), \|c - a_\vartheta\| < \epsilon\}$  is the **indifference plane** of  $\vartheta$  in the neighborhood of  $a_\vartheta$ .  $\underline{A}(a_\vartheta) = \{c \in \mathbf{R}_+^L : c_l \leq a_{\vartheta l}, \forall l \in L\}$  is the set of individual allocations, which offer weakly less quantity than individual allocation  $a_\vartheta$  for all commodities.

Overall, the economy is described by the following primitives:  $E = \{I, A, u, \Theta, \beta\}$ . This formulation of the economy allows for modeling a wide variety of economic situations. Since we impose no restrictions on  $\beta$  or the type-generating process that produces  $\beta$ , types may or may not be independently distributed. Moreover, the utility function of agents may or may not depend on the types of other agents, and so both adverse-selection problems with independent or inter-dependent valuations can be seen as special cases of our formulation. The model also allows for public goods problems, since some elements of the individual allocations can be common.

Economies with uncertainty can be easily accommodated by our model as well. For example, let  $\phi : \beta \rightarrow \Delta^S$  be the probability distribution function over states, where  $S$  the finite set of states and  $\Delta^S$  is the unit simplex  $\{\phi \in \mathbf{R}_+^S \mid \sum_{s \in S} \phi_s = 1\}$ . In this case,  $L = S \times T$ , where  $T$  is the finite set of final commodities, and the agents' expected utility function is  $u_i(a_i, \beta) = \sum_{s \in S} v_i(a_i, s) \phi_s(\beta)$ , where  $v_i(a_i, s)$  is the decision-outcome payoff in state  $s$ .

Another example, which is a special case of our formulation is the Prescott-Townsend (1984) economy when applied to adverse selection problems. In order to make our model equivalent to theirs, we simply need to specify the individual endowment  $\xi \in \mathbf{R}_+^L$ , with all agents having the same endowment, and the resource constraints  $\sum_{\vartheta} \beta(\vartheta) r_{i\vartheta}(a_i - \xi) \leq 0 \quad \forall i \in I$ , where  $r_{i\vartheta}$  is some real-valued linear function on  $L$ . Of course, Prescott and Townsend use their model to examine also moral hazard problems, while we restrict attention to problems of hidden information.

A final comment before we proceed to the next section. Even though we have not provided any restrictions on the feasible set  $A$ , which in principle could be non-convex, this does not generate problems for implementation. This is because we examine the implementation of Pareto efficient allocations, which, by definition, are feasible. Therefore, on-equilibrium path feasibility is guaranteed. For off-the-equilibrium path



feasibility we provide sufficient conditions in section 5, but the main idea is that out-of-equilibrium path individual allocations are located in the neighborhood of the efficient allocation so that feasibility is satisfied, given the feasibility of the efficient allocation.

## 4 Implementation of First Best Allocations

We now show that, under weak restrictions on allocations and preferences (Anonymity and Local Non-Common Indifference Property), there exists a mechanism which implements Pareto efficient allocations as a unique equilibrium.<sup>3</sup> The Local Non-Common Indifference Property requires that, whenever the indifference planes of any two types intersect, then this intersection is of at least one dimension less than the planes themselves. This implies that one can find allocations which satisfy incentive compatibility for the two types in the neighborhood of any allocation. Anonymity requires that the first-best allocations are such that agents of the same type receive the same individual allocation.

Formally, let  $\mathbf{a}^*$  be a Pareto efficient allocation and let  $a_{\vartheta}^*$  be the individual allocation, which an agent with type  $\vartheta$  receives in  $\mathbf{a}^*$ . In other words,  $a_{\vartheta}^*$  is the allocation which a mechanism designer would like to offer to an agent with type  $\vartheta$ , if  $\mathbf{a}^*$  were to be implemented. Then, we have the following definition:

**Definition 1:** A Social Choice Rule satisfies Anonymity if, for any two agents  $i$  and  $j$ ,  $a_i^* = a_j^* = a_{\vartheta}^*$ , whenever  $\vartheta_i = \vartheta_j = \vartheta$ .

**Assumption 1:** The Social Choice Rule satisfies Anonymity.

Under Anonymity, agents who have identical types receive identical allocations. Therefore, an agent's identity per-se has no impact on the agent's final allocation. Anonymity is a desirable property for a social choice rule. In most cases of interest, economists are concerned with the economic characteristics of agents and not with their identity. Therefore, it is reasonable to assume that, if the distribution of these characteristics remains unchanged, so does the distribution of the economically desirable outcomes. It is also a property satisfied by many commonly used social choice rules, like the Walrasian correspondence and the utilitarian social welfare function.

**Assumption 2:** Preferences satisfy the Local Non-Common Indifference Property (LNCIP).

**Definition 2:** Let  $C_{\vartheta\epsilon}(a_{\vartheta}) = \{c \in R_+^L : u_{\vartheta}(c) = u_{\vartheta}(a_{\vartheta}), \|c - a_{\vartheta}\| < \epsilon\}$ . The **Local Non-Common Indifference Property** is satisfied if  $\forall \vartheta, \eta \in \Theta$  and  $\forall a_{\vartheta} \in R_+^L$ ,

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<sup>3</sup>Of course, this result applies if a Pareto efficient allocation exists in A. If no such allocation exists then our mechanism does not apply to any allocation in A. We implicitly assume that A is such that the set of Pareto efficient allocations,  $A^*$ , is not empty.

there exists  $\bar{\epsilon}_{\vartheta, \eta} > 0$  such that  $\dim(C_{\vartheta\epsilon}(a_{\vartheta}) \cap C_{\eta\epsilon}(a_{\vartheta})) \leq L \times S - 1$ ,  $\forall \epsilon < \bar{\epsilon}_{\vartheta, \eta}$ .

