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UKQCD Collaboration

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We present the results of a lattice computation of the form factors for $B^0 \to \pi^- l^+ \nu_l$ decays near zero-recoil. These results will allow a determination of the CKM matrix element $|V_{ub}|$ when measurements of the differential decay rate become available. We also provide models for extrapolation of the form factors and rate to the full recoil range. Our computation is performed in the quenched approximation to QCD on a $24^3 \times 48$ lattice at $\beta = 6.2$, using a non-perturbatively $\mathcal{O}(a)$-improved action. The masses of all light valence quarks involved are extrapolated to their physical values.

12.15Hh 12.38Gc 13.20He

Semileptonic decays of heavy-light mesons, particularly those containing a b quark, have attracted considerable interest. They play a crucial role in determining Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. With CLEO and now BABAR and Belle taking data, the prospects for an accurate measurement of the differential rate for $B \to \pi l \nu_l$ decays are excellent[1]. Combined with a first-principles calculation of the relevant form factors, this measurement will allow a model-independent extraction of $|V_{ub}|$, currently one of the least well-determined CKM parameters.

The transition amplitude for exclusive semileptonic $b \to u$ decays factorises into leptonic and hadronic parts. The hadronic matrix elements contain the non-perturbative, strong-interaction effects and are currently the largest source of theoretical uncertainty in the determination of $|V_{ub}|$ from these decays [1,2]. We present here a lattice QCD calculation in the quenched approximation of the matrix element relevant for $B^0 \to \pi^- l^+ \nu_l$ decays. While this matrix element has already been obtained from the lattice at restricted values of q^2 [4] or for a range of q^2 with a "pion" composed of quarks with masses around that of the strange quark [5], we determine it for the physical pion and a wider range of q^2 values [2]. We perform a fully $\mathcal{O}(a)$-improved calculation to reduce discretisation errors and implement light quark mass extrapolations as suggested in [10]. The q^2-dependence of this matrix element has also been calculated using light-cone sum rules [11] and a variety of quark models [12].

The matrix element can be parameterised in terms of two form factors:

$$\langle \pi^-(\vec{k})|V^\mu|B^0(\vec{p})\rangle = f_+(q^2)(p + k - q\Delta_{m^2})^\mu + f_0(q^2)q^\mu\Delta_{m^2} \tag{1}$$

where $q = p - k$ and $\Delta_{m^2} = (M_B^2 - m_\pi^2)/q^2$. The form factors f_+ and f_0 are both real, dimensionless functions of the four-momentum transfer squared. In the limit of zero lepton mass the differential decay rate is given by

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[1]CLEO already have a measurement of the total rate [1] and have been able to obtain the differential decay rate for $B \to \rho l \nu_l$ decays in three recoil bins [2]. They are currently attempting a similar binning for $B \to \pi l \nu_l$ decays [3].

[2]Preliminary studies of these decays have already been presented by two groups [6–9].

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_F^2\,|V_{\rm ub}|^2}{192\pi^3 M_B^3}[\lambda(q^2)]^{3/2}\left|f_+(q^2)\right|^2, \tag{2}$$

where λ is a kinematic factor given by

$$\lambda(q^2) = (M_B^2 + m_\pi^2 - q^2)^2 - 4M_B^2 m_\pi^2 \tag{3}$$

and where q^2, the invariant mass-squared of the lepton pair, takes values in the range from 0 to $(M_B - m_\pi)^2$.

We work in the quenched approximation to QCD on a $24^3 \times 48$ lattice at $\beta = 6.2$, corresponding to an inverse lattice spacing of $a^{-1} = 2.54^{+5}_{-9}$ GeV as determined from the ρ mass [13]. We use 216 gauge configurations generated from a combination of the over-relaxed [14,15] and Cabibbo-Marinari [16] algorithms with periodic boundary conditions. The heavy-light mesons are composed of a propagating heavy quark with mass around that of the charm quark to keep discretisation errors in check, and a light antiquark with a mass around that of the strange quark to keep finite volume errors and the CPU time required to obtain the corresponding quark propagator under control. We then obtain results for the physical $B^0 \to \pi^- l^+ \nu_l$ decay by extrapolating in heavy and light quark masses. We employ an SW fermion action [17] that removes all $\mathcal{O}(a)$ discretisation errors from the hadron spectrum [18]. We can also remove all $\mathcal{O}(a)$ discretisation errors from matrix elements of local currents by an appropriate definition and renormalisation of these currents [18]. In the case of the vector current, we have

