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Finite-top-mass effects in NNLO Higgs production

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We construct an accurate approximation to the exact NNLO cross section for Higgs production in gluon-gluon fusion by matching the dominant finite top mass corrections recently computed by us to the known result in the infinite mass limit. The ensuing corrections to the partonic cross section are very large when the center of mass energy of the partonic collision is much larger than the Higgs mass, but lead to a moderate correction at the percent level to the total Higgs production cross section at the LHC. Our computation thus reduces the uncertainty related to these corrections at the LHC from the percent to the per mille level.

The search for the Higgs boson is one of major tasks of the forthcoming experiments at the Large Hadron Collider (LHC) at CERN. The theoretical and experimental effort which has been put into Higgs studies for LHC phenomenology is remarkable. In particular, the determination of higher–order corrections in perturbative QCD has been widely investigated. The dominant Higgs production mechanism in the Standard Model is gluon–gluon fusion through a top loop. The hadronic cross section can be obtained by convolution of the partonic cross section with parton distributions $f_i(x,\mu^2)$

$$\sigma(\alpha_s;\tau_h,y_t,m_H^2)=\sigma_0(y_t)\sum_{i,j}\int_{\tau_h}^1\frac{dx_1}{x_1}\int_{\tau_h}^1\frac{dx_2}{x_2} C_{i,j}\left(\alpha_s;\frac{\tau_h}{x_1x_2},y_t\right)f_i\left(x_1,m_H^2\right)f_j\left(x_2,m_H^2\right) \quad (1)$$

where $\sigma_0(y_t)$ is the partonic Born cross section [1] and the dimensionless variables τ_h and y_t parametrise the hadronic center-of-mass energy and the dependence on the top mass:

$$\tau_h=\frac{m_H^2}{s},\quad y_t=\frac{m_t^2}{m_H^2}. \quad (2)$$

The dimensionless coefficient function $C(\alpha_s;\tau,y_t)$ contains the QCD corrections. The NLO contribution to it was computed in [2] and recently confirmed in [3]. The dominant NLO correction comes from the radiation of soft gluons, which cannot resolve the quark loop in the ggH coupling. Therefore, at least at the inclusive level, the approximation to the exact NLO result obtained [4,5] by taking the limit $m_t\to\infty$ turns out to be very accurate. This approximation considerably simplifies the calculation because the ggH coupling becomes pointlike and the corresponding Feynman diagrams have one less loop.

Recently, the NNLO contribution to $C(\alpha_s;\tau,y_t)$ has been computed in the $m_t\to\infty$ limit [6]. The NNLO result appears to be perturbatively quite stable and it should provide a good approximation to the yet unknown exact result; it has been widely used for precision phenomenology at the LHC [7]. However, the infinite m_t approximation fails in the limit of large partonic center–of–mass energy or equivalently $\tau\to 0$. This is due to the fact that the high energy behaviour of the partonic cross section is completely different according to whether the ggH coupling is pointlike or goes through a quark loop, because in the latter case the quark loop

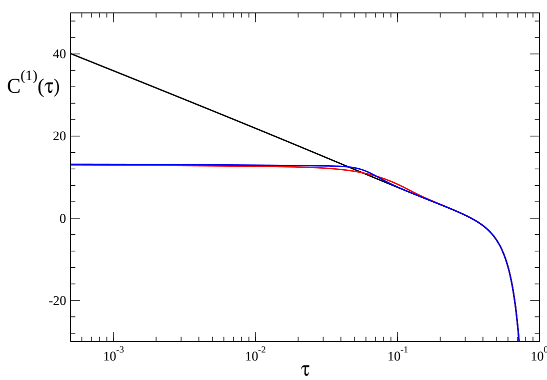

Figure 1. The NLO coefficient function for $m_H = 130$ GeV. The red curve corresponds to the exact case, the black one to $m_t \to \infty$ and the blue one to the approximation Eq. (6) with $\tau_0 = 0.057$ and $\omega = 1/100$.

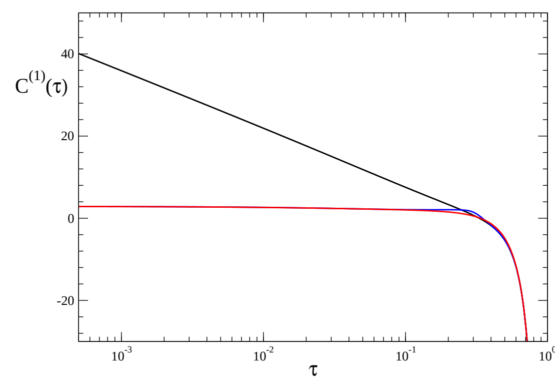

Figure 2. Same as Fig. 1, but with $m_H = 280$ GeV (here $\tau_0 = 0.315$ and $\omega = 1/20$).