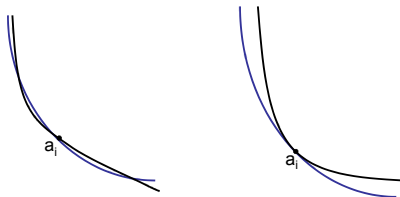


Figure 2: Indifference Curves satisfying LNCIP

LNCIP is a weaker restriction than the Single-Crossing Property (SCP) which is usually used in the literature. For example, any pair of indifference curves that has finitely many intersections satisfies the LNCIP but it violates the SCP. Also, LNCIP allows for tangent indifference planes (as long as the tangent parts “miss” at least one dimension compared to the indifference planes), while the SCP does not. On the other hand, if SCP is satisfied then LNCIP is also satisfied. Figure 2 provides two examples, which illustrate the LNCIP and distinguish it from the SCP.

The main idea of the mechanism is essentially the same as in the example of section 2. First we rank the different types according to the envy they feel for the first-best allocations of other types. Thus, types of the highest rank do not envy any other type’s allocation while types of the lowest rank are not envied by any other type. Intermediate ranks envy the allocation of at least one type with higher rank and do not envy the allocations of lower ranks.

The mechanism exploits this rank by providing allocations, on and off the equilibrium path, such that it is a strictly dominant strategy for the highest rank types (say of rank  $K$ ) to report truthfully, *irrespective of their beliefs about the reports of other types*. By always reporting truthfully, these types “signal” to the mechanism designer whether someone else misreported her type as being of rank  $K$  or not, since the mechanism designer knows the proportion of  $K$ -types in the population and the number of respective reports she should expect.

In the case that the proportion of the  $K$ -type reports is greater than the proportion of  $K$ -types in the population, the designer “punishes” the misreporting types and “rewards” the truthfully reporting types by providing allocations in the neighborhood of the first-best allocations of the misreporting types. These out-of-the equilibrium path allocations are designed to satisfy no-envy and to provide slightly higher utility to

K-types than the utility of the first-best allocations of the other types, so that K-types indeed prefer to report truthfully. This is always possible due to the LNCIP property.

Given the dominant strategy of the K-types, the best-response of types of the second highest rank is to also report truthfully, and given this the best-response of the third highest rank types is to report truthfully, and so on and so forth. Thus, by iterated elimination of strictly dominated strategies, the unique equilibrium of the mechanism is for all agents to report truthfully. We break down these arguments in a series of Lemmas, which lead to the main result in Proposition 1. All proofs are included in the appendix.

**Lemma 1:** If  $\mathbf{a}^*$  is a Pareto efficient allocation which satisfies Anonymity, then there exists at least one type  $\vartheta$ , who does not envy the individual allocation of any other agent:  $U_\vartheta(\mathbf{a}_\vartheta^*) \geq U_\vartheta(\mathbf{a}_\eta^*), \forall \eta \in \Theta$ .

**Corollary 1:** If  $\mathbf{a}^*$  is a Pareto efficient allocation which satisfies Anonymity, then Lemma 1 holds for any subset of  $\Theta$ . Namely, let  $\check{\Theta} \subseteq \Theta$  and let  $\check{A} = \{\mathbf{a}_\vartheta^* : \vartheta \in \check{\Theta}\}$ . Then, Lemma 1 holds for  $\check{\Theta}$  with regard to  $\check{A}$  as well.

Let  $\mathbf{Rank}(\mathbf{K}) = \{\vartheta \in \Theta : U_\vartheta(\mathbf{a}_\vartheta^*) \geq U_\eta(\mathbf{a}_\eta^*), \forall \eta \in \Theta\}$ , be the set of types who do not envy the allocation of any other type. By Lemma 1, we know that this set is non-empty. Then, by removing this set of types from the set  $\Theta$  and applying Corollary 1, we can define  $\mathbf{Rank}(\mathbf{K}-1) = \{\vartheta \in \Theta : U_\vartheta(\mathbf{a}_\vartheta^*) \geq U_\eta(\mathbf{a}_\eta^*), \forall \eta \in \Theta - \mathbf{Rank}(\mathbf{K})\}$ . By iteration, we can define  $K$  groups,  $1 \leq K \leq \Theta$ , such that the types in each one of them do not envy any of the types in their own group or groups with lower rank, but they envy the allocation of some type(s) in groups with higher rank<sup>4</sup>. We will also refer to group  $\mathbf{Rank}(\mathbf{K})$  as the group with the **highest rank** and group  $\mathbf{Rank}(\mathbf{1})$  as the group with the **lowest rank**.

**Definition 3:** An individual allocation  $\hat{a}_\vartheta(a_i)$  is *incentive compatible* for type  $\vartheta$  within a set of individual allocations  $\hat{A}(a_i)$  if and only if  $u_\vartheta(\hat{a}_\vartheta(a_i)) > u_\vartheta(\hat{a}(a_i)), \forall \hat{a}_\eta(a_i) \in \hat{A}(a_i)$ .

**Lemma 2:** If LNCIP holds then for any individual allocation  $a_i$  there exists a collection of  $\Theta$  individual allocations  $\hat{A}(a_i)$  such that for any type  $\vartheta \in \Theta$  there exists one individual allocation  $\hat{a}_\vartheta(a_i) \in \hat{A}(a_i)$  which is incentive compatible for this type.