$$V_\mu^I = V_\mu^L + c_V a \frac{1}{2}(\partial_\nu + \partial_\nu^\star)T_{\mu\nu}^L \tag{4}$$

where $V_\mu^L = \bar{Q}\gamma_\mu q$ and $T_{\mu\nu}^L = \bar{Q}\sigma_{\mu\nu}q$ are the local lattice vector and tensor currents respectively and where ∂_ν $(\partial_\nu^\star)$ is the forward (backward) lattice derivative. The renormalised current is

$$V_\mu = Z_V\left(1 + \frac{b_V}{2}(am_Q + am_q)\right)V_\mu^I + \mathcal{O}(a^2), \tag{5}$$

where $am_{Q,q} = 1/2\kappa_{Q,q} - 1/2\kappa_{\rm crit}$ and both b_V and the matching coefficient Z_V have been determined non-perturbatively [19]. The mixing coefficient c_V is only known to one loop in perturbation theory [20], but is small.

To quantify residual discretisation errors, we consider the effective matching coefficient, $Z_V^{\rm eff}$, obtained from the forward matrix element of the current, $V_\mu = \bar{Q}\gamma_\mu Q$, between degenerate heavy-light mesons at rest. Since, in the continuum, this current is conserved we have

$$\langle P(\vec{0})\,|V_0|P(\vec{0})\,\rangle = 2M_P = Z_V^{\rm eff}\langle P(\vec{0})\,|V_0^I|P(\vec{0})\,\rangle \tag{6}$$

where P is a heavy-light pseudoscalar meson. We calculate $Z_V^{\rm eff}$ for two values of the heavy quark mass. We compare these results with the matching coefficient, $Z_V(1 + b_V am_Q)$, evaluated for these quark masses using the non-perturbative values of the coefficients Z_V and b_V. These two measures will differ by terms of $\mathcal{O}(a^2)$. The comparison is shown in Table I. The agreement between the two procedures is excellent, suggesting that, at least for short-distance quantities, higher-order discretisation errors are small. However, we cannot exclude the possibility that discretisation errors for non-degenerate, non-forward matrix elements are larger than the above comparison suggests. Although the present calculation has been performed at one value of the lattice spacing, recent work [21] using the same improved action, suggests that for long-distance quantities such as masses the error in extrapolating to the continuum limit from $\beta = 6.2$ is around 6%.

Four values of the hopping parameter, κ, were chosen for the heavy quark ($\kappa_Q = 0.1200, 0.1233, 0.1266, 0.1299$), three values for the active light quark (A) ($\kappa_A = 0.1346, 0.1351, 0.1353$), and two for the spectator light quark (S) ($\kappa_S = 0.1346, 0.1351$). We obtain the relevant matrix elements from heavy-to-light three-point correlation functions, divided by the appropriate factors extracted from two-point functions as in Refs. [22,23]. We place the operator for the heavy-light pseudoscalar at $t = 20$, and the operator for the light-light pseudoscalar at $t = 0$. We find that contamination of the signal from other time orderings as well as from excited states is negligible.

We use eight different combinations of $\vec{p}$ and $\vec{k}$ to determine the momentum dependence of the corresponding form factors: $0 \to 0$, $0 \to 1$, $0 \to \sqrt{2}$, $1 \to 0$, $1 \to 1$, $1 \to 1_\perp$, $1 \to 1_\leftarrow$ and $1 \to \sqrt{2}_\perp$ in lattice units. There is no $0 \to 0$ channel for f_+. For each momentum channel, the form factors are obtained from a simultaneous fit to the matrix elements of the temporal and spatial components of the vector current.

