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Monopoles, vortices and confinement in $SU(3)$ gauge theory

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Abstract

We compute, in $SU(3)$ pure gauge theory, the vacuum expectation value (vev) of the operator which creates a Z_3 vortex wrapping the lattice through periodic boundary conditions (dual Polyakov line). The technique used is the same already tested in the $SU(2)$ case. The dual Polyakov line proves to be a good disorder parameter for confinement, and has a similar behaviour to the monopole condensate. The new features which characterise the construction of the disorder operator in $SU(3)$ are emphasised.

Key words: Confinement, vortices, monopoles
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1   Introduction

A creation operator $\mu(C)$ of Z_2 vortices was recently introduced for the $SU(2)$ pure Yang-Mills theory [1], in order to investigate their relevance for confinement. We recall that, for a generic $SU(N)$ theory, $\mu(C)$ creates a defect along the line C; it is defined by the following algebraic relation with the Wilson loop $W(C')$ [2]:

$$\mu(C)W(C') = W(C')\mu(C) \exp\left(\frac{i2\pi n_{CC'}}{N}\right) \tag{1}$$

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$n_{CC'}$ being the linking number of the curves C and C'. As discussed in [2], two operators satisfying Eq. 1 are dual to each other: in the phase characterised by an area law for W, μ has a perimeter behaviour and viceversa.

$\mu(C)$ can be viewed as a *dual parallel transport* along the line C. In what follows we shall choose for C a straight line, say along the z direction, closing at infinity through periodic boundary conditions (PBC). $\mu(C)$ on this line can be considered as a *dual Polyakov line*. In Ref. [1] the behaviour of $\langle\mu(C)\rangle$ for this line was studied across the $SU(2)$ deconfining phase transition, using techniques introduced in Refs. [3–5] to study monopole condensation. The result was that $\langle\mu(C)\rangle$ is a good disorder parameter for the deconfinement phase transition, and a finite size scaling analysis allowed a satisfactory determination of the critical temperature and indices. In addition it was found that $\langle\mu(C)\rangle$ coincides within errors with the disorder parameter describing monopole condensation.

In this paper, we perform a similar analysis for the $SU(3)$ pure gauge theory, and again we determine the critical temperature and the critical exponents ν and δ. As in previous works, instead of $\langle\mu(C)\rangle$, we study

$$\rho = \frac{\partial}{\partial\beta}\log\langle\mu\rangle \tag{2}$$

which is more convenient from the numerical point of view, and provides the same information as $\langle\mu(C)\rangle$. Again we compare with the analogous quantities defined by magnetic charge condensation, finding a similar behaviour.

The existing literature on the subject [6–8] deals with the $SU(2)$ case and was already compared to our approach in [1].

2 From $SU(2)$ to $SU(3)$

The definition of the vortex creation operator μ presented in [1] can be extended to the case of a Z_3 vortex in a straightforward manner. The peculiarities of this new definition are summarised in this section, while the reader is referred to [1] for a detailed discussion of the method.

The expectation value of μ in an $SU(N)$ gauge theory is defined as the ratio of two partition functions:

$$\langle\mu(t_0, x_0, y_0)\rangle = \frac{\tilde{Z}}{Z} = Z^{-1}\int[dU]e^{-\beta\tilde{S}[U]} \tag{3}$$

where Z is the usual partition function with the Wilson action,

$$S[U] = \frac{1}{N} \sum_{x,\mu\nu} \mathrm{Tr}\, \mathrm{Re}\,[1 - P_{\mu\nu}(x)] \tag{4}$$

and $\tilde{S}$ is obtained from S by multiplying a line of plaquettes in the $0y$ plane by an element of the center:

$$P_{0y}(t_0, x > x_0, y_0, z) \mapsto \mathbf{z}\, P_{0y}(t_0, x > x_0, y_0, z),\ \forall z \tag{5}$$

where $\mathbf{z} \in Z_N$.

For the $SU(3)$ case discussed in this paper

$$\mathbf{z} = \exp\{i\, n\, 2\pi/3\}, \quad \text{with} \quad n = 0, 1, 2, \tag{6}$$

corresponding to the case of zero, one vortex and one antivortex. Following [1], it can be shown by suitable changes of variables in the definition of $\tilde{Z}$ that such a prescription amounts to create a vortex string closed by PBC in the z direction at (x_0, y_0, t_0), together with an antivortex line at $(N_s - 1, y_0, t_0)$ due to PBC in the x direction.