Lemma 2 states that, if the LNCIP holds, then in the neighborhood of any individual allocation  $a_i$ , there exists a set of allocations such that each agent of a certain type prefers a particular allocation over the rest. In other words, it is possible to find incentive compatible individual allocations for any type in the neighborhood of any

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<sup>4</sup>One extreme case is when an allocation exhibits no-envy, in which case  $\mathbf{Rank}(\mathbf{K})$  contains the whole set of types (egalitarian allocations:  $K = 1$ ). The other extreme case is when each rank-group contains a single type, in which case the types form a complete hierarchy, from the one who is envied by all the other types to the one who is not envied by anyone ( $K = I$ ).

allocation, which implies that it is possible to satisfy no-envy, at least in a local sense.

**Proposition 1:** If preferences satisfy the LNCIP, then, for every Pareto efficient allocation  $\mathbf{a}^*$ , which satisfies Anonymity, there exists a mechanism, for which  $\mathbf{a}^*$  is the unique Bayes-Nash equilibrium allocation and agents report their private information truthfully.

Even though the mechanism which is used in the proof of Proposition 1 is formally presented in the appendix, we would like to comment on the advantages it presents in comparison to the existing literature (see for example, Jackson, 1991, Maskin, 1999). First, our mechanism holds even with two agents (or even in the degenerate case of one agent). Second, the required message space is minimal, since agents send messages only about their own type. Third, we do not require any ad-hoc game, which has no equilibrium in pure strategies (like an integer game), in order to rule out undesirable equilibria. This is achieved by ‘enticing’ some of the misreporting agents to report truthfully, whenever there are multiple misrepresentations. Fourth, our solution concept is iterated elimination of strictly dominated strategies and, therefore, our mechanism is not limited only to Bayesian implementation. Finally, Assumptions 1 and 2 are relatively weak and there are many cases of interest that comply with them.

## 5 Out-of-Equilibrium Path Feasibility

The mechanism which is used for the proof of Proposition 1 has one caveat. Out-of-the-equilibrium path feasibility may be violated. This is because if the reported frequency does not match the realized frequency of two types, with agents of one type envying the first-best allocation of the other type agents, then these two types receive allocations in the neighborhood of the first-best allocation of the envying type. For example, say that  $\eta$ -types envy  $a_\vartheta^*$  and the reported frequencies of  $\vartheta$  and  $\eta$  types do not match their realized frequencies. Then all agents who reported these two types receive an allocation in the neighborhood of  $a_\eta^*$ . Since we do not impose any restrictions on the Pareto-frontier, we know only that  $a_\eta^*$  allocations are feasible  $N(\eta)$  times, but not necessarily  $N(\eta) + N(\vartheta)$  times, as required (recall that  $N(\eta)$  is the number of  $\eta$ -types according to the realized frequency).

In the case described above, out-of-the-equilibrium path feasibility can be satisfied by slightly modifying the mechanism of Proposition 1, but additional restrictions on preferences or on the realized frequency may be necessary<sup>5</sup>. What is required for the uniqueness of the equilibrium is that there exist *feasible* individual allocations, say  $a'_\vartheta$  and  $a'_\eta$  such that:  $U_\eta(a_\eta^*) > U_\eta(a'_\eta) > U_\eta(a'_\vartheta)$ ,  $U_\vartheta(a'_\vartheta) > U_\vartheta(a_\eta^*)$  **and**  $U_\vartheta(a'_\vartheta) > U_\vartheta(a'_\eta)$ . That is, one can find out-of-equilibrium feasible allocations such that  $\eta$ -types prefer their first-best allocation over the out-of-equilibrium allocations, but  $\vartheta$ -types prefer one of the out-of-equilibrium allocations over the other and over  $a_\eta^*$ .

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<sup>5</sup>In fact, this is the only case in the mechanism of Proposition 1, where feasibility may be violated.

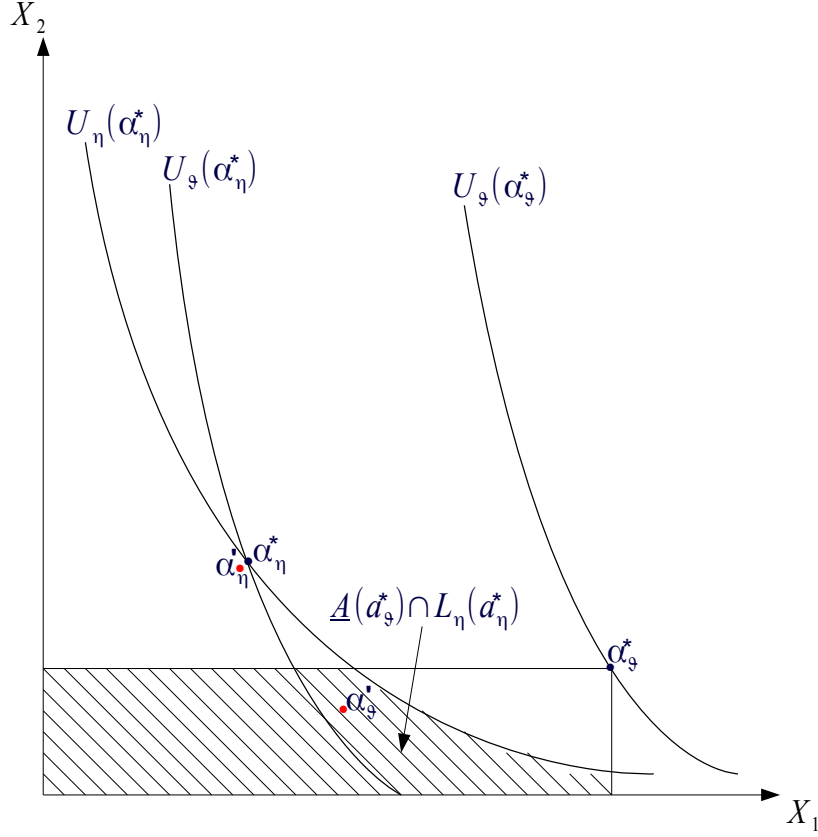


Figure 3: Feasible and Incentive Compatible Allocations. Case (i):  $\underline{A}(a_{\vartheta}^*) \cap L_{\vartheta}(a_{\eta}^*) \subset \underline{A}(a_{\vartheta}^*) \cap L_{\eta}(a_{\eta}^*)$