effectively provides a form factor which softens the interaction. Indeed, the coefficient function behaves respectively as

$$C \underset{\tau\to 0}{\sim} \begin{cases} \sum_{k=1}^{\infty} \alpha_s^k \ln^{2k-1}\left(\frac{1}{\tau}\right) & \text{if } m_t \to \infty \\ \sum_{k=1}^{\infty} \alpha_s^k \ln^{k-1}\left(\frac{1}{\tau}\right) & \text{for finite } m_t \end{cases} \qquad (3)$$

Equation (3) shows that the difference at high energy between the exact and approximate behaviour is larger at higher orders, so one might expect the relative accuracy of the infinite m_t approximation to become accordingly worse. In Ref. [10] we have recently computed the leading high energy logarithms Eq. (3) at finite m_t, using the techniques of high-energy (or k_T) factorization [8] (the corresponding coefficients in the $m_t \to \infty$ limit had been previously computed in Ref. [9]). We can use this result to construct an improvement of the NNLO result [6], by replacing its spurious double logarithmic growth with the correct high energy behaviour, Eq. (3).

This construction requires a suitable matching procedure, and it gives us an approximation to the exact NNLO result. We shall first perform this improvement on the NLO contribution, where the exact result is known: it turns out to give an approximation to the exact NLO partonic cross section which is everywhere accurate to better than 1 % for any value of the Higgs mass, thus leading to an approximation to the total cross section which is accurate to the level of 0.05%. We shall than construct a similar improvement of the NNLO term. This improvement changes the total cross section computed up to NNLO by an amount which varies between 0.2% for light Higgs ($m_H \sim 130$ GeV) to 1% for heavy Higgs ($m_H \sim 280$ GeV). This is thus the size of the error which is made if the cross section is computed at NNLO using the approximate $m_t \to \infty$ result. By varying the matching prescription, we estimate that the ambiguity on this result is at the level of the per mille.

The perturbative expansion of the coefficient function in the gluon-gluon channel is

$$C(\alpha_s; \tau, y_t) = \delta(1-\tau) + \frac{\alpha_s}{\pi} C^{(1)}(\tau, y_t) + \left(\frac{\alpha_s}{\pi}\right)^2 C^{(2)}(\tau, y_t) + \mathcal{O}\left(\alpha_s^3\right). \qquad (4)$$

The leading high energy behaviour of the NLO and NNLO contributions is given by

$$C^{(1)}(\tau, y_t) = \mathcal{C}_1^{(1)}(y_t) C_A + O(\tau) \qquad (5)$$
$$C^{(2)}(\tau, y_t) = -\mathcal{C}_2^{(2)}(y_t) C_A^2 \ln\tau + \mathcal{C}_1^{(2)}(y_t) + O(\tau).$$

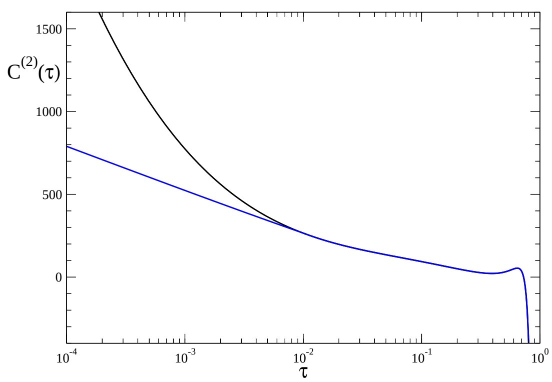

Figure 3. The NNLO coefficient function for $m_H = 130$ GeV. The black curve corrdesponds to $m_t \to \infty$ and the blue one to the approximation Eq. (6) with $\tau_0 = 0.011$ and $\omega = 1/100$. The exact case is not known at NNLO.

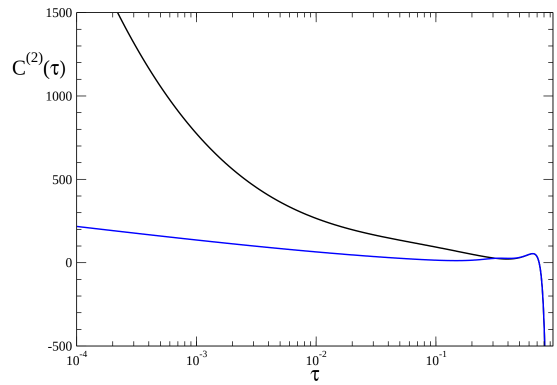

Figure 4. Same as Fig. 3, but with $m_H = 280$ GeV (here $\tau_0 = 0.317$ and $\omega = 1/20$).

The leading NNLO coefficient $\mathcal{C}_2^{(2)}(y_t)$ was computed in Ref. [10], while the subleading NNLO coefficient $\mathcal{C}_1^{(2)}(y_t)$ is unknown.