It is worthwhile emphasizing that the vortex creation operator defined here does not depend on a specific gauge choice. At finite temperature, it is related to the increase of the free energy when a vortex is produced in the vacuum of the theory [6–9]. It is gauge-invariant by construction and relies neither on the identification of the vortices locations nor on their density nor on any choice of the gauge.

Similarly to the $SU(2)$ case, the measurement of a single operator $\langle\mu\rangle$ is needed at finite temperature, with C^* boundary conditions to ensure that the net effect of μ is the creation of a single vortex as explained in [5]. The free energy of the vortex configuration is given by:

$$\langle\mu\rangle \propto \exp\{-F/T\} \tag{7}$$

In pure gauge theory, the confined phase is characterised by the area law for the Wilson loop, corresponding to a linearly rising potential between static quarks. With a compact time-dimension, the potential can also be determined from the correlation of two Polyakov lines according to:

$$\langle \mathrm{Tr}\, P(R)\, \mathrm{Tr}\, P^\dagger(0)\rangle \propto e^{-V(R)N_t} \tag{8}$$

where N_t is the lattice extension in the compact dimension. Eq. 8 and cluster property imply that the vev of a single Polyakov line has to vanish in the confined phase, that is whenever the Wilson loop has an area law. One can

argue that the dual loops exhibit a similar behaviour: a non-zero value for their vev corresponds to the confined phase, characterised by a perimeter law for the dual loop, while a vanishing vev for the dual Polyakov line corresponds to the phase where the dual loop has an area law (i.e. the deconfined phase). Perturbative calculations at high temperature show that indeed $\langle \mu \rangle$ has an area law in this phase [10], implying vanishing vev for the dual Polyakov line. In the following section, we shall present non-perturbative lattice results for $\langle \mu \rangle$ as a function of the temperature.

3 Numerical results

In this paper, we present data only for straight vortex lines wrapping in the z direction by PBC.

Due to the exponential in its definition, $\langle \mu \rangle$ has large fluctuations, which make a direct measurement of its expectation value a challenging task. As usual [1,3–5], we focus on the quantity:

$$\rho = \frac{\partial}{\partial \beta} \log \langle \mu \left(t_0, x_0, y_0 \right) \rangle = \langle S \rangle_S - \langle \tilde{S} \rangle_{\tilde{S}} \tag{9}$$

ρ is easy to compute and yields all the relevant information. At finite temperature, ρ is expected to have a sharp negative peak in the critical region [3–5], if $\langle \mu \left(t_0, x_0, y_0 \right) \rangle$ is a disorder operator for the deconfinement phase transition.

Our results for ρ are displayed in Fig. 1, for several lattice sizes. Fig. 1 shows the characteristic peak at the critical coupling, already observed for $SU(2)$. As a check of our code, we have verified on a $12^3 \times 4$ lattice, that ρ has the same behaviour for a vortex and an antivortex, as expected due to the invariance under charge conjugation. The results are shown in Fig. 2.

The results for $SU(3)$ are qualitatively very similar to the ones already presented for $SU(2)$ in [1]. The value of ρ in the deconfined phase tends to $-\infty$ in the thermodynamic limit, so that $\langle \mu \rangle$ vanishes in that limit, in agreement with the result obtained using perturbation theory in [10]. The small β behaviour of ρ is reported in Fig. 3. Since no change in ρ is observed as the volume goes large compared to the physical correlation length, we conclude that in the confined phase $\langle \mu \rangle$ has a a non-vanishing vev in the thermodynamic limit.

Finite size scaling techniques yield a quantitative description of the behaviour of ρ in the critical region. The following equation