Such allocations exist if  $L_{\vartheta}(a_{\eta}^*)$  is a subset of  $L_{\eta}(a_{\eta}^*)$  in the interior of  $\underline{A}(a_{\vartheta}^*)$  (see also Figure 3). Because  $\underline{A}(a_{\vartheta}^*) \cap L_{\vartheta}(a_{\eta}^*) \subset \underline{A}(a_{\vartheta}^*) \cap L_{\eta}(a_{\eta}^*)$ , there exists an allocation  $a'_{\vartheta}$  such that  $a'_{\vartheta} \in \underline{A}(a_{\vartheta}^*) \cap L_{\eta}(a_{\eta}^*)$  and  $a'_{\vartheta} \notin \underline{A}(a_{\vartheta}^*) \cap L_{\vartheta}(a_{\eta}^*)$ . This means that  $\vartheta$  types strictly prefer  $a'_{\vartheta}$  to  $a_{\eta}^*$  and vice versa for  $\eta$  types. Moreover, because  $a'_{\vartheta}$  is in the interior of  $\underline{A}(a_{\vartheta}^*)$ , it is feasible to provide it up to  $N(\vartheta)$  times. It is also easy to find an allocation  $a'_{\eta}$ , arbitrarily close but strictly in the interior of  $\underline{A}(a_{\eta}^*)$  such that the rest of the required inequalities are satisfied (Figure 3).

A more demanding case is when  $\underline{A}(a_{\vartheta}^*) \cap L_{\eta}(a_{\eta}^*) \subset \underline{A}(a_{\vartheta}^*) \cap L_{\vartheta}(a_{\eta}^*)$  (Figure 4). Then it is impossible to find a allocation  $a'_{\vartheta}$ , which satisfies the required inequalities and is in the interior of  $\underline{A}(a_{\vartheta}^*)$ . If one does not impose any further restriction on preference than LNCIP, then such an allocation exists for sure only in the neighborhood of  $a_{\eta}^*$ . But  $a_{\eta}^*$  is available up to  $N(\eta)$  times at most, and therefore, if  $N(\eta) < N(\vartheta)$ , there may not be enough out-of-equilibrium path allocations for  $\vartheta$  types to induce truthful reporting from their part, whenever  $\eta$  types misreport.

The problem can be solved if we impose the single-crossing property on the utility

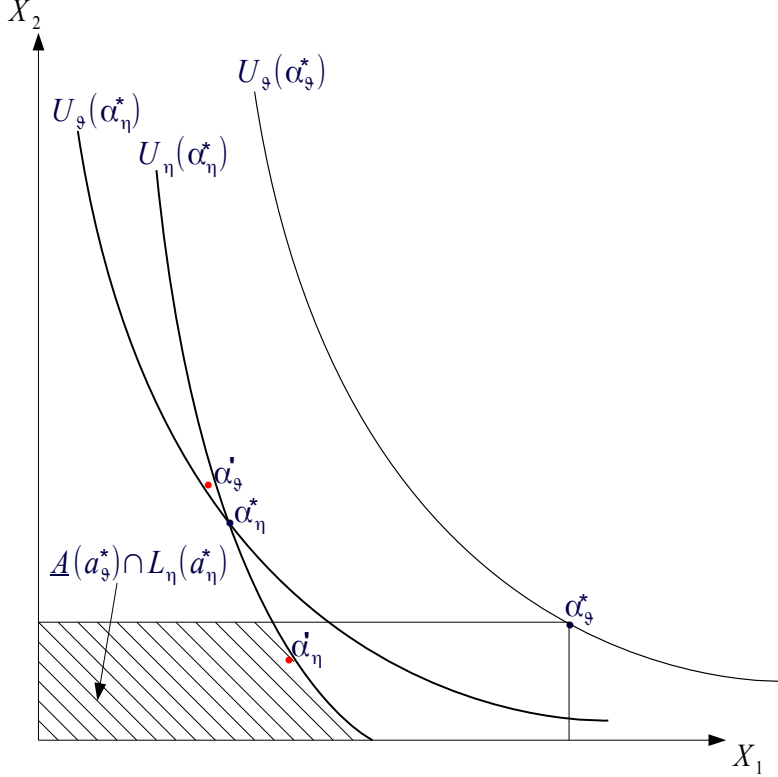


Figure 4: Feasible and Incentive Compatible Allocations. Case (ii):  $\underline{A}(a_\vartheta^*) \cap L_\eta(a_\eta^*) \subset \underline{A}(a_\vartheta^*) \cap L_\vartheta(a_\eta^*)$

functions of different types, so that the types who are envied have steeper indifference planes than those types who envy them (with respect to some commodity which is provided in greater quantity in the first best allocations of the envied types). Such a condition ensures that the situation of Figure 4 does not arise.