The chiral limit is reached at the value of the hopping parameter for which the pion mass vanishes. For the present simulation this value is $\kappa_{\rm crit} = 0.13582(1)$. The physical value of m_π/m_ρ is reached for $\kappa_n = 0.13578(1)$. In order to evaluate the form factor, f_i, $i = +$, 0, at physical light quark masses, we must consider both the explicit mass dependence of f_i and the indirect dependence arising from the change in q^2:

$$f_i = f_i(q^2_{(\kappa_A, \kappa_S)}, \kappa_A, \kappa_S). \tag{7}$$

We first interpolate, to a common set of q^2 values, the form factors corresponding to a given heavy quark and different light quark mass combinations. For the different heavy quarks, these sets are chosen such that the corresponding sets of heavy quark velocities are the same for all heavy quarks, for reasons that will become clear below. The interpolation is performed at fixed light quark mass using a form motivated by pole dominance models:

$$f_i(q^2) = \frac{f_i(0)}{1 - q^2/M_i^2} \ , \tag{8}$$

with the kinematical constraint, $f_+(0) = f_0(0)$, imposed. Any consequent model dependence is mild as we use this ansatz only to interpolate the form factors in the range of q^2 for which we have data. We have also tried other ansätze including dipole/pole for $f_+(q^2)$ and $f_0(q^2)$ respectively, with and/or without the kinematical constraint. This is included in our estimate of systematic errors and is shown in Figure 1.

These interpolated data points are then extrapolated in κ_A and κ_S to κ_n at *fixed* q^2, according to

$$f(\kappa_S, \kappa_A) = \alpha + \beta \left(\frac{1}{\kappa_S} - \frac{1}{\kappa_{\text{crit}}} \right) + \gamma \left(\frac{1}{\kappa_S} + \frac{1}{\kappa_A} - \frac{2}{\kappa_{\text{crit}}} \right). \tag{9}$$

The extrapolation is two-dimensional in the active and spectator light quark masses.

In previous UKQCD collaboration analyses [6,22] a term linear in the pion mass was added to account for the indirect q^2 contribution. Here we hold q^2 fixed as the light quark masses are changed [10]. This yields a more reliable extrapolation by separating the q^2 and the explicit light quark mass dependences.

Heavy quark symmetry (HQS) is used to model the form factors' dependence on heavy meson mass at fixed four-velocity, v. We thus work with the recoil variable

$$v \cdot k = \frac{M_P^2 + m_\pi^2 - q^2}{2M_P}. \tag{10}$$

The scaling relations for f_+ and f_0 at fixed $v \cdot k$, as given by heavy quark effective theory are [24],

$$C(M_P, M_B) f_i(v \cdot k) M_P^{s_i/2} = \gamma_i \left(1 + \frac{\delta_i}{M_P} + \frac{\epsilon_i}{M_P^2} + \cdots \right) \tag{11}$$

where the ellipsis denotes higher order terms in the heavy quark expansion and $s_i = \{-1, 1\}$ when $i = \{+, 0\}$. The coefficient C is the logarithmic matching factor [25],

$$C(M_P, M_B) = \left(\frac{\alpha_s(M_B)}{\alpha_s(M_P)} \right)^{2/\beta_0} \tag{12}$$

where $\beta_0 = 11$ in quenched QCD and α_s is the one loop running coupling with $\Lambda_{\overline{\text{MS}}}^{(4)} = 295$ MeV.

We choose a set of five values of $v \cdot k$ in a region accessible to our calculation for all of our values of heavy and light quark mass. For each initial heavy quark this yields a distinct set of q^2 values. It is these sets of q^2 values which we use in the light quark mass extrapolation described above. From that extrapolation, we obtain the form factors for all M_P at the chosen values of $v \cdot k$. We then extrapolate f_+ and f_0 at fixed $v \cdot k$ to obtain the form-factors at the B-meson mass. Figure 2 shows both a quadratic fit to all four data points and a linear fit to the three heaviest. The difference between these two procedures is used to estimate the systematic uncertainty in this extrapolation. Table II shows the heavy quark and meson masses used in this work.

The resulting $B^0 \to \pi^- l^+ \nu_l$ form factors and differential decay rate are shown as functions of q^2 in Figures 3 and 4 and their values are summarised in Table III. It is important to note that these results can be used for a determination of $|V_{\text{ub}}|$, without model-dependent assumptions about the q^2 dependence of form factors, once experiments measure the differential or partially integrated rate in the range of q^2 values reached by our calculation. Future lattice calculations in full, unquenched QCD will permit a completely model-independent determination of $|V_{\text{ub}}|$.