$$\frac{\rho}{N_s^{1/\nu}} = f \left(N_s^{1/\nu} \left(\beta_c - \beta \right) \right) \tag{10}$$
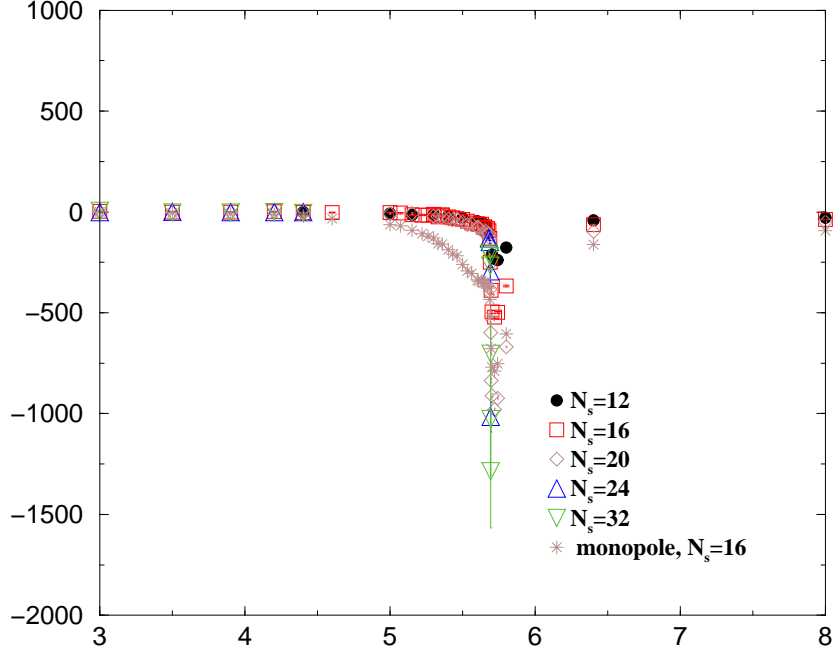
4

Fig. 1. ρ vs. β for the $n = 1$ vortex on $N_s^3 \times 4$ lattices. The analogous result for monopoles[5] is superimposed for comparison.
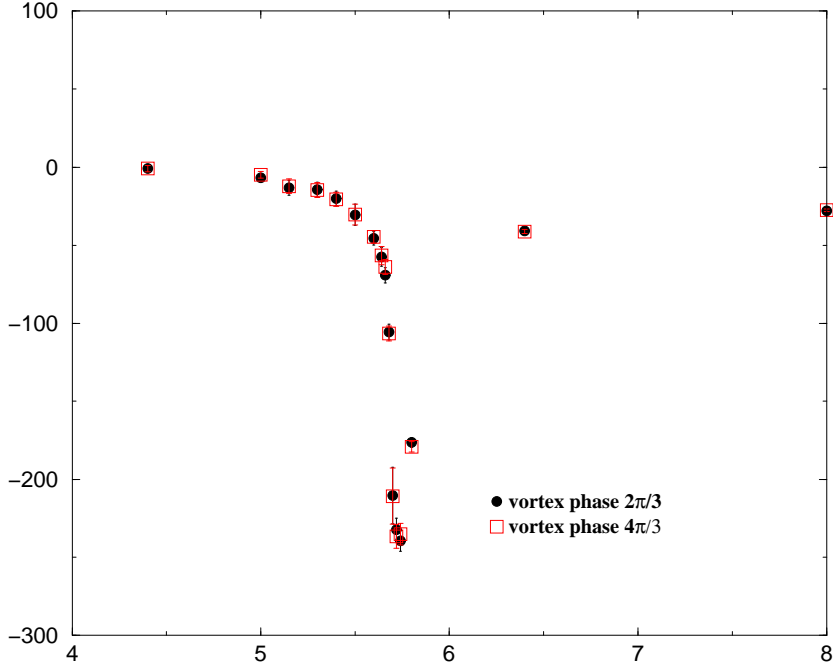

Fig. 2. Comparison of ρ defined for a vortex and an antivortex configuration.

with N_s being the lattice size, summarises the fact that the rescaled quantity $\rho/N_s^{1/\nu}$ is a universal function of the scaling variable

$$x = N_s^{1/\nu} \left(\beta_c - \beta \right) \tag{11}$$

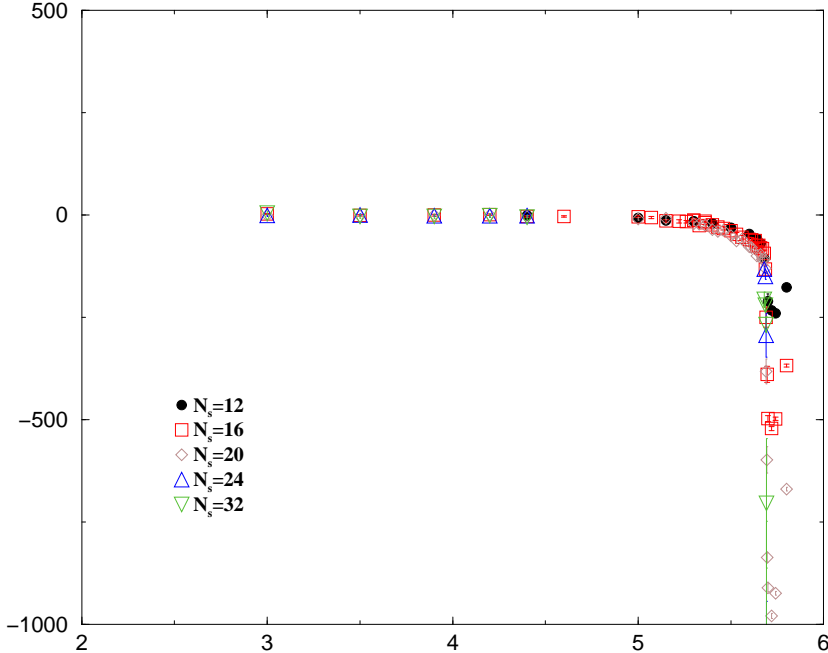For a detailed discussion of the finite size scaling of ρ we refer to [3,5,1]. In the