Formally, we require that, if  $U_\eta(a_\vartheta^*) > U_\eta(a_\eta^*)$  **and**  $a_{\eta k}^* > a_{\vartheta k}^*$  for some commodity  $k$ , then there exists at least one commodity  $l$  such that  $a_{\vartheta l}^* > a_{\eta l}^*$  **and**  $-\frac{\partial U_\vartheta / \partial l}{\partial U_\vartheta / \partial k} < -\frac{\partial U_\eta / \partial l}{\partial U_\eta / \partial k}$ .

A more general condition than single-crossing, which also ensures that the situation of Figure 4 does not arise, is to assume that, in the interior of the set  $\underline{A}(a_\vartheta^*)$ , the lower-contour set of the envied type is a subset of the lower-contour set of the type who feels envy (the contour sets are taken with respect to the first best allocations of the type who feels envy). Formally, whenever  $U_\eta(a_\vartheta^*) > U_\eta(a_\eta^*)$ , then the restriction is  $\underline{A}(a_\vartheta^*) \cap L_\vartheta(a_\eta^*) \subset \underline{A}(a_\vartheta^*) \cap L_\eta(a_\eta^*)$ .

Finally, a third way to satisfy off-the-equilibrium path feasibility is to impose a condition on the realized frequency of types. The condition requires that the number of incentive compatible allocations in the neighborhood of  $a_\eta^*$  is large enough such that it is feasible to provide each of  $\vartheta$ -type agents with one of these allocations, whenever it is not possible to provide incentive compatible allocations in the interior of  $\underline{A}(a_\vartheta^*)$ . Formally,

whenever  $U_\eta(a_\vartheta^*) > U_\eta(a_\eta^*)$  and  $\underline{A}(a_\vartheta^*) \cap L_\eta(a_\eta^*) \subset \underline{A}(a_\vartheta^*) \cap L_\vartheta(a_\eta^*)$ , then  $N(\eta) \geq N(\vartheta)$ .

If any of the above additional assumptions is imposed, then one can still use the mechanism of Proposition 1 to fully implement first best allocations and satisfy out-of-equilibrium path feasibility at the same time. The only difference is a change in the implemented allocations in case 2.(b) of the mechanism (see page 18). All other parts of the mechanism and the proof remain unaffected.

## Conclusion

In this paper we consider a general hidden-type economy and, under relatively weak conditions, we show that it is possible to construct a mechanism which has a unique Bayes-Nash equilibrium, where all agents reveal their type truthfully and they receive a first-best allocation. If the realized frequencies of types are known, then one can aggregate the messages that all agents are sending out and uncover any misreport(s), even if the identity of the liar is not known.

Truth-telling, however, requires appropriately designed punishments for lying. If the punishment from detecting a lie is too severe, then some agents may deliberately lie about their type in order to force other agents to also do so. The lies cancel out and the former agents “steal” the allocations of the latter, who are forced to lie under the fear of the extreme punishments. This can lead to coordination failures and multiplicity of equilibria. Therefore, uniqueness of the equilibrium requires a careful construction of the allocations when lies are detected. We show that such punishments exist when the indifference curves of different types are not locally identical, meaning that in the neighborhood of any allocation one can find other allocations such that each type prefers one of these over the rest.

It should be stressed that we obtain our equilibrium by using iterated elimination of strictly dominated strategies and, hence, it is also a Bayes-Nash equilibrium. This contrasts with most of the existing papers, where the Bayesian equilibrium concept is used. Finally, even though our assumption regarding the realized distribution of types may appear as strong, in a companion paper (Boukouras and Koufopoulos, 2013) we relax it. Then, we show that first-best allocations can be implemented arbitrarily close as the number of agents increases and the law of large numbers applies.

# Appendix

## Proof of Lemma 1

Consider a Pareto efficient allocation  $\mathbf{a}^*$ , which satisfies Anonymity with type-dependent individual allocations  $a_{\vartheta}^*$  and suppose that Lemma 1 does not hold. Then, all types envy at least another type:  $\forall a_{\vartheta}^*, \exists \eta \in \Theta, \eta \neq \vartheta : U_{\vartheta}(a_{\eta}^*) > U_{\vartheta}(a_{\vartheta}^*)$ . But, since this holds for all types, then there exists at least one reassignment of individual allocations among the I individuals such that some of them are made strictly better-off and the rest remain as well-off as under  $\mathbf{a}^*$ .

In order to find one such reassignment, use the following algorithm. Pick an arbitrary  $\vartheta \in \Theta$  and let  $\bar{\vartheta} = \{\eta \in \Theta : U_{\vartheta}(a_{\eta}^*) > U_{\vartheta}(a_{\vartheta}^*)\}$ , be the set of types whom  $\vartheta$ -types envy. Reassign one individual allocation  $a_{\eta}^*$ , for some  $\eta \in \bar{\vartheta}$ , to one agent of type  $\vartheta$ . If  $\vartheta \in \bar{\eta}$ , then reassign  $a_{\vartheta}^*$  (from the  $\vartheta$ -type individual who received  $a_{\eta}^*$ ) to  $\eta$  (to the specific agent whose  $a_{\eta}^*$  allocation was reassigned) and stop the reassignment.

If  $\vartheta \notin \bar{\eta}$ , then reassign some allocation  $a_{\zeta}^*$ ,  $\zeta \in \bar{\eta}$  to  $\eta$  and then proceed to the individual whose allocation  $a_{\zeta}^*$  was reassigned. Iterate the procedure until you reach some agent of type  $\lambda$ , such that there exists some type  $\kappa \in \bar{\lambda}$ , whose allocation  $a_{\kappa}^*$  has already being reassigned. In this case, ignore all reassignments preceding the individual of type  $\kappa$  (these agents retain their original allocations), reassign to  $\lambda$  the allocation  $a_{\kappa}^*$  and stop the reassignments (all reassignments between  $\kappa$  and  $\lambda$  are not modified).