The approximate pointlike determination of the coefficient function can be improved by subtracting its spurious small τ growth and replacing it with the exact behaviour:

$$C^{\text{app.}}(\tau, y_t) = C(\tau,\infty) + T(\tau,\tau_0) \times \left[C(\tau, y_t) - \lim_{\tau\to 0} C_0(\tau,\infty)\right], \tag{6}$$

where $C_0(\tau,\infty)$ is the sum of contributions to the infinite m_t result $C(\tau,\infty)$ which do not vanish when $\tau \to 0$, as given at NLO and NNLO by the terms listed in Eq. (5). Also, $T(\tau,\tau_0)$ is a matching function, which is introduced in order to tune the point τ_0 where the small τ behaviour sets in. We choose

$$T(\tau,\tau_0) = \frac{1}{2}\left[1 + \tanh\left(\frac{\tau_0 - \tau}{\omega}\right)\right], \tag{7}$$

which in the limit of vanishing width ω becomes the step function: $\lim_{\omega\to 0} T(\tau,\tau_0) = \Theta(\tau - \tau_0)$.

At NLO, because the exact asymptotic behaviour Eq. (5) is the constant $\mathcal{C}_1^{(1)}(y_t)$, the matching point τ_0 is naturally determined as the value of τ where the pointlike approximation equals this constant. It is clear from Fig. 1 that this choice leads to an excellent approximation to the exact result: in fact, it is accurate to better than 1% for all τ.

At NNLO the exact asymptotic behaviour Eq. (5) is a linear rise in $\ln\tau$. Hence, reasoning as at NLO, we are led to choose τ_0 as the point where the log derivative of the $m_t \to \infty$ curve matches the asymptotic value $\mathcal{C}_2^{(2)}(y_t)$:

$$\frac{d}{d\ln\tau} C^{(2)}(\tau,\infty)\Big|_{\tau=\tau_0} = -9\mathcal{C}^{(2)}(y_t). \tag{8}$$

This does not fix completely the approximate result at NNLO however, because the subleading constant $\mathcal{C}_1^{(2)}(y_t)$ is unknown. We fix it by requiring that the approximate curve Eq. (6) be continuous at $\tau = \tau_0$ even when the matching function $T(\tau,\tau_0) = \Theta(\tau - \tau_0)$. The NNLO approximation determined thus is compared to the exact result in Figs. 3, 4 for heavy and light Higgs, respectively. A more conservative matching might consist instead of taking for τ_0 the value found at NLO, and then determining again the subleading constant by continuity. The result found in this way is actually very close to the previous one, and in fact indistinguishable from it in the case of Fig. 4.

	$K^{\rm NLO}$	$K^{\rm NNLO}$
$m_H = 130$ GeV		
pointlike	1.800	2.140
exact	1.797	n.a.
appr.	1.796	2.136
$m_H = 280$ GeV		
pointlike	1.976	2.420
exact	1.958	n.a.
appr.	1.959	2.394

Table 1
The NLO and NNLO K factors Eq. (9), computed with center-of-mass energy $s = 14$ TeV.

Let us now turn to the inclusive cross section. We define a K factor

$$K(\tau_h; y_t) \equiv \frac{\sigma_{gg}(\tau_h, y_t, m_H^2)}{\sigma_{gg}^0(\tau_h; y_t, m_H^2)}, \quad (9)$$

where σ_{gg}^0 is the LO cross-section Eq. (1), computed with LO parton distributions and LO coupling constant. The value of the NLO and NNLO K factors, determined using the MRST2002 [11] gluon distribution in Eq. (1) are given in Table 1, at the LHC center-of-mass energy $s = 14$ TeV. In the table the pointlike, exact and approximate cases are shown.

At NLO the discrepancy between the infinite top mass approximation and the exact result is tiny, less than 1% even for a fairly heavy Higgs. If the improved (approximate) NLO result is used, this discrepancy is reduced by a factor three. At NNLO the inclusion of the correct small τ dependence of the partonic coefficient function changes the K factor by an amount which varies between 0.3% for $m_H = 130$ GeV and 1% for $m_H = 280$ GeV. If we modify the matching prescription by using the NLO value of τ_0 also at NNLO the approximate NNLO results of table 1 change by 0.1 %. We can take this as the error which is made by use of the infinite m_t NNLO formula (there is also a dependence on m_t in the contribution to $C^{(2)}$ which is proportional to $\delta(1-\tau)$, but at NNLO this contribution is relatively small, unlike at NLO). Dominant uncertainties on the total Higgs cros-section are typically at the percent level [7]. Varying ω in the matching function Eq. (7) between 1/20 and 1/100 the NNLO results change by about 0.1 %. We conclude that use of the improved approximate NNLO reduces the error due to finite mass terms at NNLO to the per mille level. This may be relevant in view of recent progress on the computation of electroweak corrections to this process [12].

Finally, we observe that less inclusive quantities which depend on the τ shape of the partonic cross section can be rather more sensitive to finite–mass effects, in particular if they probe the small τ tail of a coefficient function. An interesting case in point, which deserves further investigation, is the Higgs rapidity distribution [13].

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