Fig. 3. Small β ehaviour of ρ.

$SU(3)$ case, we know that the transition is first order and there are scaling violations which we parametrise in the form [5]

$$\frac{\rho}{N_s^{1/\nu}} = f\left(N_s^{1/\nu}(\beta_c - \beta)\right) + \frac{d}{N_s^3} \tag{12}$$

Assuming $\nu = 1/3$, one can fit the data to Eq. 12 to extract β_c, δ, c and d. We find $\beta_c = 5.6924(5)$ in agreement with the result $\beta_c = 5.6925$ quoted in [11]. A value for δ can also be extracted. It is however less stable towards variations of the fitting range and also depends on the value of β_c. A conservative estimate, including a large systematic error, is $\delta = 0.51(5)$, which is in agreement with the value $0.54(4)$ obtained in [5] from the monopole condensate. The systematic error is estimated by comparing different fits to the same data, with a subset of the parameters being kept fixed, or from the same fit to a subset of the data. The variation in the fitted parameters is used to estimate the error. The data together with the fitted curve are displayed in Fig. 4.

The data in Fig. 1 display a difference in the shapes for the monopole and the vortex peaks on the left of the phase transition. However, the behaviour inside the peak is described by the same critical indices.

4 Discussion

Similarly to $SU(2)$, the dual Polyakov line is a good disorder parameter for deconfinement also in the case of $SU(3)$. It can be viewed as the dual of the
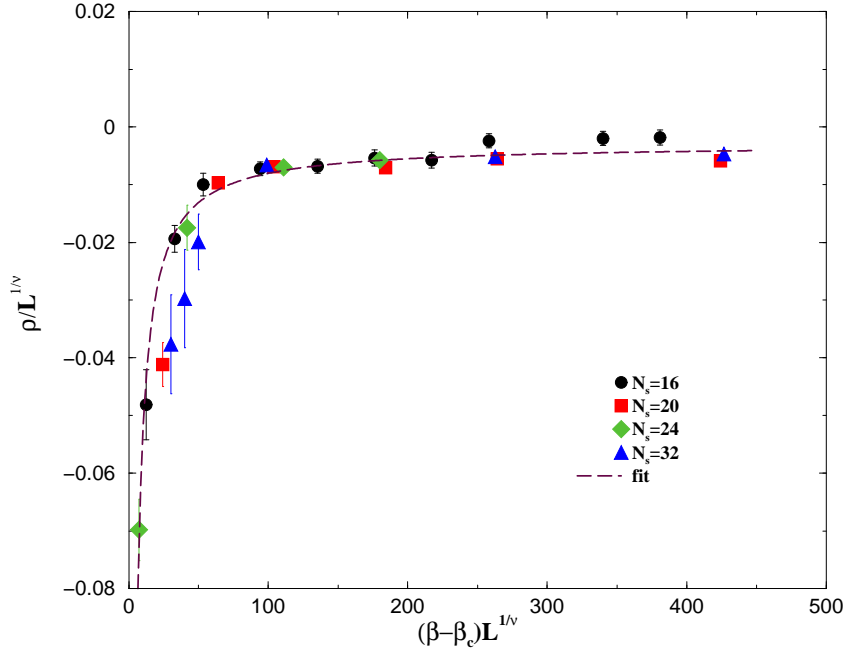
Fig. 4. Plot of rescaled data.

usual Polyakov line, the order parameter. Once again, its behaviour is similar to that of the disorder parameter describing condensation of magnetic charges, which in turn is independent of the choice of the Abelian projection.

We consider these results a step forward in the understanding of the yet unknown dual fields describing the confined phase.

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