Since the set of agents is finite and all types envy at least one allocation, after at most I reassignments, the algorithm above will end-up in some agent, whose allocation has already been reassigned. In this case, a reassignment of allocations has been found, which makes some agents in I better-off (from agent of type  $\kappa$  until agent  $\lambda$ ) while the rest remain equally well-off. This constitutes a Pareto improvement and violates the initial assumption that  $\mathbf{a}^*$  is Pareto efficient. ■

## Proof of Corollary 1

Take any subset of agents  $\check{\Theta}$  of the set  $\Theta$ . Suppose that Lemma 1 does not hold over the set  $\check{A}$ , which is the set of individual allocations of the agents with types in  $\check{\Theta}$ . But if Lemma 1 does not hold, then it is possible to find a reassignment of allocations between the agents in  $\check{\Theta}$ , such that some of them will be made better-off while the rest remain as well-off. This is a Pareto-improvement for some agents in I, which contradicts the assumption that  $\mathbf{a}^*$  is Pareto efficient. ■

## Proof of Lemma 2

Recall that  $C_{\vartheta\epsilon}(a_i) = \{c \in R_+^{S \times L} : U_{\vartheta}(c) = U_{\vartheta}(a_i), \|c - a_i\| < \epsilon\}$ . Also, recall that  $L_{\vartheta}(a_i)$  is the lower-contour set of type  $\vartheta$  associated with individual allocation  $a_i$  and  $V_{\vartheta}(a_i)$  is the corresponding upper-contour set (throughout the proof we use the



subscript  $i$  on allocations in order to make clear that we are considering individual allocations).

$\mathbf{H}_{a_i}$  is an  $L - 1$  hyper-plane, which passes through  $a_i$ , and is perpendicular to the marginal rate of substitution of some type's indifference plane through  $a_i$ .  $\mathbf{H}_{a_i}$  splits the space of allocations in two sub-spaces,  $A_1$  and  $A_2$ . In each of these sub-spaces, and due to the LNCIP, there exists some  $\bar{\epsilon}(a_i) > 0$  such that for every  $\epsilon < \bar{\epsilon}(a_i)$ , within the open ball  $B_\epsilon(a_i)$ , the upper contour set of each type is a subset of the upper contour set of some other type (see also the figure below).

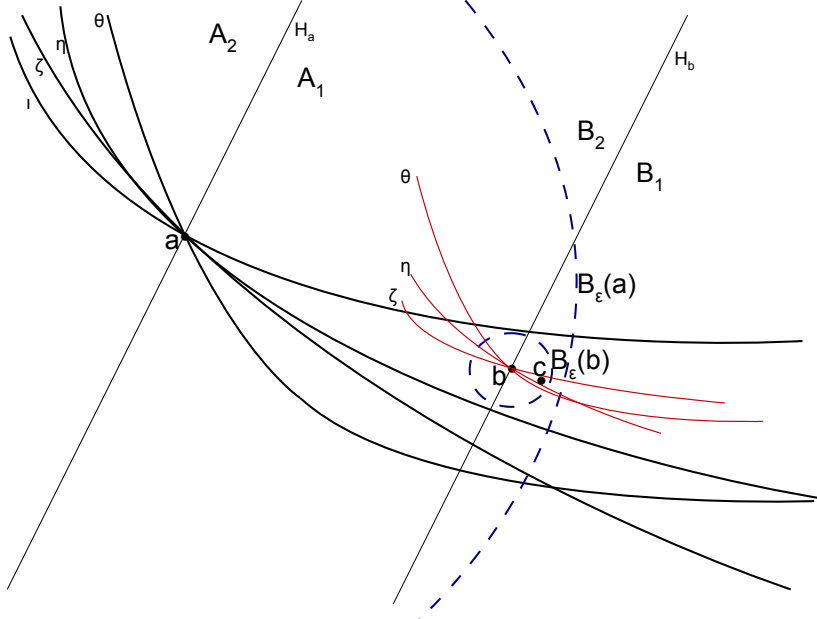


Figure 5: LNCIP and Local Incentive Compatibility

Say that an agent of type  $\iota$  is the one with the smallest upper contour set within ball  $B_\epsilon(a_i)$  and subspace  $A_1$ :  $V_\iota(a_i) \cap B_\epsilon(a_i) \cap A_1 \subset V_\eta(a_i) \cap B_\epsilon(a_i) \cap A_1, \forall \eta \in \Theta, \eta \neq \iota$ . Then, by LNCIP, there exists some allocation  $b_i \in B_\epsilon(a_i)$  such that  $a_i$  is strictly preferred to  $b_i$  by agents of type  $\iota$ , but the agents of all other types strictly prefer  $b_i$  to  $a_i$ :  $b_i \in L_\iota(a_i)$  and  $b_i \in V_\eta(a_i), \forall \eta \in \Theta, \eta \neq \iota$ .