The central values and statistical, bootstrap errors in the figures and table are obtained with the kinematically constrained pole/pole q^2-interpolation function of Eq. (8), quadratic heavy-quark-mass extrapolations and with the lattice spacing set by m_ρ. Systematic errors are determined by considering the change in our results due to: different choices of q^2-interpolation function, as discussed after Eq. (8); using r_0 [26] to set the lattice spacing; and using a linear, instead of quadratic, heavy extrapolation for the heaviest three quarks. Using the KLM quark field normalisation [27] produced negligible change in these results, suggesting that higher order mass-dependent discretisation effects are

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small. We also assign a 6% error representing the possible effect of a continuum extrapolation as mentioned above. The quoted error is the quadratic sum of the maximum variation from all sources of systematics.

To extrapolate these results to a larger range of q^2, we make model assumptions [3]. Pole dominance models suggest the following momentum dependence for the form factors,

$$f_i(q^2) = \frac{f(0)}{(1 - q^2/M_i^2)^{n_i}}, \tag{13}$$

where $i = +$, 0, n_i is an integer exponent and the kinematical constraint $f_+(0) = f_0(0)$ has already been imposed. Combining this with the HQS scaling relations of Eq. (11) implies $n_+ = n_0 + 1$. Light-cone sum-rules scaling further suggests $n_0 = 1$ [28] [4]. Another pole/dipole parameterisation for f_0 and f_+, which accounts for the B^* pole in f_+ correctly, has been suggested by Becirevic and Kaidalov (BK) [30]:

$$f_+(q^2) = \frac{c_B(1 - \alpha)}{(1 - q^2/M_{B^\star}^2)(1 - \alpha q^2/M_{B^\star}^2)}$$
$$f_0(q^2) = \frac{c_B(1 - \alpha)}{(1 - q^2/\beta M_{B^\star}^2)}. \tag{14}$$

We fit both models to our form factors extrapolated to the B-meson mass. The results are shown in Figure 3 for our central value procedure. Though both parameterisations fit the data equally well, we favour the more physical BK description.

These models can then be used to compute the total decay rate by integrating Eq. (2) with respect to q^2. This rate of course depends on the choice of model and is sensitive to uncertainties in the lattice results, since it is dominated by the low q^2 region, only reached by extrapolation. We find

$$\Gamma/|V_{\text{ub}}|^2 = \left(9^{+3+2}_{-2-2}\right) \text{ ps}^{-1}. \tag{15}$$

The first error is statistical and the second is the systematic error estimated, as for the form factors, from the effects decribed above as well as the difference between the two models. Combining this result with CLEO's exclusive $|V_{\text{ub}}| = (3.25 \pm 0.14^{+0.21}_{-0.29} \pm 0.55) \times 10^{-3}$ [2] and the B^0 lifetime, $\tau_{B^0} = 1.54 \pm 0.03$ ps [31], we find a branching ratio $\Gamma/\Gamma_{\text{total}} = \left(1.5^{+0.5+0.3}_{-0.3-0.3} \pm 0.6\right) \times 10^{-4}$, where the first error is the lattice statistical error, the second systematic and the third, the experimental errors on the mean combined in quadrature. This result is consistent with the measurement by the CLEO collaboration [1], $\Gamma/\Gamma_{\text{total}} = (1.8 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4}$, where the errors are statistical, systematic and from model dependence.

In this letter we have reported a lattice computation of the form factors and differential decay rate for the decay $B^0 \to \pi^- l^+ \nu_l$. We have used a fully $\mathcal{O}(a)$-improved action to minimise discretisation errors. We are repeating the calculation at a different lattice spacing to probe the effect of improvement. It should also be remembered that the computation was performed in the quenched approximation; we are currently generating configurations with two flavours of light dynamical quarks and aim to quantify the quenching effect.

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[3] In [10], dispersive bound techniques were used to extrapolate lattice results to the full kinematical range in a model-independent way. Such an extrapolation, however, is beyond the scope of the present letter.

