Comment on ‘‘Exact calculations of quasibound states of an isolated quantum well with uniform electric field: Quantum-well Stark resonance’’

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It is shown that the method of Airy functions of complex argument and the scattering approach based on a phase-shift analysis yield Stark quantum-well resonances which are in agreement in both low- and high-field limits. Furthermore, the scattering approach provides a unified view of resonances lying both below and above the semiclassical confining barrier.

In a recent Rapid Communication,[1] Ahn and Chuang presented very interesting results concerning Stark resonances in semiconductor quantum wells. Their work is the first study of the problem by the method of Airy functions of complex argument and offers universal curves which can be used to compute Stark shifts of confined states for any well parameters and over a wide range of electric fields. They compared their Stark shifts with those predicted by Austin and Jaros[2] and concluded that there is a discrepancy between the two methods in the high-field limit. This might be taken to imply that the scattering approach employed by Austin and Jaros fails to provide a useful alternative for investigations of Stark resonances. The purpose of this Comment is to point out that no significant discrepancy exists. In fact, the scattering method provides a unified view of the Stark phenomenon over a wide range of energies. Although much of the existing literature on the subject has been preoccupied with levels lying deep in the confining wells, there is a wealth of new field-induced effects in the hot electron range of energies.[3]

The work of Ahn and Chuang rests on the recognition that the occurrence of a resonance implies the existence of an asymptotic purely outgoing wave solution of the Schrödinger equation associated with complex energy $E-i\Gamma/2$ (the so-called Siegert state). Imposing the outgoing wave boundary condition with the use of Airy functions of complex argument thus yields E and Γ directly. In general, the Stark resonances are defined in scattering theory as complex poles of the S matrix and a variety of approaches can be adopted to locate these poles. In Ref. 2 it was proposed to combine the use of Airy functions with the method of phase-shift analysis. In particular, the phase shift was used to obtain the Stark resonance position in two ways: from the Breit-Wigner formula and from the position of the maximum of the (field-induced) change in the density of states. These results are displayed in Fig. 3 of Ref. 2. The Breit-Wigner resonance position shows anomalous (positive) Stark shift at high fields, whereas the density-of-states peak shows a consistent negative shift. Rescaling of our results for the density of states in accordance with the method of Ref. 1 shows that these are in agreement with those presented in Fig. 3 of that work. Thus the two calculations are quite consistent: the complex-Airy-function method yields the same position of the maximum of the change in the density of states as that predicted by the phase-shift method.

The discrepancy in the resonance positions obtained from the Breit-Wigner formula in the high-field regime is neither new nor surprising. The formula represents a low-field sharp-resonance approximation and is known to have a limited validity when applied to broad resonances. Our results showing the deviation of the simple Breit-Wigner fit from the peak of the density of states mark the important and physically meaningful point where the sharp-resonance approximation breaks down. Although the turn-around behavior cannot be observed directly as a positive shift in the energy of the corresponding quantum state, the related change in the density of states associated with the antiresonance at higher energies should be observable.

Both the phase shift and the complex-Airy-function calculations exhibit a remarkable feature which has not been emphasized before and which is related to the fact that the quadratic shift as predicted from perturbation theory prevails even at high fields. This means that the particle is becoming trapped in a triangular well whose depth increases linearly with the applied field. Evidence for this interpretation comes from our calculations of high-field lifetimes and line shapes which indicate that the system fails to "remember" its zero-field structure.

Although the complex-Airy-function method offers a direct determination of the Stark resonance positions, by focusing on the Siegert solution it discards off-resonance information. The phase-shift method involves a more lengthy calculation of the resonance parameters but gives solutions of the scattering problem at all energies. It is then possible to relate the width of the lowest Stark resonance to the antiresonance features in the range of higher energies. The relationship between the resonance and antiresonance widths and positions gives a quantitative meaning to observables such as line shapes and transit times. Such relations become particularly interesting in superlattices where quantum states below as well as above the semiclassical barrier are grouped into bands separated by forbidden gaps. For example, we have recently extended our calculations to systems of two to five quantum wells with overlapping wave functions.[3] It transpires that the position and bandwidths of the quasi-confined quantum states in the hot electron range are also affected by the electric field. Under certain circumstances wave-function localization effects cause high-energy states to have smaller width than their low-energy counterparts. Such anomalous field-induced localization corresponds to sharp features in the first derivative of the phase shift with respect to energy which is equivalent to the time delay (i.e., field-induced changes in the time of flight). It is difficult to conceive of other than scattering techniques that would provide a suitable means for dealing with this class of problems.

[1] D. Ahn and S. L. Chuang, Phys. Rev. B **34**, 9034 (1986).
[2] E. J. Austin and M. Jaros, Phys. Rev. B **31**, 5569 (1985).
[3] E. J. Austin and M. Jaros, J. Appl. Phys. **62**, 558 (1987).