Likewise, taking  $b_i$  as a starting point, there exists  $\mathbf{H}_{b_i}$  and an open ball  $B_\epsilon(b_i)$ , with  $B_\epsilon(b_i) \subset B_\epsilon(a_i) \cap L_\iota(a_i) \cap \bigcap_{\eta \in \Theta - \iota} V_\eta(a_i)$ , such that some type  $\zeta \in \Theta - \iota$  has the smallest upper contour set in the intersection of  $B_\epsilon(a_i)$  and subspace  $B_1$ . Thus, by LNCIP, there exists some allocation  $c_i \in B_\epsilon(b_i)$  such that  $c_i \in L_\zeta(b_i)$  and  $c_i \in V_\eta(b_i), \forall \eta \in \Theta - \{\iota, \zeta\}$ . Thus,  $a_i$  is strictly preferred to  $b_i$  and  $c_i$  by type  $\iota$ ,  $b_i$  is strictly preferred to  $a_i$  and  $c_i$  by type  $\zeta$  and all other types prefer  $c_i$  to  $a_i$  and  $b_i$ .

By using  $c_i$  as a starting point and by iterating the above steps, one can construct  $\Theta$  individual allocations in the  $\epsilon$ -neighborhood of  $a_i$  (including  $a_i, b_i$  and  $c_i$ ), such that the agents of one type strictly prefer one allocation over all the other. The properties described by Lemma 2 and definition 3 follow immediately. ■

## Proof of Proposition 1

Let  $\mathbf{a}^*$  be a Pareto efficient allocation which satisfies Anonymity. Let  $\hat{a}_\vartheta(a_i)$  denote an individual allocation in the neighborhood of individual allocation  $a_i$  which satisfies Lemma 2. This means that  $\vartheta$  prefers  $\hat{a}_\vartheta(a_i)$  to  $a_i$  and to any other allocation  $\hat{a}_\eta(a_i)$ , which is provided for any other type in the neighborhood of  $a_i$ .

$a^p$  is a “punishment” feasible individual allocation:  $a^p \in \bigcap_\vartheta \underline{A}(a_\vartheta^*)$ . That is,  $a^p$  is an individual allocation in the interior of the first best individual allocations of all types. Finally, recall that  $\beta(\vartheta)$  is the realized proportion of agents with type  $\vartheta$  and  $\mu(\vartheta)$  is the proportion of agents who report  $\vartheta$ .

The proof is done by construction. Before we proceed, we need to provide the description of a sub-game, which is induced by the main mechanism if the number of reports do not match the realized frequencies for two types. The sub-game is one where the agents of the two misreported types choose an allocation from a “pool” of incentive compatible individual allocations.

More specifically, if two types,  $\vartheta$  and  $\eta$  are misreported ( $\mu(\vartheta) \neq \beta(\vartheta), \mu(\eta) \neq \beta(\eta)$ ), then the agents who reported these two types are sequentially drawn at random to choose an individual allocation from a collection of allocations. The collection contains  $N(\vartheta)$  times an identical individual allocation, which is incentive compatible for type  $\vartheta$ , and  $N(\eta)$  times an individual allocation, which is incentive compatible for type  $\eta$ , where  $N(\vartheta)$  is the number of individuals in the economy with type  $\vartheta$ :  $N(\vartheta) = \beta(\vartheta)I$ . Each time an agent chooses an allocation, this allocation is removed from the collection and the next agent chooses from the remaining allocations. Since this sub-game is induced when only two types are misreported, the number of agents, who are involved, is exactly equal to the number of individual allocations of the collection ( $N(\vartheta) + N(\eta)$ ). Formally, the sub-game described above is represented by  $G(\vartheta, \eta, N(\vartheta) \times a_\vartheta, N(\eta) \times a_\eta)$ , where  $N(\vartheta) \times a_\vartheta$  denotes the number of times ( $N(\vartheta)$ ) the individual allocation  $a_\vartheta$  is provided. Since in each stage of the sub-game there is only a single player taking an action, the sub-game has  $N(\vartheta) + N(\eta)$  stages. Due to the incentive compatibility of the allocations, it is easy to check that the unique sub-game perfect equilibrium of the sub-game  $G(\vartheta, \eta, N(\vartheta) \times a_\vartheta, N(\eta) \times a_\eta)$  is for each agent to receive his most preferred allocation. With this in mind, we present the mechanism.

Each agent reports his type  $m_i$  and a final allocation is received according to the following mechanism  $M(\mathbf{m}, \mathbf{a})$ :

1. If  $\mu(\vartheta) = \beta(\vartheta)$  for all  $\vartheta$ , then  $a(\vartheta, \mathbf{m}_{-i}) = a_\vartheta^*$ .
2. If for two types,  $\vartheta, \eta$ ,  $\mu(\vartheta) > \beta(\vartheta)$  and  $\mu(\eta) < \beta(\eta)$ , then:
  - (a) If  $U_\vartheta(a_\vartheta^*) > U_\vartheta(a_\eta^*)$ ,  $U_\eta(a_\eta^*) > U_\eta(a_\vartheta^*)$ , then, for the agents who reported types  $\vartheta, \eta$ , the mechanism induces the game  $G(\vartheta, \eta, N(\vartheta) \times (a_\vartheta^* - \epsilon), N(\eta) \times (a_\eta^* - \epsilon))$ .  $\epsilon$  is strictly positive for all commodities and it is such that  $U_\vartheta(a_\vartheta^* - \epsilon) > U_\vartheta(a_\eta^*)$  and  $U_\eta(a_\eta^* - \epsilon) > U_\eta(a_\vartheta^*)$ .