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FIG. 1. Interpolations of $f_+(q^2)$ and $f_0(q^2)$ for the heaviest of the heavy quarks and the lightest of the light quarks. The support of the curves shows the range of q^2 which results in interpolation for all light quark mass combinations. The fits shown enforce the kinematic constraint $f_+(0) = f_0(0)$.
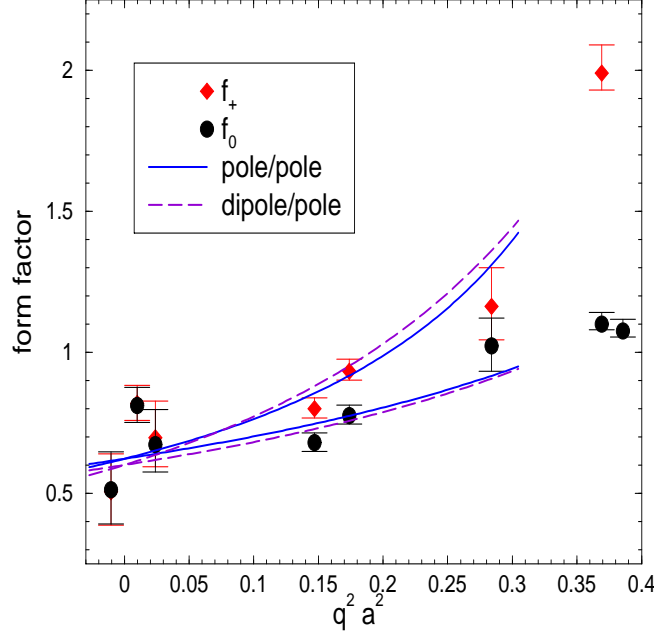

FIG. 2. Heavy-quark-mass extrapolation of $f_+(q^2)$ at fixed $v \cdot k$ corresponding to $q^2 = 22.3$ GeV^2 at the B meson scale.
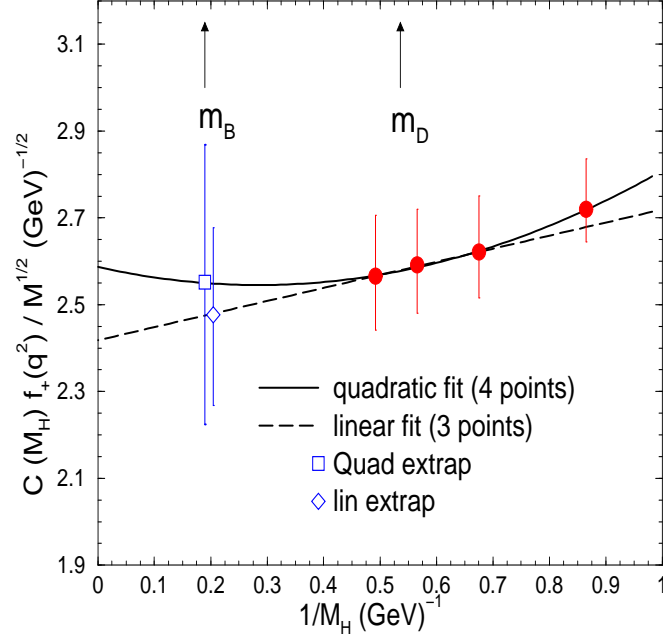
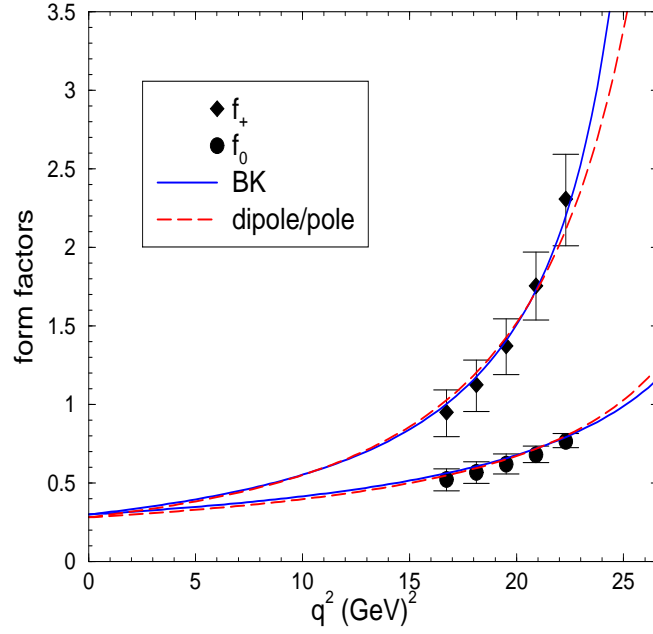FIG. 3. Momentum dependence of the form factors. The data shows statistical errors only.