- (b) If  $U_{\vartheta}(a_{\vartheta}^*) > U_{\vartheta}(a_{\eta}^*)$ ,  $U_{\eta}(a_{\eta}^*) < U_{\eta}(a_{\vartheta}^*)$ , then, for the agents who reported types  $\vartheta, \eta$ , the mechanism induces  $G(\vartheta, \eta, N(\vartheta) \times \hat{a}_{\vartheta}(a_{\eta}^*), N(\eta) \times \hat{a}_{\eta}(a_{\eta}^*))$ .
- (c) If  $U_{\vartheta}(a_{\vartheta}^*) < U_{\vartheta}(a_{\eta}^*)$ ,  $U_{\eta}(a_{\eta}^*) > U_{\eta}(a_{\vartheta}^*)$ , agents who report type  $\eta$  receive allocation  $a_{\eta}^*$  and agents who report type  $\vartheta$  receive allocation  $a^p$ .
- (d) For all  $\kappa \neq \{\vartheta, \eta\}$ ,  $a(\kappa, \mathbf{m}_{-i}) = a_{\kappa}^*$ .

3. For any other case,  $a(\vartheta, \mathbf{m}_{-i}) = \hat{a}_{\vartheta}(a^p)$ ,  $\forall \vartheta \in \Theta$ .

Under the mechanism above, it is a strictly dominant strategy for all agents with types of rank(K) to report their type truthfully. To see this consider the different beliefs of an agent of rank(K) (say  $i$  of type  $\vartheta$ ) about the messages that other agents will send. If  $i$  believes that all other agents will report their type truthfully, then the best-response for him is to report truthfully. This is because  $U_{\vartheta}(a_{\vartheta}^*) > U_{\vartheta}(a_{\vartheta}^* - \epsilon)$ , in the case he reports another type, who does not envy  $a_{\vartheta}^*$ , and  $U_{\vartheta}(a_{\vartheta}^*) > U_{\vartheta}(a^p)$ , in the case he reports a type, who envies  $a_{\vartheta}^*$  (recall that a rank(K) agent does not envy anyone.).

If  $i$  believes that only one other agent will misreport, then  $i$  still prefers to report his type truthfully, irrespectively of who misreports. Say that  $i$  believes that  $j$  is of different type (say  $\eta$ ), does not envy  $a_{\vartheta}^*$  and that  $j$  will misrepresent her preferences as being of type  $\vartheta$ . If  $i$  reports that he is of type  $\eta$ , then the two lies will cover each other and  $i$  receives  $a_{\eta}^*$ . But if he reports  $\vartheta$ , then  $\mu(\vartheta) > \beta(\vartheta)$  and  $\mu(\eta) < \beta(\eta)$ . In the latter case,  $G(\vartheta, \eta, N(\vartheta) \times (a_{\vartheta}^* - \epsilon), N(\eta) \times (a_{\eta}^* - \epsilon))$  is induced and  $i$  receives  $a_{\vartheta}^* - \epsilon$ . Since  $a_{\vartheta}^* - \epsilon$  is constructed to be strictly preferred by  $i$  to  $a_{\eta}^*$ ,  $i$  strictly prefers to report truthfully.

The same argument holds if  $i$  believes that  $j$  is of type  $\eta$ , that  $j$  envies  $a_{\vartheta}^*$  and that  $j$  will report  $\vartheta$ . Since  $i$  strictly prefers  $\hat{a}_{\vartheta}(a_{\eta}^*)$  to  $\hat{a}_{\eta}(a_{\eta}^*)$ , he prefers to report truthfully. Also, note that whenever  $i$  believes that  $j$  misreports,  $i$  strictly prefers to report truthfully than to send any other message  $\vartheta \neq \{\vartheta, \eta\}$ , because, in the latter case,  $i$  receives  $\hat{a}_{\vartheta}(a^p)$ , which makes him strictly worse-off.

In the case where  $i$  believes that multiple misrepresentations will take place, then, irrespectively of his message,  $\mu(\vartheta) \neq \beta(\vartheta)$  (if all reports but one cancel out then we go back to the analysis of the previous cases). This means that his message, alone, can not hide the fact that some agent(s) misrepresents(misrepresent) her(their) type(s). His best response remains to report truthfully:  $U_{\vartheta}(\hat{a}_{\vartheta}(a^p)) > U_{\vartheta}(\hat{a}_{\eta}(a^p))$ ,  $\forall \eta \neq \vartheta$ . We conclude that, under all possible beliefs,  $i$  strictly prefers to report truthfully.

Given this, it is a best response for an agent of rank(K-1) to report his type truthfully as well. Say that agent  $i$ , who is of rank(K-1) and type  $\eta$ , envies the allocation of some type  $\vartheta$  of rank(K). Of course, if  $i$  believes that some agent of type  $\vartheta$  will report as being of type  $\eta$ , then the best response for  $i$  is  $m_i = \vartheta$ , but, as we showed, this cannot be an equilibrium<sup>6</sup>. Hence, if  $i$  believes that all agents will report truthfully, he prefers to report truthfully as well. If he believes that only one agent of the same or lower rank

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<sup>6</sup>This argument also makes clear that our paper is not one of dominant strategy implementation, as only rank(K) individuals have dominant strategies.

will misreport their types as his own, he will still prefer to reveal his type truthfully, for the same type of reasoning as in the case of an agent of rank( $K$ ). Finally, if he believes that many agents will misreport their types, he still prefers to receive an incentive compatible allocation (by construction) than misrepresenting his own type. Therefore, given that rank( $K$ ) agents report truthfully, agents of rank( $K-1$ ) also report truthfully.

By induction, we conclude that for an agent of Rank( $\kappa$ ), if all agents of higher rank are expected to report truthfully their types, his best-response is to report truthfully, irrespectively of the actions of agents of the same or lower rank. Since it is a dominant strategy for rank( $K$ ) agents to report truthfully, then, by iterated elimination of strictly dominated strategies, the only possible equilibrium is when all agents report truthfully.

■

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