FIG. 4. The differential decay rate as a function of q^2. The outer error bars show the systematic and statistical errors added in quadrature. The curves are model fits to both f_+ and f_0.
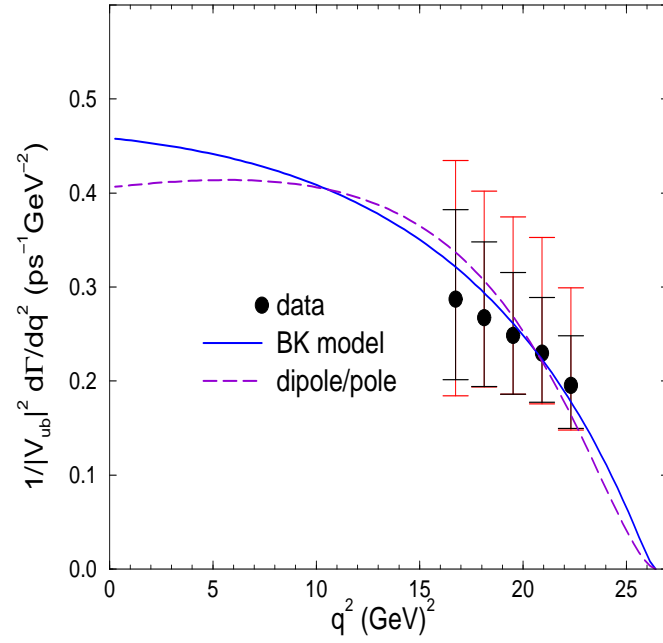
TABLE I. The effective matching coefficient for different values of the heavy quark mass and spectator hopping parameter. The current does not depend on the spectator quark.

am_Q	$Z_V^{\mathrm{eff}}, \kappa_S = 0.1346$	$Z_V^{\mathrm{eff}}, \kappa_S = 0.1351$	$Z_V(1 + b_V am_Q)$
0.485	1.315^{+3}_{-3}	1.316^{+4}_{-5}	1.332^{+4}_{-4}
0.268	1.087^{+2}_{-2}	1.085^{+3}_{-2}	1.090^{+3}_{-3}

TABLE II. Heavy quark and meson masses used in this work. The table shows the *bare* quark mass in lattice units, the *renormalisation group invariant* quark mass defined by $m_Q^{\mathrm{RI}} = Z_M(1 + b_M am_Q)m_Q$ [32] and the heavy-light(κ_n) pseudoscalar meson mass. The scale is set by m_ρ and b_m is obtained using the one-loop result of [20]

κ	am_Q	m_Q^{RI} (GeV)	M_P (GeV)
0.1200	0.485	1.52	2.035 ± 5
0.1233	0.374	1.23	1.771 ± 5
0.1266	0.268	1.02	1.483 ± 5
0.1299	0.168	0.69	1.157 ± 5

TABLE III. Form factors and differential decay rate as functions of q^2. The central value comes from the constrained pole/pole interpolation procedure, the first error is statistical, the second is systematic as described in the text.

q^2 (GeV)^2	16.7	18.1	19.5	20.9	22.3
$f_+(q^2)$	$0.9^{+1}_{-2}{}^{+2}_{-1}$	$1.1^{+2}_{-2}{}^{+2}_{-1}$	$1.4^{+2}_{-2}{}^{+3}_{-1}$	$1.8^{+2}_{-2}{}^{+4}_{-1}$	$2.3^{+3}_{-3}{}^{+6}_{-2}$
$f_0(q^2)$	$0.57^{+6}_{-6}{}^{+6}_{-20}$	$0.61^{+6}_{-6}{}^{+7}_{-19}$	$0.66^{+5}_{-5}{}^{+7}_{-17}$	$0.72^{+5}_{-4}{}^{+7}_{-15}$	$0.79^{+5}_{-4}{}^{+8}_{-12}$
$1/|V_{\mathrm{ub}}|^2 \mathrm{d}\Gamma/\mathrm{d}q^2$ (ps^{-1}GeV^{-2})	$0.29^{+10}_{-\ 9}{}^{+11}_{-\ 6}$	$0.27^{+8}_{-7}{}^{+11}_{-\ 2}$	$0.25^{+7}_{-6}{}^{+11}_{-\ 2}$	$0.23^{+6}_{-5}{}^{+11}_{-\ 2}$	$0.20^{+5}_{-5}{}^{+\ 9}_{-\ 2}$